Distributed Co-Phasing for General Signal Constellations over Wireless Sensor Networks

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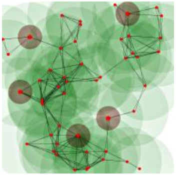
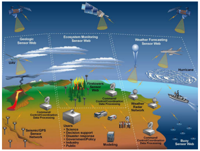
Introduction

- A Wireless Sensor Network (WSN) is a collection of sensors (nodes) to perform a specialized task.
- Applications
  - Defense: Battlefield surveillance and intrusion detection
  - Health: Patient monitoring
  - Industry: Machine health monitoring and quality control
  - Other: Traffic control, Environment and habitat monitoring





Characteristics of WSN

- Inexpensive and small nodes
- Self powered (Battery or Harvested Energy)
- Limited processing power at nodes
- A powerful master controller called Fusion Center (FC)
- Possibility of node failure
- Large scale deployment
- Mobility of nodes (Time varying channel characteristics)

Problem Set up

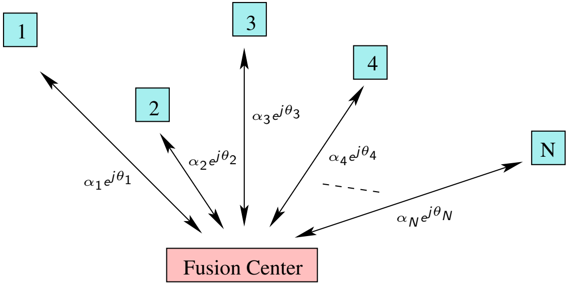


Figure: Wireless Sensor Network

- N sensors having correlated information
- Simultaneous transmission from all sensors to Fusion Center (FC)
- Nodes intend to transmit in such a way that signals combine coherently at the receiver (Transmit Beamforming)

Distributed Co-Phasing

- Reciprocal channel between sensors and FC, $g_k = \alpha_k e^{j\theta_k}$
- Two phases of transmission
  - Pilot : From FC to all nodes

    $$r_k[n] = g_k\sqrt{E_P} + \eta_k[n], n = 1, \ldots, N_P$$

    N_P: No of pilot transmission; E_P: Power of pilot symbols
    All nodes estimate their respective channel phase angles $\hat{\theta}_k$ from pilot symbols
  - Data : From all nodes simultaneously to FC.
    All nodes pre rotate their transmission by $\hat{\theta}_k$ so that transmitted signals combine coherently at the FC

    $$r[n] = \sum_{k=1}^{N} x_k[n]e^{-j\hat{\theta}_k} g_k + \nu[n], n = 1, \ldots, N_D$$

    N_D: No of data transmission; E_S: Power of data symbols
- Channel from all sensors to FC remain constant for $N_P + N_D$ symbol duration.

Contributions From This Work

- This system is studied for BPSK signaling in a previous work. BPSK signaling does not require CSI at FC (Threshold is always ‘0‘, independent of the channel)
- We use Mutual Information (MI) between sensors and FC to show that performance improves with a higher order constellation
- Higher order constellation requires CSI at the FC
- We propose two blind methods to estimate CSI at the FC and analyze their performance
- We study the *channel corruption* problem and propose a solution for the same

Mutual Information

Assuming perfectly correlated transmission, data transmission from sensors to FC may be written as

$$r[n] = x[n]H_{\text{DCP}} + \nu[n]$$

where $H_{\text{DCP}} = \sum_{k=1}^{N} \alpha_k e^{j\theta_{ek}}$ is the effective DCP channel
We compute the MI with and without CSI at FC as,

- With CSI

$$I(x:r) = \mathbb{E}\left\{ \sum_i p(x_i) \int_{r\in\Re} p(r/H_{\text{DCP}}, x_i) log \frac{p(r/H_{\text{DCP}}, x_i)}{p(r/H_{\text{DCP}})} dr \right\}.$$

- Without CSI

$$I(x:r) = \sum_i p(x_i) \int_{r\in\Re} p(r/x_i) log \frac{p(r/x_i)}{p(r)} dr.$$

where $p(r/x_i) = \mathbb{E}\{p(r/x_i, H_{\text{DCP}})\}$

MI for Different Constellations

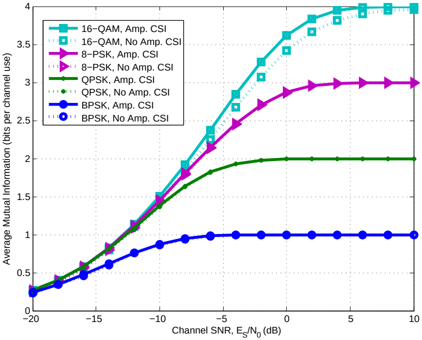


Figure: Comparison of Mutual Information for Ideal DCP with no CSI and perfect CSI for non constant modulus constellations; N=5

DCP for General Signal Constellations

- Higher order constellations require CSI at FC.
- ML estimate of the effective channel, H_{DCP} is

$$\hat{H}_{\text{DCP},ML} = \arg\max_{H \in \mathcal{C}} \frac{1}{M^{N_D}} \frac{1}{(\pi\sigma^2)^{\frac{N_D}{2}}} \sum_{x \in \mathcal{X}} e^{-\sum_{k=1}^{N_D} \frac{|r[k] - Hx_k|^2}{\sigma^2}}$$

- Hard to compute in closed form

Power Method

We propose an estimate of $|H_{\text{DCP}}|$ defined as,

$$|\hat{H}_{\text{DCP}}| = \left[\frac{\frac{1}{N_D} \sum_{n=1}^{N_D} |r[n]|^2 - N_0}{E_S} \right]^{\frac{1}{2}}$$

- Motivated by the power of the received signal
- It can be shown that $\mathbb{E}\{ |\hat{H}_{\text{DCP}}|^2\} = |H_{\text{DCP}}|^2$

K-means Algorithm for Blind Detection

ML estimate of the effective DCP channel is,

$$\hat{H}_{\mathrm{DCP},ML} = \arg\max_{H \in \mathcal{C}} \frac{1}{M^{N_D}} \frac{1}{(\pi\sigma^2)^{\frac{N_D}{2}}} \sum_{x \in \mathcal{X}} e^{-\sum_{k=1}^{N_D} \frac{|r[k] - Hx_k|^2}{\sigma^2}}$$

- ML estimate of $\hat{H}_{\mathrm{DCP}}$ should minimize the squared distance of the received data points to a scaled version of the transmitted constellation.
- The K means algorithm is an iterative method that partitions a set of vectors in to K groups such that their squared distance to K centroids are minimized

K-means Algorithm

- Randomly select M points as the centroids μ
- Nearest Neighbor condition: Assign each point in the data set to its nearest (in squared euclidean distance) centroid (μ).

$$R_k(i) = \{x_j : [x_j - \mu(i)]^2 \leq [x_l - \mu(i)]^2, \forall\ l \neq i\}$$

- Centroid condition: Once all the groups are identified, find the centroid of each group to update the vector μ. This is μ_{k+1}

$$\mu(i) = \frac{\sum_{j=1}^{M_i} x_j}{M_i}$$

Example of K-means Algorithm

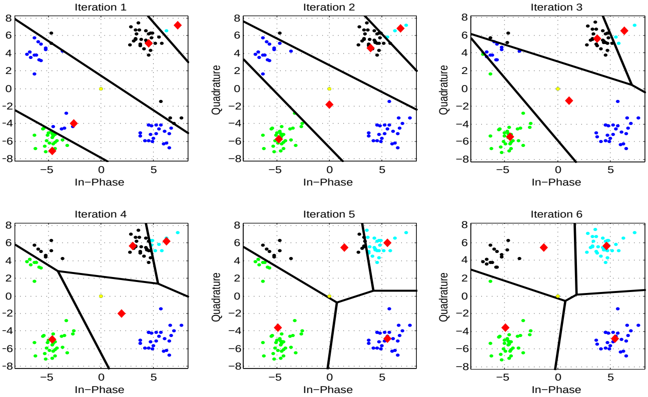


Figure: K-means algorithm operating on a 4-QAM constellation

Modified K-means Algorithm

- The centroids μ are a scaled version of the transmitted constellation
- Centroid update step in K-means algorithm reduces to finding an optimum scaling factor β that minimizes the following cost function

$$\hat{\beta} = \arg\min_{\beta} J(\beta) \triangleq \sum_{k=1}^{M} \sum_{l=1}^{M_k} |\beta s_k - r_{lk}|^2$$

$$\frac{\partial J}{\partial \beta} = 0 \Rightarrow \hat{\beta} = \frac{\sum_{k=1}^{M} s_k^* \sum_{l=1}^{M_k} r_{lk}}{\sum_{k=1}^{M} M_k |s_k|^2}$$

M: Number of points in the transmitted constellation
M_k: Number of points in the k^{th} group
s_k: Elements of the transmitted constellation.

Performance Analysis

- Two causes of decoding error
  - Noise and fading
  - Channel Corruption: When angle of the effective channel(H_{DCP}) is more than half the rotational symmetry of the transmitted constellation, it leads to a phase ambiguity at the FC causing catastrophic decoding errors.
- Assumptions
  K-means algorithm based method converges to a $\hat{H}_{\text{DCP}}$ such that,
  - $|\hat{H}_{\text{DCP}}| = |H_{\text{DCP}}|$.
  - $\hat{\phi}_H$ is the minimum possible.
- The expression for probability of error may be written as,

$$P_e = P_{cc} + (1 - P_{cc})P_{e,CSIR}$$

Probability of Error for M-PAM signaling

- P_e with perfect CSIR

$$P_{e,CSIR,PAM} = \frac{2(M-1)}{M}\mathbb{E}_{H_{\text{DCP}}}\left\{Q\left(\sqrt{\frac{E_S}{N_0}}|H_{\text{DCP}}|\right)\right\}$$

- We approximate $|H_{\text{DCP}}|$ as a Nakagami-m random variable with same first and second moments.

$$P_{e,CSIR,PAM} = \frac{\phi_\gamma(1)}{2\sqrt{\pi}}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)}\;{}_2F_1\left(m,\frac{1}{2};m+1;\frac{1}{1+\frac{\bar{\gamma}}{m}}\right)$$

$$m = \frac{\mathbb{E}\{|H_{\text{DCP}}|^2\}}{var\{|H_{\text{DCP}}|^2\}} \text{ and } \Omega = \frac{E_S}{N_0}\mathbb{E}\{|H_{\text{DCP}}|^2\}$$

where $\bar{\gamma} = \mathbb{E}\{R^2\}$, $\phi_\gamma(1) = \left(1+\frac{\bar{\gamma}}{m}\right)^{-m}$ and ${}_2\mathrm{F}_1$ is the Gauss hypergeometric function

Probability of Error for M-PAM signaling contd...

- Probability of channel corruption

$$
\begin{aligned}
P_{cc,PAM} &= \{\phi_H > \frac{\pi}{2}\} = P\{H_{R,\mathrm{DCP}} < 0\} \\
&= Q\left(\frac{\mu_R}{\sigma_R}\right)
\end{aligned}
$$

where $\mu_R = \mathbb{E}\{H_{R,\mathrm{DCP}}\}$ and $\sigma_R = std\{H_{R,\mathrm{DCP}}\}$

- μ_R and σ_R are obtained as

$$
\begin{aligned}
\mu_R &= N\sqrt{\frac{2E_S}{N_0}}\sqrt{\frac{\pi\Omega_k}{4}\frac{\gamma_p\Omega_k}{(1+\gamma_p\Omega_k)}} \\
\sigma_R^2 &= N\,\frac{2E_S}{N_0}\Omega_k\left[\frac{1+2\gamma_p\Omega_k}{2(1+\gamma_p\Omega_k)}\right] + N(N-1)\frac{\pi E_S}{2N_0}\frac{\gamma_p\Omega_k^2}{(1+\gamma_p\Omega_k)} - \mu_R^2
\end{aligned}
$$

Probability of Error for M-QAM signaling

- P_e with perfect CSIR
  Probability of error for M-QAM is obtained by decomposing it to two M-PAM constellations.

$$P_{e,CSIR,QAM} = 2P_{e,CSIR,PAM} - P^2_{e,CSIR,PAM}$$

- Probability of channel corruption

$$\begin{aligned} P_{cc,QAM} &= P\{\phi_H > \frac{\pi}{4}\} \\ &= P\{H_{R,\mathrm{DCP}} < 0\} + P\{|H_{I,\mathrm{DCP}}| > H_{R,\mathrm{DCP}} | H_{R,\mathrm{DCP}} > 0\} \end{aligned}$$

Derivation of Moments of $|H_{\text{DCP}}|$

- Second moment of $|H_{\text{DCP}}|$

$$\begin{aligned}\mathbb{E}\{|H_{\text{DCP}}|^2\} &= \mathbb{E}\left\{\left[\sum_{k=1}^{N}\alpha_k\cos\theta_{ek}\right]^2+\left[\sum_{k=1}^{N}\alpha_k\sin\theta_{ek}\right]^2\right\}\\ &= N\mathbb{E}\{\alpha_k^2\cos^2\theta_{ek}\}+N(N-1)\left[\mathbb{E}\{\alpha_k\cos\theta_{ek}\}\right]^2\\ &\quad +N\mathbb{E}\{\alpha_k^2\sin^2\theta_{ek}\}\end{aligned}$$

- Joint moments of α_k, $\cos\theta_{ek}$ and $\sin\theta_{ek}$ can be obtained from the pdf of θ_{ek} [?]

Derivation of Moments of $|H_{\text{DCP}}|$ contd...

- Fourth moment of $|H_{\text{DCP}}|$

$$\begin{aligned}
\mathbb{E}\{|H_{\text{DCP}}|^4\} &= \mathbb{E}\{H_{R,\text{DCP}}^4\} + \mathbb{E}\{H_{I,\text{DCP}}^4\} + 2\mathbb{E}\{H_{R,\text{DCP}}^2 H_{I,\text{DCP}}^2\} \\
\mathbb{E}\{H_{R,\text{DCP}}^4\} &= \mathbb{E}\left\{\left[\sum_{k=1}^{N} \alpha_k \cos\theta_{ek}\right]^4\right\} \\
&= N\mathbb{E}\left\{\alpha_k^4 \cos^4\theta_{ek}\right\} + 3N(N-1)\left[\mathbb{E}\left\{\alpha_k^2 \cos^2\theta_{ek}\right\}\right]^2 \\
&\quad + N(N-1)(N-2)(N-3)\left[\mathbb{E}\left\{\alpha_k \cos\theta_{ek}\right\}\right]^4 \\
&\quad + 6N(N-1)(N-2)\mathbb{E}\left\{\alpha_k^2 \cos^2\theta_{ek}\right\}\left[\mathbb{E}\left\{\alpha_k \cos\theta_{ek}\right\}\right]^2 \\
&\quad + 4N(N-1)\mathbb{E}\left\{\alpha_k^3 \cos\theta_{ek}^3\right\}\mathbb{E}\left\{\alpha_k \cos\theta_{ek}\right\}
\end{aligned}$$

Constellations Immune to Channel Corruption

- Channel corruption is due to the rotational symmetry of the transmitted constellation
- Can use asymmetric constellations to avoid channel corruption
- Asymmetric constellations have lower separation between points compared to a symmetric constellation (Ex: BPSK and OOK)
  - Separation between points give immunity to noise and fading
  - Asymmetry gives immunity to channel corruption
- There is a trade-off between symmetric and asymmetric constellations over DCP

Constellations Immune to Channel Corruption: Case Study

- We compare two simple signaling schemes,
  - BPSK: symmetric (CSI not required)
  - OOK : Asymmetric (Require CSI at FC)
- Probability of error for BPSK and OOK may be written as

$$
\begin{aligned}
P_{e,BPSK} &= P_{cc,PAM} + (1 - P_{cc,PAM})P_{e,CSIR,2PAM} \\
P_{e,OOK} &= P_{e,CSIR,2PAM}
\end{aligned}
$$

- Plotting $P_{e,BPSK}$ against $P_{e,OOK}$ reveals which constellation performs better.

Comparison of Power Method and K-means Algorithm

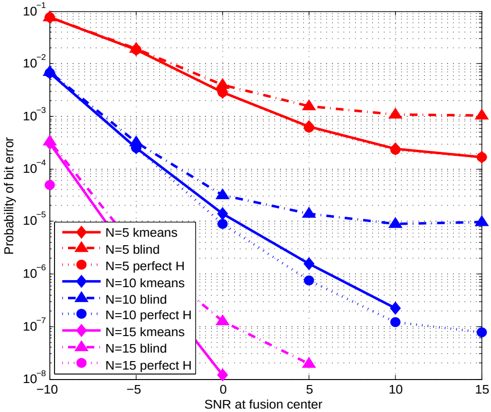


Figure: P_e vs SNR for 4-PAM; Pilot SNR=5dB

Performance of 4-PAM

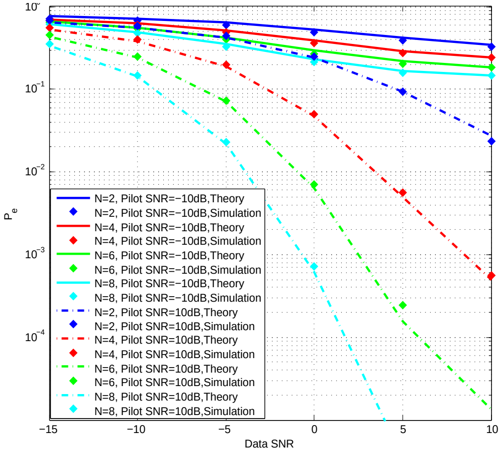


Figure: P_e vs SNR for 4-PAM using k means at different pilot SNR

Effect of N_D on the Performance of K-means

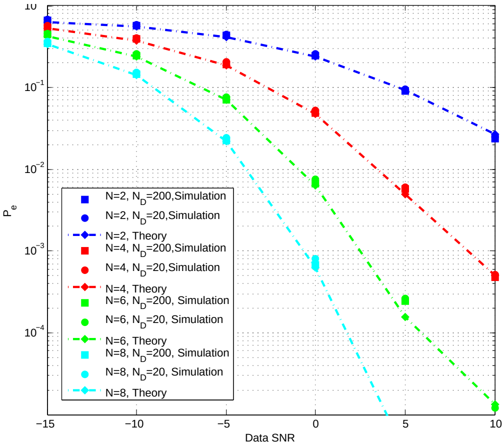


Figure: P_e vs SNR for 4-PAM using k means for different values of N_D

Performance of 16-QAM

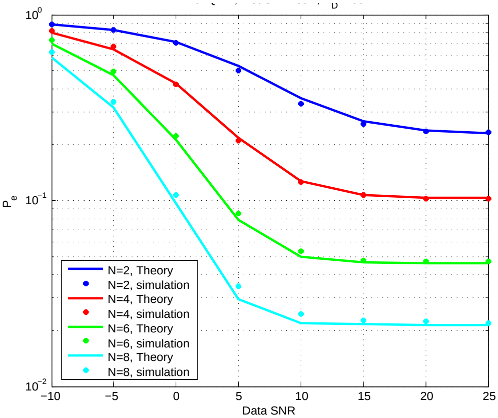


Figure: P_e vs SNR for 16-QAM using K-means; Pilot SNR=0dB

Comparison of BPSK and OOK

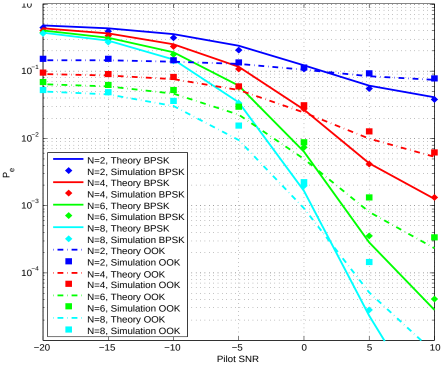


Figure: Performance of symmetric BPSK and OOK with varying Pilot SNR and number of sensors, $DataSNR = 0dB$

Channel Corruption for PAM

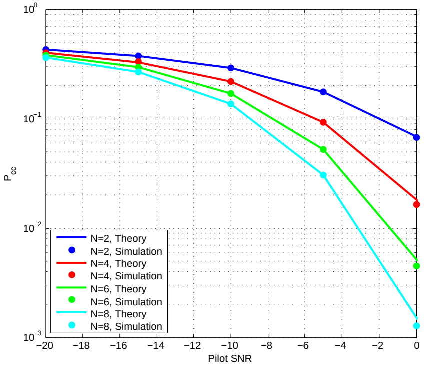


Figure: Probability of channel corruption with varying Pilot SNR and number of sensors for 4-PAM

Comparison of Conventional and Modified K-means Algorithm

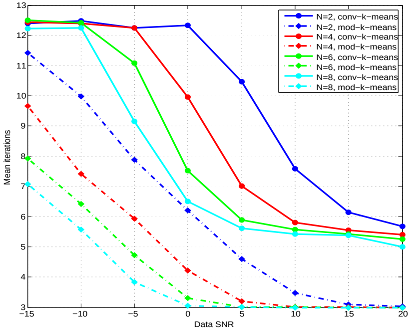


Figure: Comparison of mean number of iterations for modified K-means algorithm and conventional K-means algorithm

Conclusion

- MI analysis shows a higher order constellation performs better
- Proposed two blind channel estimation methods to use general signal constellations over DCP
- Analyzed the performance of the proposed schemes and derived expressions for
  - Probability of channel corruption for M-PAM and M-QAM signaling
  - Probability of error for M-PAM and M-QAM signaling
- Studied the problem of channel corruption and proposed solutions
- Validated the results through simulation

THANK YOU

APPENDIX-A

Derivation of $\mathbb{E}\{\alpha_k^4 \cos^4 \theta_{ek}\}$

$$
\begin{aligned}
\mathbb{E}\{\alpha_k^4 \cos^4 \theta_{ek}\} &= \frac{1}{8}\left[\mathbb{E}\left\{\alpha_k^4 \; \mathbb{E}\{\cos 4\theta_{ek}/\alpha_k\} + 4\alpha_k^4 \; \mathbb{E}\{\cos 2\theta_{ek}/\alpha_k\} + 3\alpha_k^4\right\}\right] \\
\mathbb{E}\{\cos 4\theta_{ek}/\alpha_k\} &= \mathbb{E}\{\cos|4\theta_{ek}|/\alpha_k\} \\
&= \int_{\theta=0}^{\pi} \cos 4\theta_{ek} \; f(|\theta_{ek}|/\alpha_k) d\theta_{ek} \\
&= \int_{\theta=0}^{\pi} \cos 4\theta \left[\frac{e^{-\gamma_p \alpha_k^2}}{\pi} + \frac{\gamma_p \alpha_k^2 \sin 2\theta}{\pi} \int_{x=0}^{\pi-\theta} \frac{e^{-\frac{\gamma_p \alpha_k^2 \sin^2\theta}{\sin^2 x}}}{\sin^2 x} dx\right] \\
&= \frac{\gamma_p \alpha_k^2}{\pi} \int_{x=0}^{\pi} \frac{1}{sin^2 x} \int_{\theta=0}^{\pi-x} \cos 4\theta \sin 2\theta e^{-\frac{\gamma_p \alpha_k^2 \sin^2\theta}{\sin^2 x}} d\theta dx \\
&= \frac{\gamma_p \alpha_k^2}{\pi} \int_{x=0}^{\pi} \frac{1}{sin^2 x} \int_{t=0}^{\sin^2 x} \left(8t^2 - 8t + 1\right) e^{-\frac{\gamma_p \alpha_k^2 t}{\sin^2 x}} dtdx \\
&= \frac{6}{\gamma_p^2 \alpha_k^4} - \frac{4}{\gamma_p \alpha_k^2} + 1 - \frac{6}{\gamma_p^2 \alpha_k^4} e^{-\gamma_p \alpha_k^2} - \frac{2}{\gamma_p \alpha_k^2} e^{-\gamma_p \alpha_k^2}
\end{aligned}
$$

Derivation of $\mathbb{E}\{\alpha_k^4 \cos^4\theta_{ek}\}$ contd...

$$\mathbb{E}\{\alpha_k^4 \cos 4\theta_{ek}\} = \mathbb{E}\left\{\frac{6}{\gamma_p^2} - \frac{4\alpha_k^2}{\gamma_p} + \alpha_k^4 - \frac{6}{\gamma_p^2}e^{-\gamma_p\alpha_k^2} - \frac{2\alpha_k^2}{\gamma_p}e^{-\gamma_p\alpha_k^2}\right\}$$

For Rayleigh fading channel,

$$\begin{aligned}\mathbb{E}\{e^{-u\gamma_k}\} &= \mathcal{L}_{\gamma_k}(u) = \frac{1}{1+u\Omega_k} \\ \mathbb{E}\{\gamma_k e^{-u\gamma_k}\} &= -\frac{\partial}{\partial u}\mathcal{L}_{\gamma_k}(u) = \frac{\Omega_k}{(1+u\Omega_k)^2}\end{aligned}$$

$$\begin{aligned}\Rightarrow \mathbb{E}\{\alpha_k^4 \cos 4\theta_{ek}\} &= 8\sigma_r^4 + \frac{6}{\gamma_p^2} - \frac{4\Omega_k}{\gamma_p} - \frac{6}{\gamma_p^2}\frac{1}{1+\gamma_p\Omega_k} - \frac{2}{\gamma_p}\frac{\Omega_k}{(1+\gamma_p\Omega_k)^2} \\ \mathbb{E}\{\alpha_k^4 \cos 2\theta_{ek}\} &= \mathbb{E}\left\{\alpha_k^4\mathbb{E}\{\cos 2\theta_{ek}/\alpha_k\}\right\} = 8\sigma_r^4 - \frac{\Omega_k}{\gamma_p} + \frac{\Omega_k}{\gamma_p(1+\gamma_p\Omega_k)^2} \\ \Rightarrow \mathbb{E}\{\alpha_k^4 \cos^4\theta_{ek}\} &= 8\sigma_r^4 - \frac{\Omega_k^2}{4}\frac{5+4\gamma_p\Omega_k}{(1+\gamma_p\Omega_k)^2}\end{aligned}$$