Distributed Co-phasing For Wireless Sensor Networks

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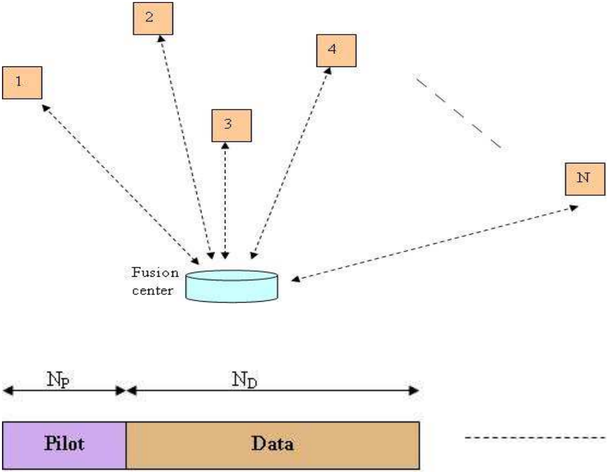
System Model



System Model

- N_p pilot symbols sent from FC to sensors
- N_d simultaneous data transmissions from each sensor to FC
- complex channel from kth sensor to Fusion center is $g_k = \alpha_k e^{j\theta_k}$
- Received signal at the kth sensor during pilot transmission $r_k[n] = g_k \sqrt{E_p} + \eta_k[n]$
- received signal at the fusion center is
$$r[n] = \sum_{k=1}^{n} x[n] e^{-j\hat{\theta}_k} g_k + v[n]$$

- Performance analysis already done for BPSK
- Works for only constant modulus constellations
- Performance could be improved with higher order constellations
- Channel estimate required at the fusion center.

Mutual Information

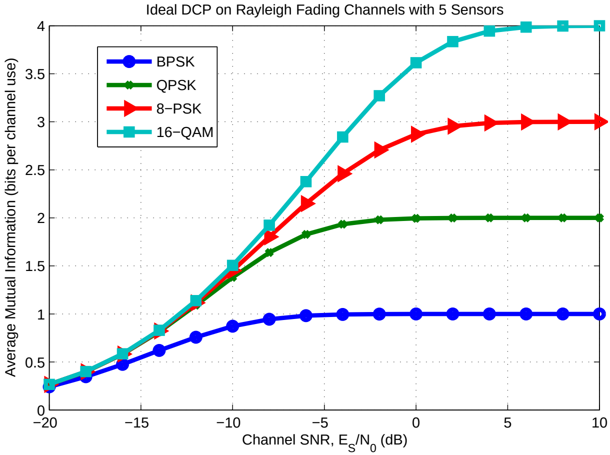
$$
\begin{aligned}
Y &= HX + N \\
I(X:Y) &= \sum_i p(x_i) \int_y p(y/x_i) log \frac{p(y/x_i)}{p(y)} dy \\
p(y) &= \sum_i p(y/x_i) p(x_i) \\
p(y/x_i) &= \mathbb{E}_H \{ p(y/h, x_i) \} \\
\int_y p(y/x_i) log \frac{p(y/x_i)}{p(y)} dy &= \mathbb{E}_{p(y/x_i)} \{ log \frac{p(y/x_i)}{p(y)} \}
\end{aligned}
$$

Mutual Information

- Difficult to evaluate in closed form
- Use Monte carlo simulation
- Example shown for BPSK.

$$\begin{aligned}
I(X:Y) &= p(x=1)\int_y p(y/x=1) log \frac{p(y/x=1)}{p(y)} dy + \\
&\quad p(x=-1)\int_y p(y/x=-1) log \frac{p(y/x=-1)}{p(y)} dy \\
&= \int_y p(y/x=1) log \frac{p(y/x=1)}{p(y)} dy \\
&\quad \{\text{due to symmetry and assuming p(x=-1)=p(x=1)}\} \\
&= \mathbb{E}_{p(y/x=1)}\left\{ log \frac{p(y/x=1)}{p(y)} \right\} \\
\text{Also, } p(y/x=1) &= \mathbb{E}_H\{p(y/h, x=1)\} \quad \text{and} \\
p(y) &= p(y/x=1)p(x=1) + p(y/x=-1)p(x=-1)
\end{aligned}$$

Mutual Information



- Blind estimate

$$\begin{aligned}
Y &= HX + N \\
\mathbb{E}\{Y^2/H\} &= H^2\mathbb{E}\{X^2\} + \mathbb{E}\{N^2\} \\
&= H^2 P_X + \sigma^2 \\
& \quad P_X \text{ is the average power in X. } P_X = 5 \text{ for 4PAM} \\
H^2 &= \frac{\mathbb{E}\{Y^2/H\} - \sigma^2}{P_X} \\
& \quad \text{For sufficiently large value of } N_D, \\
\frac{1}{N_D}\sum Y^2 &\rightarrow \mathbb{E}\{Y^2/H\}
\end{aligned}$$

- k-means algorithm
  - Need to identify k groups in the received sequence
  - k is known
  - k means algorithm can be used for blind demodulation

Modified k means algorithm

- Nearest neighbor condition
- Centroid condition
- Centroid update could be improved by using the structure of the constellation

$$\hat{\beta} = arg \min_{\beta} \sum_{k=1}^{M} \sum_{l=1}^{M_k} |\beta S_k - X_{lk}|^2$$

$$\text{Let } J = \sum_{k=1}^{M} \sum_{l=1}^{M_k} |\beta S_k - X_{lk}|^2$$

$$\frac{\partial J}{\partial \beta} = 0 \Rightarrow$$

$$\hat{\beta} = \frac{\sum_{k=1}^{M} S_k^* \sum_{l=1}^{M_k} X_{lk}}{\sum_{k=1}^{M} M_k |S_k|^2}$$

Assumptions

- Perfect DCP is achieved
- Channel is perfectly estimated by k-means
- $P_e = \mathbb{E}_H\left[P\{error/H\}\right]$ where $H = \sum \alpha_k$

Improved Gaussian Approximation

$$\begin{aligned} P(\theta) &= P(\mu) + (\theta-\mu)P^{'}(\mu) + \frac{1}{2}(\theta-\mu)^2 P^{''}(\mu) + \ldots \\ \mathbb{E}\{P(\theta)\} &= P(\mu) + \frac{1}{2}\sigma^2 P^{''}(\mu) \\ &\approx P(\mu) + \frac{1}{2}\sigma^2 \frac{P(\mu+h) - 2P(\mu) + P(\mu-h)}{h^2} \\ &\approx \frac{2}{3}P(\mu) + \frac{1}{6}[P(\mu+\sqrt{3}\sigma) + P(\mu-\sqrt{3}\sigma)] \end{aligned}$$

Pe for 4PAM

$$
\begin{aligned}
P\{err/H\} &= \frac{1}{M}\sum_{i=1}^{M} P(err/X = x_i, H);\ \ x_i \in \{\pm 1, \pm 3, \pm 5 \ldots\} \\
&= \frac{1}{M}\left[(M-2)2Q\left(\sqrt{\frac{E_S}{N_0}}H\right) + 2Q\left(\sqrt{\frac{E_S}{N_0}}H\right)\right] \\
&= \frac{2(M-1)}{M} Q\left(\sqrt{\frac{E_S}{N_0}}H\right) \\
P\{err\} &= \mathbb{E}_H\left\{\frac{2(M-1)}{M} Q\left(\sqrt{\frac{E_S}{N_0}}H\right)\right\}
\end{aligned}
$$

Pe for 4PAM

$$
\begin{aligned}
\text{Let } R &= \sqrt{\frac{E_S}{N_0}} H \\
\mu &= \mathbb{E}[R] = \sqrt{\frac{E_S}{N_0}} \sum_{k=1}^{N} \mathbb{E}\{\alpha_k\} \\
&= N\sqrt{\frac{E_S \pi}{2N_0}} \\
var\{R\} &= \frac{E_S}{N_0} N \; var\{\alpha_k\} = \frac{E_S}{N_0} N(2 - \frac{\pi}{2}) \\
\therefore \; \sigma &= \sqrt{\frac{E_S}{N_0} N(2 - \frac{\pi}{2})} \\
\text{Using IGA,} \quad P\{err\} &\approx \frac{M-1}{3M}\left[4Q(\mu) + Q(\mu + \sqrt{3}\sigma) + Q(\mu - \sqrt{3}\sigma)\right]
\end{aligned}
$$

Pe for 4QAM

$$
\begin{aligned}
P\{err/H\} &= \frac{1}{M}\sum_{i=1}^{M} P(err/X = x_i, H);\ x_i \in \{\pm 1 \pm i\} \\
&= \frac{1}{M}\sum_{i=1}^{M} 2Q\left(\sqrt{\frac{E_S}{N_0}}H\right) \\
&= 2Q\left(\sqrt{\frac{E_S}{N_0}}H\right) \quad \text{by symmetry} \\
P\{err\} &= \mathbb{E}_H\left\{2Q\left(\sqrt{\frac{E_S}{N_0}}H\right)\right\} \\
P\{err\} &\approx 2\left[Q(\mu) + \frac{1}{6}Q(\mu + \sqrt{3}\sigma) + \frac{1}{6}Q(\mu - \sqrt{3}\sigma)\right]
\end{aligned}
$$

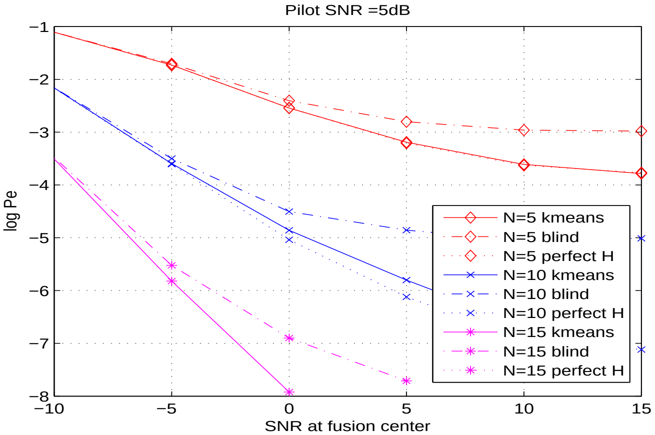


Figure: Comparison of blind estimate and k means

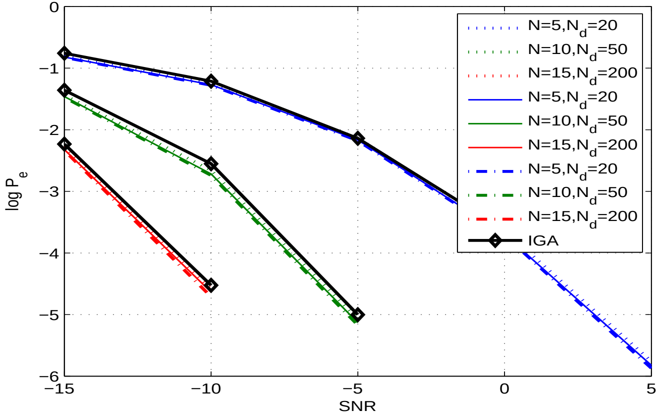


Figure: P_e for different values of N_d

Way ahead

- Variable power allocations schemes
- Censoring sensors
- Multiple receive antennas at Fusion center

Thank You