Addendum to Packet Drop Probability Analysis of Dual and Mono Energy Harvesting Links

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Abstract

This report serves as an addendum to [1], and presents the PDP analysis of dual EH links with zero size energy buffers, for ARQ and HARQ-CC protocols. In addition to this, expressions for the transition probabilities of the discrete-time Markov chain (DTMC) model of the dual and mono energy harvesting links are provided [1]. The notations used here assumes the same meaning as defined in [1].

I. Introduction

To analyze the PDP of dual EH links with retransmission, the system evolution can be modeled as a DTMC [1]. This report serves as an addendum to the PDP analysis presented in [1]. In Sec. II we present the PDP analysis of dual EH links with no energy buffers at both the EHNs, while transition probabilities are provided in Sec. III.

II. PDP of batteryless EHNs

In this section we analyze the PDP of dual EH links with zero energy buffer EHNs. The PDP analysis for ARQ-based links with slow fading channels is provided in [1]. Also, the PDP analysis of HARQ-CC based zero buffer dual EH links is provided. First, we present the results for fast fading channels.

For ARQ-based dual EH links with fast fading channels, the PDP is given as

$$P_{\mathrm{D}}(K) = \left(1 - \rho_t \rho_r e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{E_s \sigma_c^2}}\right)^K, \tag{1}$$

where (1) is written using (4).

In the case of HARQ-CC, the PDP of dual EH links without energy buffer is given as

$$P_{\mathrm{D}}(K)=\sum_{m=0}^{K}\binom{K}{m}(\rho_t\rho_r)^m(1-\rho_t\rho_r)^{K-m}p_{\mathrm{D}_m} \tag{2}$$

where p_{D_m} is the probability that the transmitted packet is dropped, when both nodes simultaneously harvest energy in exactly m out of the K slots in the current frame. For the slow fading case, p_{D_m} is given by (6), while for fast fading channels it is given by (22) in [1], with $i=j=0, m_t=m_r=\Psi_1=m,$ and $L_n=1 \quad \forall \quad n$. Next, we present the transition probabilities for DTMC model of dual and mono EH links.

III. Transition Probability Matrix, G, for dual EH links

The probability of transition from state (i_1,j_1,r) to (i_2,j_2,s) is $G_{i_1,j_1,r}^{i_2,j_2,s}=\Pr\left(B_{n+1}^t=i_2,B_{n+1}^r=j_2,U_{n+1}=s\middle|B_n^t=i_1,B_n^r=j_1,U_n=r\right)$, where $i_1,i_2\in\{0,1,\ldots,B_{\max}^t\}$, $j_1,j_2\in\{0,1,\ldots,B_{\max}^r\}$ and $r,s\in\{-1,0,\ldots,K\}$. For $r\in\{0,\ldots,K-1\}$, $i_1\geq L_r$ and $j_1\geq R$, the $G_{i_1,j_1,r}^{i_2,j_2,s}$ is written as follows

$$G_{i_1,j_1,r}^{i_2,j_2,s}=\begin{cases}\rho_t\rho_r Pr\left[\gamma_n<\gamma_0\right], & i_2=i_1-L_r+1, j_2=j_1-R+1, s=r+1,\\ \rho_t\rho_r Pr\left[\gamma_n\geq\gamma_0\right], & i_2=i_1-L_r+1, j_2=j_1-R+1, s=-1,\\ (1-\rho_t)\rho_r Pr\left[\gamma_n<\gamma_0\right], & i_2=i_1-L_r, j_2=j_1-R+1, s=r+1,\\ (1-\rho_t)\rho_r Pr\left[\gamma_n\geq\gamma_0\right], & i_2=i_1-L_r, j_2=j_1-R+1, s=-1,\\ \rho_t(1-\rho_r) Pr\left[\gamma_n<\gamma_0\right], & i_2=i_1-L_r+1, j_2=j_1-R, s=r+1,\\ \rho_t(1-\rho_r) Pr\left[\gamma_n\geq\gamma_0\right], & i_2=i_1-L_r+1, j_2=j_1-R, s=-1,\\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n<\gamma_0\right], & i_2=i_1-L_r, j_2=j_1-R, s=r+1,\\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n\geq\gamma_0\right], & i_2=i_1-L_r, j_2=j_1-R, s=-1,\\ 0, & \text{otherwise.}\end{cases} \tag{3a}$$

For $r \in \{0, \ldots, K-1\}$, $L_r - 1 \leq i_1 < L_r$ and $j_1 \geq R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ \rho_t (1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = r + 1, \\ \rho_t (1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3b}$$

For $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $R - 1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t) \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, , s = r + 1, \\ (1-\rho_t) \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3c}$$

For $r \in \{0, \ldots, K-1\}$, $L_r - 1 \leq i_1 < L_r$ and $R - 1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - L_r + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = 0, j_2 = 0, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, , s = r, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3d}$$

For $r \in \{0, \ldots, K-1\}$, $0 \le i_1 \le L_r - 2$ or $0 \le j_1 \le R-2$

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r, & i_2 = i_1+1, j_2 = j_1+1, s = r, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1+1, s = r, \\ (1-\rho_r)\rho_t, & i_2 = i_1+1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3e}$$

For $r = -1$, $i_1 \ge 0$ and $j_1 \ge 0$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r, & i_2 = i_1+1, j_2 = j_1+1, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1+1, s = -1, \\ (1-\rho_r)\rho_t, & i_2 = i_1+1, j_2 = j_1, s = -1, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{3f}$$

In (3a) – (3d), $\Pr[\gamma_n < \gamma_0]$ for the slow and the fast Rayleigh fading channels and ARQ is given as

$$p_{\text{out},\ell} \triangleq \Pr[\gamma_\ell < \gamma_0] = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{L_\ell E_s \sigma_c^2}}, \tag{4}$$

where $\mathcal{N}_0$ denotes the power spectral density of the AWGN at the receiver. While for HARQ-CC with slow fast fading channels it is obtained using $\Psi_1 = n$ in the following equation For *slow fading* channels

$$p_{\text{D}}(i,j,m_t,m_r) = \Pr\left[|h|^2 < \frac{\gamma_0 \mathcal{N}_0}{\sum_{m=1}^{\Psi_1} P_m}\right], \tag{5}$$

$$= 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_c^2 \sum_{m=1}^{\Psi_1} P_m}}, \tag{6}$$

and for fast fading channel it is obtained using $\Psi_1 = n$ in (16) of [2].

The terms in the above transition probability expressions are obtained by considering the events that need to occur for the particular transition to happen. For e.g., in (3a) transition mentioned in first case happens if both transmitter and receiver harvest the energy in current slot, and the decoding failure occurs in current attempt. Note that, in the above, for simplicity, the transition probabilities are written for infinite buffer size at both transmitter and receiver. However, the above expressions can be trivially modified for finite battery case.

A. Transition Probability Matrix $\boldsymbol{G}_m$

The probability of transition from state (i,r) to (j,s) is $G^{rs}_{(m)ij} = Pr(B_{(M+1)K} = j, U_{n+1} = s | B_{MK} = i, U_n = r)$, where $i,j \in \{0,1,\ldots,\infty\}$ and $r,s \in \{-1,0,\ldots,K\}$.

For $r \in \{0,\ldots,K-1\}$ and $i \geq L_r$

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r+1, \\ \rho_t Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho_t) Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r, s = r+1, \\ (1-\rho_t) Pr\left[\gamma_n > \gamma_0\right], & j = i - L_r, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{7a}$$

For $r \in \{0,\ldots,K-1\}$ and $L_r - 1 \leq i < L_r$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r+1, \\ \rho_t Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho_t), & j = i, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{7b}$$

For $r \in \{0,\ldots,K-1\}$ and $0 \leq i \leq L_r - 2$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t, & j = i+1, s = r, \\ (1-\rho_t), & j = i, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{7c}$$

For $r = -1$ and $i \geq 0$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t, & j = i+1, s = -1, \\ (1-\rho_t), & j = i, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{7d}$$

In (7a) and (7b), $\Pr[\gamma_n < \gamma_0]$, for ARQ is written using (4), while for HARQ-CC with slow fading channels it can be computed using (6) with $\Psi_1 = n$. For fast fading channels one need to use (16) in [2] with $\Psi_1 = n$.

References

[1] M. K. Sharma and C. R. Murthy, "Packet drop probability analysis of dual energy harvesting links with retransmissions," submitted for publication.

[2] M. Sharma and C. Murthy, "Packet drop probability analysis of ARQ and HARQ-CC with energy harvesting transmitters and receivers," in *Proc. IEEE GlobalSIP*, Dec. 2014, pp. 148–152.