Effective Capacity of Energy Harvesting Wireless Links

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Outline of Talk

- Motivation
- Preliminaries : Effective Bandwidth
- Effective Capacity
- Effective Capacity Properties
- Effective Capacity of EH Links.

Motivation

- Given a PHY - Layer infrastructure, what QoS Gurantees can be provided?
- *Water-filling* is better than *total channel inversion* from information-theoratic point of view?
- Whether the former is also better than the latter in terms of QoS gurantees?
- Essentially, want to model the wireless channel/ PHY-layer service process in terms of connection level QoS metric eg.
  - delay,
  - delay-violation probability,
  - packet loss probability etc.

QoS guarantees in wired networks: constant capacity

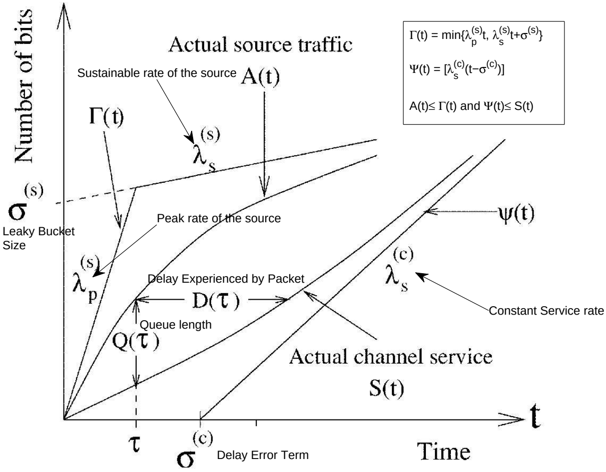

Figure : Traffic and service characterization.

Extension to wireless networks

- Need accurate model of the time-varying capacities of wireless channels.
- Providing a strict lower bound will most likely result in extremely conservative guarantees.
- Hence, *statistical service characterization (SC)* i.e.,

$$\sup_t \Pr\{\tilde{S}(t) < \Psi(t)\} \leq \epsilon \tag{1}$$

- But, $\Psi(t) = [\lambda_s^c(t - \sigma^c)]^+$
- Statistical SC requires a relation between $\{\lambda_s^c, \sigma^c, \epsilon\}$ and fading channel.
- for AWGN : $\{r_{\text{awgn}}, 0, 0\}$

Effective Bandwidth

- It models the stochastic bhaviour of source traffic
- For an arrival process $\{A(t), t \geq 0\}$, effective bandwidth is

$$\alpha(u) \triangleq \frac{\Lambda(u)}{u}, \quad \forall \quad u \geq 0 \tag{2}$$

Where, $\Lambda(u)$ is, asymptotic log-moment generating function, defined as

$$\Lambda(u) \triangleq \lim_{t \to \infty} \frac{1}{t} \log E\left[e^{uA(t)}\right] \tag{3}$$

and, $A(t)$: amount of source data over the time interval $[0, t)$

Effective Bandwidth (contd.)

- For a queue of infinite size, served by a channel of constant service rate r

$$\sup \Pr\{Q(t) \geq B\} \backsim e^{-\theta_B(r)B} \quad \text{as} \quad B \to \infty \tag{4}$$

where, $f(x) \backsim g(x)$ means that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$.

- While for smaller B,

$$\sup \Pr\{Q(t) \geq B\} \approx \gamma(r) e^{-\theta_B(r)B} \quad \text{as} \quad B \to \infty \tag{5}$$

where,

$$\gamma(r) = \Pr\{Q(t) \geq 0\}, \text{ Prob. of nonempty buffer, and}$$

$$\text{QoS exponent, } \theta_B, \text{ is the solution of } \alpha(\theta_B) = r$$

Effective Bandwidth (contd.)

- The pair $\{\gamma(r), \theta_B(r)\}$, model the source.
- For a source modeled by the pair $\{\gamma(r), \theta(r)\}$, and delay bound D_{max}

$$\sup_t \Pr\{D(t) \geq D_{\text{max}}\} \approx \gamma(r)e^{-\theta(r)D_{\text{max}}}, \quad (6)$$

$D(t)$: delay experienced by a packet arriving at time t, and

- For a delay-bound violation probability at most ϵ, the constant capacity should be r, such that,

$$\epsilon = \gamma(r)e^{-\theta(r)D_{\text{max}}}$$

- In terms of traffic envelope $\Gamma(t) = \min\{\lambda_p^{(s)}t, \lambda_s^{(s)}t + \sigma^{(s)}\}$,

$$\lambda_s^{(s)} = r \ \text{ and } \ \sigma^{(s)} = rD_{\text{max}}.$$

Effective bandwidth

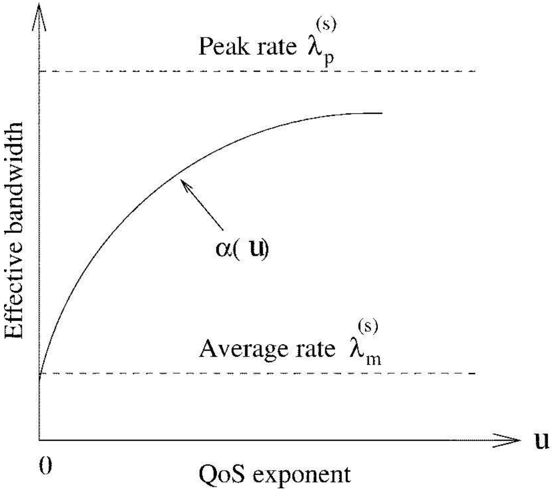

Figure : Effective bandwidth function $\alpha(u)$.

Effective capacity

- For a channel with instantaneous capacity $r(t)$, the effective capacity is

$$E_C(\theta) = \frac{-\Lambda^C(-\theta)}{\theta}, \quad \forall \ \theta \geq 0 \tag{7}$$

where, $\Lambda^C(-\theta) = \lim_{t\to\infty} \frac{1}{t} \log E\left[e^{-\theta \tilde{S}(t)}\right]$, is Gartner- Ellis limit, and

$$\tilde{S}(t) = \int_0^t r(\tau) d\tau \text{ : service provided by channel}$$

- For an infinite buffer, supplied by a source of constant data rate μ,

$$\sup_t \Pr\{D(t) \geq D_{\max}\} \approx \gamma^{(c)}(\mu) e^{-\theta^{(c)}(\mu) D_{\max}} \tag{8}$$

Effective capacity

$$\gamma^{(c)}(\mu) = Pr\{Q(t) \geq 0\}, \text{probability of nonempty buffer at random } t$$

$$\theta^{(c)}(\mu) = \mu E_C^{-1}(\mu), \text{QoS exponent}$$

- A link modeled by $\{\gamma^c(\mu), \theta^c(\mu)\}$, can support a source with rate μ, s.t.

$$\epsilon = \gamma^{(c)}(\mu) e^{-\theta^{(c)}(\mu) D_{\max}}$$

  where, ϵ is the maximum tolerable delay-violation probability.
- In terms of SC $\Psi(t) = [\lambda_s^{(c)}(t - \sigma^{(c)})]^+$,
  - $\lambda_s^{(c)}$ is the channel sustainable rate and,
  - $\sigma^{(c)} = D_{\max}$

Effective capacity Vs QoS exponent

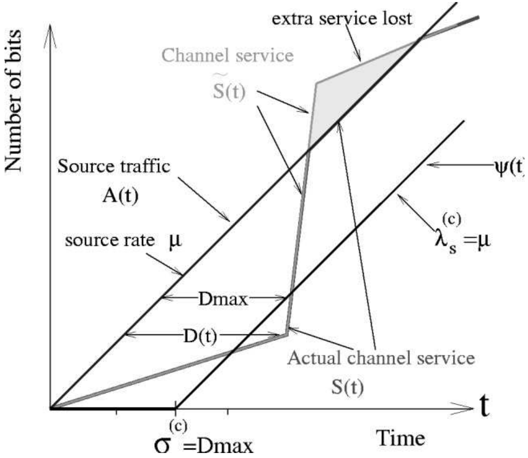

Figure : Relation between SC and delay-bound violation.

Effective capacity

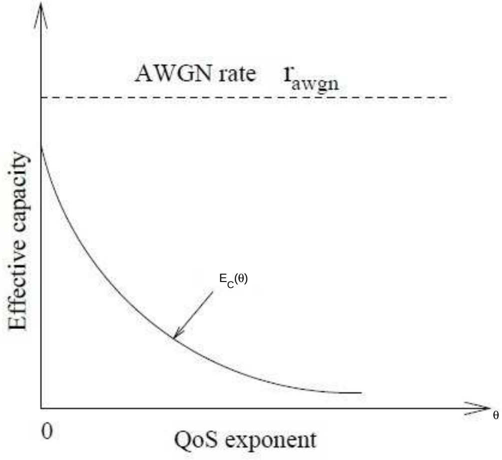

Figure : Effective capacity function $E_C(u)$.

QoS performance of generalized porcesses

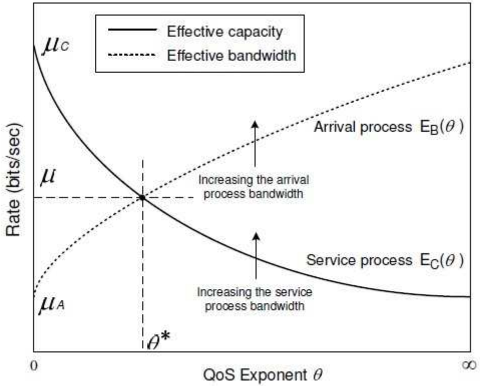

Figure : Relationship between effective capacity and effective bandwidth as a function of QoS exponent θ

Calculation of delay-bound violation prob.

1. Calculate effective bandwidth and effective capacity $E_C(\theta)$ using the statistical properties of arrival and service processes.
2. Determine rate and QoS-exponent pair (μ, θ^*) s.t.

$$\alpha(\theta) = E_C(\theta) = \mu$$

3. Approximate the buffer non empty prob. as,

$$\gamma = \frac{\mu_A}{\mu_C}$$

   or estimate it.
4. Calculate the delay violation probability as,

$$\Pr\{D(t) > D_{\max}\} = \gamma e^{-\theta \mu D_{\max}}$$

EC of FSMC-based wireless-channel service process

- Let sequence $\{R(t), t = 1, 2, 3, \ldots\}$ denote the attainable service rates of the discrete service process.
- $R(t)$ takes value in a discrete set $\mathcal{R} \triangleq \{\mu_1, \mu_2, \ldots, \mu_K\}$.
- Gartner-Ellis limit of the time-cumulated service process $S(t) \triangleq \sum_{i=1}^{t} R(i)$ defined as

$$\Lambda_C(\theta) \triangleq \lim_{t \to \infty} \frac{1}{t} \log \left(E\{e^{\theta S(t)}\} \right)$$

- Assume that $\Lambda_C(\theta)$ is a convex function and differentiable function for all real θ.

EC of FSMC-based wireless-channel service process

Theorem

Let $\{\mu_k, k = 1, 2, 3, \ldots\}$ *be the number of bit transmitted in state* k *of the FSMC-based service process and define* $\mathbf{\Phi}(\theta) \triangleq diag\{e^{-\mu_1\theta}, e^{-\mu_2\theta}, \ldots, e^{-\mu_K\theta}\}$*, then the effective capacity of the FSMC-based service process is determined by*

$$E_C(\theta) = -\frac{1}{\theta}\log\left(\rho\{\mathbf{P\Phi}(\theta)\}\right)$$

, where,

$\mathbf{P}$: *transition probability matrix*

$\rho\{.\}$: *spectral radius of the matrix*

EC of FSMC proof

Let $v_i(\theta,t) \triangleq E\{e^{-\theta S(t)}|R(1)=\mu_i\}$ and $\mathbf{v}(\theta,t) \triangleq \{v_1(\theta,t),\ldots,$ $v_K(\theta,t)\}$, respectively. Hence,

$$
\begin{aligned}
v_i(\theta,t) &= \{e^{-\theta S(t)}|R(1)=\mu_i\}E\{e^{-\theta(S(t)-R(1))}|R(1)=\mu_i\} \\
&= e^{-\mu_i\theta}\sum_{j=1}^{K} E\{e^{-\theta(S(t)-S(1))}|R(2)=\mu_j, R(1)=\mu_i\} \\
&\qquad \Pr\left\{R(2)=\mu_j|R(1)=\mu_i\right\} \\
&= e^{-\mu_i\theta}\sum_{j=1}^{K} E\{e^{-\theta(S(t)-S(1))}|R(2)=\mu_j\}p_{ij} \\
&= e^{-\mu_i\theta}\sum_{j=1}^{K} E\{e^{-\theta(S(t-1))}|R(1)=\mu_j\}p_{ij} \\
&= e^{-\mu_i\theta}\sum_{j=1}^{K} v_i(\theta,t-1)p_{ij}
\end{aligned}
$$

EC of FSMC proof (contd.)

In matrix form,

$$\mathbf{v}(\theta,t)^T = (\boldsymbol{\Phi}(\theta)\mathbf{P})^{t-1}\boldsymbol{\Phi}(\theta)\mathbf{1}^T \tag{9}$$

where, $\mathbf{1} : K -$ dimensional row-vector of $\mathbf{1} = [1,\dots,1]$. Now,

$$E\{e^{\theta S(t)}\} = \boldsymbol{\pi}\mathbf{v}(\theta,t)^T = \boldsymbol{\pi}(\boldsymbol{\Phi}(\theta)\mathbf{P})^{t-1}\boldsymbol{\Phi}(\theta)\mathbf{1}^T = \boldsymbol{\pi}(\mathbf{P}\boldsymbol{\Phi}(\theta))^t\mathbf{1}^T$$

Since $\mathbf{P}\boldsymbol{\Phi}(\theta)$ is a *primitive nonnegative matrix*, hence,

$$\lim_{t\to\infty}\left(\boldsymbol{\pi}(\mathbf{P}\boldsymbol{\Phi}(\theta))^t\mathbf{1}^T\right) = (\rho\{\mathbf{P}\boldsymbol{\Phi}(\theta)\})^t\,\boldsymbol{\pi}\mathbf{y}(\theta)\mathbf{x}(\theta)\mathbf{1}^T$$

where, $\mathbf{y}(\theta)$ and $\mathbf{x}(\theta)$ are, respectively the column and row eigenvectors of the matrix $\mathbf{P}\boldsymbol{\Phi}(\theta)$, corresponding to $\rho\{\mathbf{P}\boldsymbol{\Phi}(\theta)\}$ and satisfying $\mathbf{y}(\theta)\mathbf{x}(\theta) = 1$. Hence,

$$E_C(\theta) = -\lim_{t\to\theta}\frac{1}{\theta t}\log E\{e^{\theta S(t)}\} = -\frac{1}{\theta}\log\left(\rho\{\mathbf{P}\boldsymbol{\Phi}(\theta)\}\right)$$

Properties of effective capacity

Theorem

Following claims hold for the effective-capacity function $E_C(\theta)$ of the FSMC-based service process:

$$\frac{dE_C(\theta)}{d\theta} \leq 0, \quad , \forall \ \theta > 0$$

$$\sup_{\theta > 0} E_C(\theta) = \lim_{\theta \to 0} E_C(\theta) = \bar{\mu}$$

$$\inf_{\theta > 0} E_C(\theta) = \lim_{\theta \to \infty} E_C(\theta) = \mu_{\text{min}}$$

where, $\bar{\mu}$ and μ_{min} is average and minimum no. of bits transmitted per attempt.

The above results essentially indicate the delay-throughput tradeoff.

Proof: EC properties

1. Let $f(\theta) \triangleq -\Lambda_C(-\theta)$. Due to concavity of $f(\theta)$ we have $f''(\theta) \leq 0$. Then,

$$E_C'(\theta) = \left(\frac{f(\theta)}{\theta}\right) = \frac{\theta f'(\theta) - f(\theta)}{\theta^2}$$

As $f(0) = 0$. Hence,

$$(\theta f'(\theta) - f(\theta))|_{\theta=0} = 0$$

. Moreover, for all $\theta > 0$,

$$(\theta f'(\theta) - f(\theta)) = \theta f''(\theta) \leq 0$$

Hence, $E_C'(\theta) \leq 0$ for all $\theta < 0$. Therefore the $E_C(\theta)$ is a *monotonically decreasing* function of θ.

Proof: claim 2 (EC properties)

Let $\triangleq \rho\{\mathbf{P\Phi}(\theta)\}$. Hence,

$$\lambda(\theta)\mathbf{x}(\theta) = \mathbf{x}(\theta)\mathbf{P\Phi}(\theta)$$

Since, $\Lambda_C(\theta)$ is differentiable, $\mathbf{P\Phi}(\theta)$ is also differentiable. Hence, using the Taylor series expansion at $\theta = 0$

$$\lambda(\theta) = \lambda_0 + \lambda_1\theta + o(\theta)$$

$$\mathbf{x}(\theta) = \mathbf{x}_0 + \mathbf{x}_1\theta + o(\theta)$$

Since, $1.\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$, at $\theta = 0$, we obtain $\lambda_0 = 1$ and $\mathbf{x}_0 = \boldsymbol{\pi}$. Hence,

$$\lambda(\theta)\mathbf{x}(\theta) = \lambda_0\mathbf{x}_0 + (\lambda_0\mathbf{x}_1 + \lambda_1\mathbf{x}_0)\theta + o(\theta) = \boldsymbol{\pi} + (\mathbf{x}_1 + \lambda_1\boldsymbol{\pi})\theta + o(\theta)$$

Let $\mathbf{U} \triangleq \text{diag}\{\mu_1, \mu_2, \ldots, \mu_K\}$. Hence,

$$\mathbf{x}(\theta)\mathbf{P\Phi}(\theta) = (\boldsymbol{\pi} + \mathbf{x}_1\theta)\mathbf{P}(\mathbf{I} - \theta\mathbf{U}) + o(\theta) = \boldsymbol{\pi}\mathbf{P} + (\mathbf{x}_1\mathbf{P} - \boldsymbol{\pi}\mathbf{P}\mathbf{U})\theta + o(\theta)$$

$$= \boldsymbol{\pi} + (\mathbf{x}_1\mathbf{P} - \boldsymbol{\pi}\mathbf{U})\theta + o(\theta)$$

Proof: claim 2 and 3(EC properties)

On comparing above two equations,

$$\mathbf{x}_1 + \lambda_1 \boldsymbol{\pi} = \mathbf{x}_1 \mathbf{P} - \boldsymbol{\pi} \mathbf{U}$$

Solving above equation for λ_1,

$$\lambda_1 = -\sum_{k=1}^{K} \pi_k \mu_k = -\bar{\mu}$$

Using basic definition of EC,

$$\lim_{\theta \to 0} E_C(\theta) = -\lim_{\theta \to 0} \frac{1}{\theta} \log \lambda(\theta) = -\lambda_1 \lim_{\theta \to 0} \frac{1}{\lambda_1 \theta} \log (1 + \lambda_1(\theta) + o(\theta))$$
$$= -\lambda_1 = \bar{\mu}$$

As $E_C(\theta)$ is a monotonically decreasing function hence the proof follows. *Claim 3:* Let $j = \arg\min_{1 \le i \le K} \mu_i$, hence

$$\lim_{\theta \to \infty} \lambda(\theta) = p_{jj} e^{-\theta \mu_{\min}}.$$

Scaling property of EC

Theorem

If *$E_{C_a}(\theta)$ is the EC function of the service process $R_a(t)$,* ***then*** *the EC of the service process $R_b(t) = \chi R_b(t)$, denoted by $E_{C_b}(\theta)$, is*

$$E_{C_b}(\theta) = \chi E_{C_a}(\chi\theta),$$

where $\chi \in \mathbb{R}$.

Proof : By definition,

$$\begin{aligned} E_{C_b}(\theta) &= -\lim_{t\to\infty}\frac{1}{\theta t}\log\left(E\left\{e^{-\theta\sum_{i=1}^{t}R_b(i)}\right\}\right) \\ &= -\lim_{t\to\infty}\frac{1}{\theta t}\log\left(E\left\{e^{-\chi\theta\sum_{i=1}^{t}R_a(i)}\right\}\right) \\ &= -\chi\lim_{t\to\infty}\frac{1}{(\chi\theta)t}\log\left(E\left\{e^{-(\chi\theta)\sum_{i=1}^{t}R_a(i)}\right\}\right) = \chi E_{C_a}(\chi\theta) \end{aligned}$$

Effect of channel correlation

Theorem

For all of the FSMC-based wireless-channel service processes with the same marginal pdf,

(1).*The uncorrelated channel process achieves the maximum EC*

(2).*The fully correlated channel process leads to minimum EC*

References

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