Bayesian Learning for Joint Sparse OFDM Channel Estimation and Data Detection

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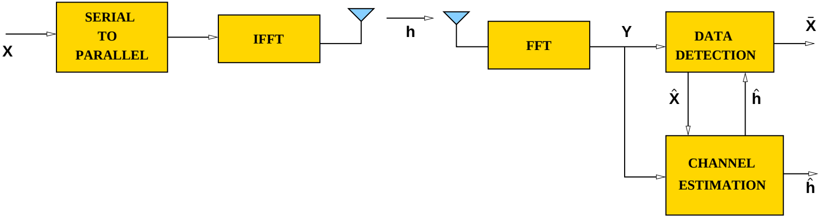
Outline

OFDM System Model



- The received signal **Y** is given by,

$$\mathbf{Y} = \mathbf{XFh} + \mathbf{V}$$

- The system model considering the pilots can be written as

$$\mathbf{Y}_T = \mathbf{X}_T\mathbf{F}_T\mathbf{h} + \mathbf{V}_T$$

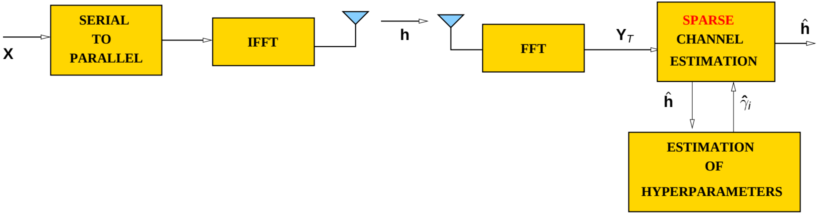
EM Algorithm for Channel Estimation and Data Detection

- $\mathbf{h}$ is treated as the hidden variable

$$\text{E-step}: Q(\mathbf{X}/\mathbf{X}^{(p)}) = E_{\mathbf{h}/\mathbf{Y},\mathbf{X}^{(p)}}(\log \mathcal{P}(\mathbf{Y},\mathbf{h}/\mathbf{X})/\mathbf{Y},\mathbf{X}^{(p)})$$
$$\text{M-step}: \mathbf{X}^{(p+1)} = \arg\max_{\mathbf{X}} Q(\mathbf{X}/\mathbf{X}^{(p)})$$

- $\log \mathcal{P}(\mathbf{Y},\mathbf{h}/\mathbf{X}) = \underbrace{\log \mathcal{P}(\mathbf{Y}/\mathbf{h},\mathbf{X})}_{\text{Log Likelihood, func. of } \mathbf{X}} + \underbrace{\log \mathcal{P}(\mathbf{h})}_{\text{not a func. of } \mathbf{X}}$

SBL for Channel Estimation



- **h** is sparse in time domain
- $\mathbf{h}(i) \sim \mathcal{CN}(0, \gamma_i)$, where γ_i is a deterministic but unknown hyperparameter
- Estimate of the sparsity profile is given by $\Gamma = \text{diag}(\gamma_1, \gamma_2 \ldots \gamma_M)$, i.e., if the diagonal entries $\gamma_i = 0$, $h_i = 0$

Estimation of Hyperparameters

- Prior for the sparse vector:

$$\mathcal{P}(\mathbf{h};\Gamma) = \prod_{i=1}^{M} (2\pi\gamma_i)^{-\frac{1}{2}} exp\left(-\frac{h_i^2}{2\gamma_i}\right)$$

- ML estimate of the hyperparameters obtained by maximizing over the marginalized pdf $\mathcal{P}(\mathbf{Y};\Gamma)$,

$$\mathcal{P}(\mathbf{Y};\Gamma) = (2\pi)^{-\frac{N}{2}} |\Sigma_Y|^{-\frac{1}{2}} exp\left[-\frac{1}{2}\mathbf{Y}^H|\Sigma_Y|^{-1}\mathbf{Y}\right]$$

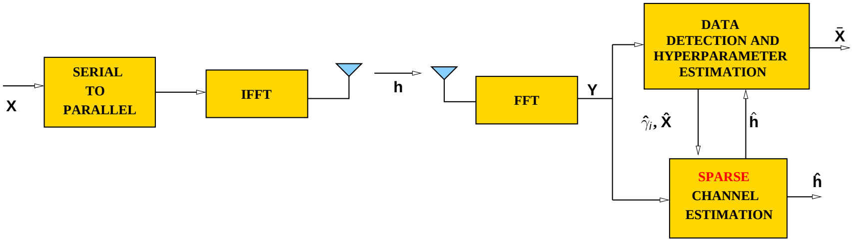
where $\Sigma_Y = \sigma^2\mathbf{I} + \mathbf{XF}\Gamma\mathbf{F}^H\mathbf{X}^H$, $\Gamma = \text{diag}(\gamma_1, \gamma_2 \ldots \gamma_M)$

- EM algorithm used to find the ML estimate of the hyperparameters

SBL for Basis Selection

- E step: $Q(\Gamma/\Gamma^{(p)}) = E_{\mathbf{h}/\mathbf{Y};\Gamma^{(p)}}(\log \mathcal{P}(\mathbf{Y}, \mathbf{h}; \Gamma))$
- The posterior density of the hidden variable is given by $\mathcal{P}(\mathbf{h}/\mathbf{Y}; \Gamma^{(p)}) = \mathcal{N}(\mu, \Sigma_h)$ where $\mu = \sigma^{-2}\Sigma_h \mathbf{A}^H \mathbf{Y}$ and $\Sigma_h = (\sigma^{-2}\mathbf{A}^H\mathbf{A} + \Gamma^{-1})^{-1}$, $\mathbf{A} \triangleq \mathbf{XF}$
- M-step: $\Gamma^{(p+1)} = \arg\max_{\gamma_i > 0} Q(\Gamma/\Gamma^{(p)})$
- $\log \mathcal{P}(\mathbf{Y}, \mathbf{h}; \Gamma) = \underbrace{\log \mathcal{P}(\mathbf{Y}/\mathbf{h})}_{\text{not a func. of } \gamma_i} + \underbrace{\log \mathcal{P}(\mathbf{h}; \Gamma)}_{\text{func. of } \gamma_i}$
- Upon convergence, many of the γ_i are driven to zero

Proposed Algorithm



- The posterior pdf of the hidden variable **h** is estimated in the E-step
- In the M-step, $\log \mathcal{P}(\mathbf{Y}/\mathbf{h}, \mathbf{X})$ is used to find the ML estimate of $\mathbf{X}$ and $\log \mathcal{P}(\mathbf{h}; \Gamma)$ is used to find the ML estimate of γ_i

Combined Algorithm

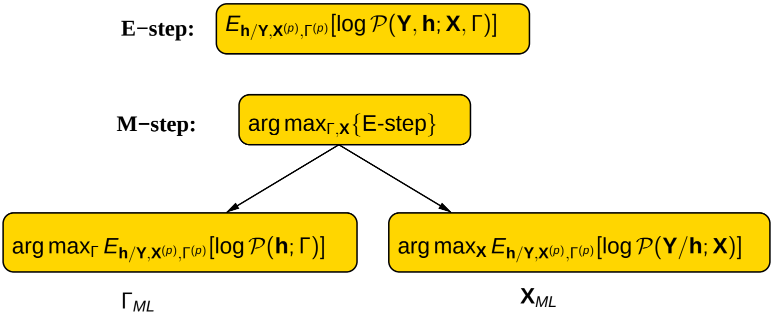


Figure: Proposed Algorithm

Computing the Bayesian CRB

- The estimation error is bounded as

$$E\left[(\mathbf{h}-\hat{\mathbf{h}})(\mathbf{h}-\hat{\mathbf{h}})^H\right] \geq \mathbf{J}^{-1},$$

Here $J_{ij} = E_{\mathbf{Y},\mathbf{h}}\left[-\frac{\partial^2 \log(\mathcal{P}(\mathbf{Y},\mathbf{h}))}{\partial h_i \partial h_j}\right]$

- The Fisher Information Matrix is written as

$$\mathbf{J} = \mathbf{J}_D + \mathbf{J}_{Pr},$$

where $\mathbf{J}_D$: Data information matrix, $\mathbf{J}_{Pr}$: Prior information matrix, $\mathbf{J}_{Pr} = \text{diag}\left(\frac{1}{\gamma_i}\right)$

- BCRB on the MSE in $\mathbf{h}$ becomes

$$E\left[\|\mathbf{h}-\hat{\mathbf{h}}\|^2\right] \geq \sum_{i=1}^{M}\left(\frac{P}{\sigma^2}+\frac{1}{\gamma_i}\right)^{-1} \triangleq \text{BCRB}_\Gamma$$

Pruned EM-SBL

- In practice, γ_i do not exactly go to zero in a finite number of iterations
- These small non-zero values contribute to the MSE
- MSE can be reduced by performing a pruning operation:

$$\Gamma_{EM-SBL} = \begin{cases} \gamma_i & \text{if} \quad |\hat{\mathbf{h}}(i)|^2 > \nu \\ 0 & \text{otherwise} \end{cases}$$

- Threshold : $\nu = K' \cdot$ BCRB

Simulation Results

- An ODFM system with 64 subcarriers and QPSK transmit symbols is considered
- The sparse fading channel (number of non zero taps $K = 6$) is assumed to be constant for one symbol
- The delay spread (M) assumed to be equal to the length of Cyclic Prefix ($M = 32$)
- MSE, SER and the Support recovery performance of the algorithm is simulated

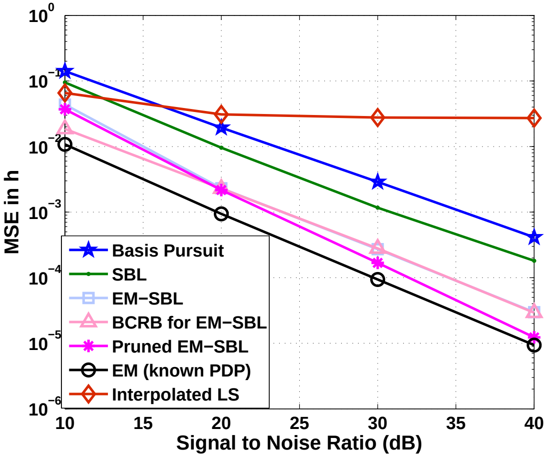


Figure: MSE with 17 pilot subcarriers

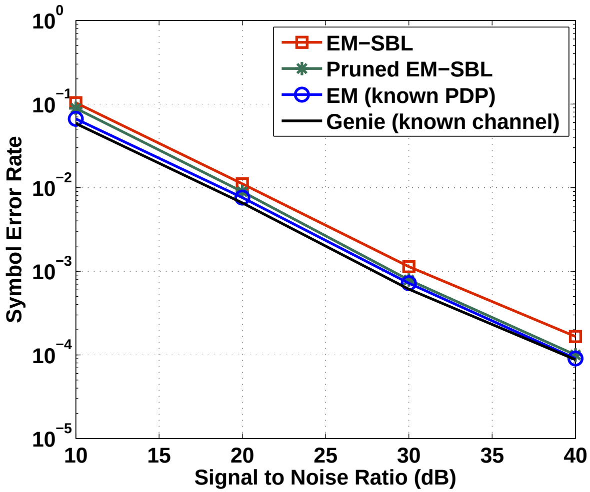


Figure: SER *vs.* SNR with 17 pilot subcarriers

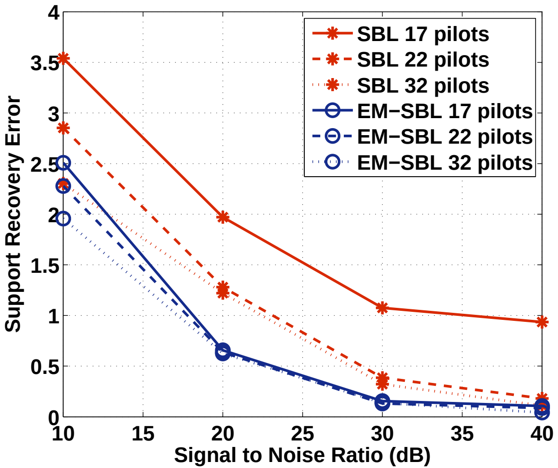


Figure: Support Recovery Error *vs.* SNR for the EM-SBL and the SBL algorithms

Conclusions

- SBL techniques were applied for sparse OFDM channel estimation
- The SBL algorithm was enhanced to obtain the EM-SBL algorithm
  - Joint channel estimation and data detection
- A pruning technique was proposed to further improve on the EM-SBL algorithm
- Simulations demonstrated the improved performace in MSE, SER and the support recovery of the sparse channel

Thank You

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