Interference Alignment for the K User Constant MIMO Interference Channel

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Outline

1. Preliminaries
2. IA Performance Measures
3. Precoder design for IA
4. Simulation Results
5. Conclusion

1. Preliminaries
2. IA Performance Measures
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Definition: Degrees of Freedom

- If at high SNR, the capacity scales with SNR as

$$C_\Sigma(SNR) = d_\Sigma \log(SNR) + o(\log(SNR))$$

- The DOF is defined as

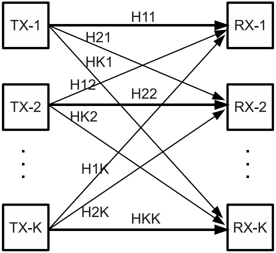
$$d_\Sigma \triangleq \lim_{\text{SNR}\to\infty} \frac{C(\text{SNR})}{\log(\text{SNR})}$$

- Special case: equal DOF per user: symmetric per-user DOF

$$d_{sym} = \lim_{SNR\to\infty} \frac{C_{sym}(SNR)}{\log(SNR)}$$

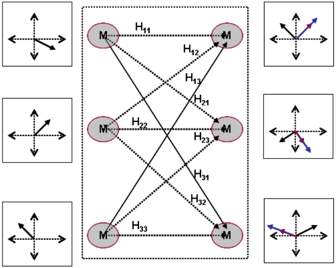
- Note: single user MIMO channel: d is the *rank* of channel matrix.

K-User interference channel



- $\mathbf{H}_{kj} \in \mathbb{C}^{N \times M}$: channel matrix from Tx j to Rx k
- Symbol extension: Concatenate S successive Tx/Rx symbols to get the $NS \times MS$ block diagonal *extended channel matrix* with the diagonals containing $\mathbf{H}_{kj}(t)$
- Time varying channel: $\mathbf{H}_{kj}(t)$ are different at each symbol time t
- Constant channel: $\mathbf{H}_{kj}(t) = \mathbf{H}_{kj}$ for S symbol transmit durations

IA for $K = 3$ user MIMO interference channel



- $S_k \in \mathbb{C}^{d \times 1}$: symbols to be transmitted from Tx k
- $V_k \in \mathbb{C}^{M \times d}$: precoding matrix used at Tx k
- The channel output at Rx-1:

$$\mathbf{y}_1 = \mathbf{H}_{11}\mathbf{V}_1\mathbf{s}_1 + \mathbf{H}_{12}\mathbf{V}_2\mathbf{s}_2 + \mathbf{H}_{13}\mathbf{V}_3\mathbf{s}_3 + \mathbf{z}_1$$

Necessary & Sufficient Conditions for IA ($M \times M$ Case)

Necessary conditions for IA:

$$\begin{aligned} span(\mathbf{H}_{12}\mathbf{V}_2) &= span(\mathbf{H}_{13}\mathbf{V}_3) \\ span(\mathbf{H}_{21}\mathbf{V}_1) &= span(\mathbf{H}_{23}\mathbf{V}_3) \\ span(\mathbf{H}_{31}\mathbf{V}_1) &= span(\mathbf{H}_{32}\mathbf{V}_2) \end{aligned}$$

Sufficient conditions for IA:

$$span(\mathbf{V}_1) = span(\mathbf{E}\mathbf{V}_1), \quad \mathbf{V}_2 = \mathbf{F}\mathbf{V}_1, \quad \mathbf{V}_3 = \mathbf{G}\mathbf{V}_1$$

where

$$\mathbf{E} = \mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21}$$

$$\mathbf{F} = \mathbf{H}_{32}^{-1}\mathbf{H}_{31} \text{ and } \mathbf{G} = \mathbf{H}_{23}^{-1}\mathbf{H}_{21}$$

Additional Condition: Linear Independence

- Previous conditions only guarantee alignment of *interference*
- In addition, need signal to be linearly independent of interference, i.e.,

$$\begin{aligned} \text{rank}([H_{11}V_1 \quad H_{12}V_2]) &= M \text{ at Rx-1} \\ \text{rank}([H_{22}V_2 \quad H_{23}V_3]) &= M \text{ at Rx-2} \\ \text{rank}([H_{33}V_3 \quad H_{32}V_2]) &= M \text{ at Rx-3} \end{aligned}$$

- Assumptions:
  - H_{kj} drawn from continuous distribution
  - Global channel knowledge at all nodes

Achievable DOF: Accounting for Dimensionalities

Let the desired DOF per user $= d$

- Total dimension available at j^{th} receiver $= N$
- Dimension occupied by the desired signal $= d$
- Dimension remaining for interference $= N - d$
- Dimension occupied by interference without IA $= (K - 1)d$
- If n users are aligned, the dim. of intf. subspace $= (K - n - 1)d$

Therefore, to achieve d DOF per user, it is sufficient that

$$(K - n - 1)d \leq N - d, \text{ i.e., } n \geq K - \frac{N}{d}.$$

Feasibility of IA

IA with d DOF per user is feasible if there exist precoding matrices $V_k \in \mathbb{C}^{M \times d}$, and receive filtering matrix $W_k \in \mathbb{C}^{N \times d}$ such that,

$$\begin{aligned} \mathbf{W}_k^H \mathbf{H}_{kj} \mathbf{V}_j &= \mathbf{0},\ j = 1, 2, \ldots, K, j \neq k, \\ \text{rank}\left(\mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k \right) &= d \end{aligned}$$

Extension to case with symbol extensions is straightforward.

Sufficient conditions for IA

- A sufficient condition for IA

$$\begin{aligned} \mathbf{H}_{12}\mathbf{V}_2 &= \mathbf{H}_{13}\mathbf{V}_3 && \text{(at RX-1)} \\ \mathbf{H}_{21}\mathbf{V}_1 &= \mathbf{H}_{23}\mathbf{V}_3 && \text{(at RX-2)} \\ \mathbf{H}_{31}\mathbf{V}_1 &= \mathbf{H}_{32}\mathbf{V}_2 && \text{(at RX-3)} \end{aligned}$$

- $K(K-2)Nd$ linear equations with KMd unknowns.
- There exists a solution with probability one if $M \geq (K-2)N$
- If n interfering users are aligned, the achievable DOF per user

$$d = \frac{N}{K-n}$$

- Thus, we can construct *linear* IA conditions for n user IA

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Performance Measure – 1

1. Fraction of the interfering signal power in the desired signal subspace, denoted p_k:

$$p_k \triangleq \frac{\sum_{j=1}^{d_k} \lambda_j[\mathbf{Q}_k]}{\mathrm{trace}\left(\mathbf{Q}_k\right)},$$

where $\mathbf{Q}_k$ is the interference covariance matrix at receiver k, and $\lambda_j[\mathbf{Q}_k]$ is the j^{th} smallest eigenvalue of $\mathbf{Q}_k$.

- When the IA is perfect, $p_k = 0$
- A small value of p_k indicates a better IA

Performance Measure – 2

2. Relative power of the weakest desired data stream, denoted q_k:

$$q_k \triangleq \frac{\sigma_{d_k}^2\left[\mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k\right]}{\sum_{l=1}^{d_k} \sigma_l^2\left[\mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k\right]}$$

where, $\sigma_l\left[\mathbf{A}\right]$ represents the l^{th} largest singular value of $\mathbf{A}$.

- $0 \leq q_k \leq 1/d_k$
- A non-zero value of q_k ensures the desired DOF per user is attained
- Loosely speaking, q_k close to $1/d_k$ results in the same data rate on all the desired data streams

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Eigen Beamforming

A sufficient condition for *one user* IA for the $K = 4$ user interference channel

$$\begin{aligned} \mathbf{H}_{41}\mathbf{V}_1 &= \mathbf{H}_{42}\mathbf{V}_2 + \mathbf{H}_{43}\mathbf{V}_3 \\ \mathbf{H}_{32}\mathbf{V}_2 &= \mathbf{H}_{31}\mathbf{V}_1 + \mathbf{H}_{34}\mathbf{V}_4 \\ \mathbf{H}_{23}\mathbf{V}_3 &= \mathbf{H}_{21}\mathbf{V}_1 + \mathbf{H}_{24}\mathbf{V}_4 \\ \mathbf{H}_{14}\mathbf{V}_4 &= \mathbf{H}_{12}\mathbf{V}_2 + \mathbf{H}_{13}\mathbf{V}_3 \end{aligned}$$

which can be written as

$$\tilde{\mathbf{H}}\mathbf{V} = 0$$

Eigen Beamforming Algorithm

- $\mathbf{V} \triangleq [\mathbf{V}_1; \mathbf{V}_2; \mathbf{V}_3; \mathbf{V}_4]^T \in \mathbf{C}^{KM \times d}$ and

$$\tilde{\mathbf{H}} \triangleq \begin{bmatrix} -\mathbf{H}_{41} & \mathbf{H}_{42} & \mathbf{H}_{43} & \underline{\mathbf{0}} \\ \mathbf{H}_{31} & -\mathbf{H}_{32} & \underline{\mathbf{0}} & \mathbf{H}_{34} \\ \mathbf{H}_{21} & \underline{\mathbf{0}} & -\mathbf{H}_{23} & \mathbf{H}_{24} \\ \underline{\mathbf{0}} & \mathbf{H}_{12} & \mathbf{H}_{13} & -\mathbf{H}_{14} \end{bmatrix}$$

- Instead of exact solution to $\tilde{\mathbf{H}}\mathbf{V} = 0$, can look for MMSE
- MMSE solution to V is given by

$$V^o = eig_{min}[\mathbf{Q}]$$

where $Q = \tilde{\mathbf{H}}^H \tilde{\mathbf{H}}$.

Eigen Beamforming Algorithm

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- Instead of exact solution to $\tilde{\mathbf{H}}\mathbf{V} = 0$, can look for MMSE
- MMSE solution to V is given by

$$V^o = eig_{min}[\mathbf{Q}]$$

  where $Q = \tilde{\mathbf{H}}^H \tilde{\mathbf{H}}$.
- However, this solution does not guarantee that the dimension of the desired signal is d_S!

Constrained Least Squares Formulation

- To preserve the desired signal dimension, we solve the constrained optimization problem:

$$\min_{\mathbf{V}} \|\tilde{\mathbf{H}}\mathbf{V}\|_F^2 \quad \text{sub. to} \quad \text{rank}(\mathbf{H}_{kk}\mathbf{V}_k) = d$$

- That is,

$$\min_{\mathbf{V}} J(\mathbf{V}) \triangleq \text{trace}\left(\mathbf{V}^H\mathbf{Q}\mathbf{V}\right) \text{ sub. to } \mathbf{H}\mathbf{V} = \mathbf{b},\ k = 1, 2, \ldots, K$$

where $\mathbf{b} \triangleq [\mathbf{b}_1; \mathbf{b}_2; \ldots; \mathbf{b}_K]^T$, with $\mathbf{b}_k \in \mathbb{C}^{N \times d}$ being full rank matrices

Solution to the Constrained Least Squares Problem

- The matrix $\mathbf{H} \in \mathbb{C}^{KN \times KM}$ is a full rank matrix given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & 0 & \dots & 0 \\ 0 & \mathbf{H}_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{H}_{kk} \end{bmatrix},$$

- The solution to the optimization problem is

$$\mathbf{V}_0 = \mathbf{Q}^{-1}\mathbf{H}^H\mathbf{T}^\dagger\mathbf{b}$$

where $\mathbf{T} \triangleq \left[\mathbf{H}\mathbf{Q}^{-1}\mathbf{H}^H\right]$

Proof of the Solution

- Suppose the opt. soln. is $\mathbf{V}_n = \mathbf{V}_0 + \Delta$, such that $\mathbf{H}\mathbf{V}_n = \mathbf{b}$
- The objective function is given by:

$$J(\mathbf{V}_n) = \text{trace}\left([\mathbf{V}_0 + \Delta]^H \mathbf{Q}[\mathbf{V}_0 + \Delta]\right)$$
$$= \text{trace}\left(\mathbf{V}_0^H \mathbf{Q}\mathbf{V}_0 + \Delta^H \mathbf{Q}\Delta + \Delta^H \mathbf{Q}\mathbf{V}_0 + \mathbf{V}_0^H \mathbf{Q}\Delta\right)$$

- From the constraint $\mathbf{H}\mathbf{V}_n = \mathbf{b}$, clearly, $\mathbf{H}\Delta = \mathbf{0}$, and

$$\Delta^H \mathbf{Q}\mathbf{V}_0 = \Delta^H \mathbf{Q}\mathbf{Q}^{-1}\mathbf{H}^H \mathbf{T}^\dagger \mathbf{b} = \mathbf{0}$$

- Similarly,

$$\mathbf{V}_0^H \mathbf{Q}\Delta = \mathbf{b}^H \mathbf{T}^{\dagger H} \mathbf{H}\mathbf{Q}^{-1}\mathbf{Q}\Delta = \mathbf{0}$$

- Note that $\text{trace}(\Delta^H \mathbf{Q}\Delta) \geq 0$ since $\mathbf{Q}$ is positive definite.
- Hence, $J(\mathbf{V}_0) \leq J(\mathbf{V}_n)$ and $\mathbf{V}_0$ is the optimum solution.

Constrained Least Squares – 2

- Recall, the constrained optimization problem to be solved

$$\min_{\mathbf{V}} \|\tilde{\mathbf{H}}\mathbf{V}\|_F^2 \quad \text{subject to} \quad \mathbf{H}\mathbf{V} = \mathbf{b}.$$

- From the results of Generalized SVD (GSVD) of $[\tilde{\mathbf{H}}; \mathbf{H}]$,

$$\begin{aligned} \tilde{\mathbf{H}} &= \mathbf{U}\mathbf{C}\mathbf{X}^H \\ \mathbf{H} &= \mathbf{G}\mathbf{D}\mathbf{X}^H \end{aligned}$$

  - $\mathbf{U}, \mathbf{G} \in \mathbb{C}^{KN \times KN}$ are unitary matrices
  - $\mathbf{X} \in \mathbb{C}^{KM \times KM}$ is a full rank matrix
  - $\mathbf{C}$ and $\mathbf{D} \in \mathbb{C}^{KN \times KM}$ are diagonal matrices.
- Using the unitary transformation, we restate the problem as

$$\min_{\mathbf{V}} \|\mathbf{U}^H\tilde{\mathbf{H}}\mathbf{V}\|_F^2 \quad \text{subject to} \quad \mathbf{G}^H\mathbf{H}\mathbf{V} = \mathbf{G}^H\mathbf{b}$$

Constrained Least Squares: Solution – 2

- From the above, we have

$$\min_{\mathbf{V}} \|\mathbf{C}\mathbf{X}^H\mathbf{V}\|_F^2 \quad \text{subject to} \quad \mathbf{D}\mathbf{X}^H\mathbf{V} = \mathbf{G}^H\mathbf{b}$$

- The GSVD solution from [Golub]

$$\mathbf{V}_{0,j} = \sum_{i=1}^{KMS} \frac{\mathbf{G}_i^H\mathbf{b}_j}{\mathbf{D}_{ii}}\mathbf{W}_i, \quad j = 1, 2, \ldots, d_S,$$

where $\mathbf{A}_i$ is the i^{th} column of $\mathbf{A}$ and $\mathbf{W} \triangleq [\mathbf{X}^H]^{-1}$

- Note: for the proposed Eigen beamforming
  - Global knowledge of all K^2 links is required, i.e.,
  - Centralized algorithm
  - The iterative algorithm (discussed next) addresses this

Distributed Iterative Algorithm

- Consider the $K-$user interference channel. The interference + noise term at *Rx*-*k* is

$$\mathbf{IR}_k = \sum_{j=1, j\neq k}^{K} \mathbf{W}_k{}^H \mathbf{H}_{kj} \mathbf{V}_j \mathbf{s}_j + \mathbf{W}_k{}^H \mathbf{Z_k}$$

- When IA is feasible, we can find a $\mathbf{W}_k$. such that

$$\sum_{j=1, j\neq k}^{K} \mathbf{W}_k{}^H \mathbf{H}_{kj} \mathbf{V}_j = 0$$

- With dimensionality constr. rank $\left(\mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k\right) = d_k$, we have

$$\min_{\mathbf{W}_k} J_k \triangleq \text{trace}(\mathbf{W}_k^H \mathbf{Q}_k \mathbf{W}_k) \quad \text{subject to} \quad \mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k = \alpha \mathbf{I}_{d_k}$$

where α is selected to obtain: trace $\left(\mathbf{W}_k^H \mathbf{W}_k\right) = 1$

Precoder Design: Distributed Iterative Algorithm

- In the above, $\mathbf{Q}_k$ is the interference plus noise covariance matrix at receiver k, defined as,

$$\mathbf{Q}_k = \sum_{j=1, j\neq k}^{K} [\mathbf{H}_{kj}\mathbf{V}_j][\mathbf{H}_{kj}\mathbf{V}_j]^H + \mathbf{I}_N$$

- The solution that minimizes J_k is given by,

$$\mathbf{W}_k^{opt} = \alpha \mathbf{Q}_k^{-1}\mathbf{U}_k[\mathbf{U}_k^H \mathbf{Q}_k^{-1}\mathbf{U}_k]^{-1}$$

$\mathbf{U}_k \triangleq \mathbf{H}_{kk}\mathbf{V}_k$ is the desired signal subspace of the k^{th} user

$$\alpha = \frac{1}{\sqrt{\text{trace}\left([\mathbf{Q}_k^{-1}\mathbf{U}_k[\mathbf{U}_k^H\mathbf{Q}_k^{-1}\mathbf{U}_k]^{-1}]^H[\mathbf{Q}_k^{-1}\mathbf{U}_k[\mathbf{U}_k^H\mathbf{Q}_k^{-1}\mathbf{U}_k]^{-1}]\right)}}$$

Iterative Algorithm: Optimum Receive Filter

Lemma

For the constrained optimization problem,

$$\min_{\mathbf{G}} J(\mathbf{G}) \triangleq \text{trace}\left(\mathbf{G}^H\mathbf{Q}\mathbf{G}\right) \quad \text{subject to} \quad \mathbf{A}\mathbf{G} = \alpha\mathbf{I_d}$$

$\mathbf{G} \in \mathbb{C}^{M\times d}$, $\mathbf{Q} \in \mathbb{C}^{M\times M} > 0$ and $\mathbf{A} \in \mathbb{C}^{d\times M}$. The optimum solution is

$$\mathbf{G_0} = \alpha\mathbf{Q}^{-1}\mathbf{A}^H[\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^H]^{-1}.$$

Proof: Omitted.

Iterative Algorithm: Optimum Precoder Matrices

- The interfering signal due to transmitter k at the unintended receivers,

$$\mathbf{IT}_k = \mathbf{W}_j^H \mathbf{H}_{jk} \mathbf{V}_k \mathbf{s}_k, \quad j = 1, 2, \ldots, K, j \neq k$$

- Feasibility condition for perfect IA,

$$\mathbf{W}_j^H \mathbf{H}_{jk} \mathbf{V}_k = 0, \quad j = 1, 2, \ldots, K, j \neq k$$

- The interference power due to transmitter k at receiver j,

$$\mathbf{LI}_{kj} = \text{trace}\left(\mathbf{V}_k^H [\mathbf{W}_j^H \mathbf{H}_{jk}]^H [\mathbf{W}_j^H \mathbf{H}_{jk}] \mathbf{V}_k\right)$$

$\Rightarrow$ Obtain $\mathbf{V}_k$ which minimizes the total interference power due to transmitter k

Iterative Algorithm: Optimum Precoder Matrices

The optimization problem is given by,

$$\min_{\mathbf{V}_k} L_k = \text{trace}\left(\mathbf{V}_k^H \mathbf{R}_k \mathbf{V}_k\right), \quad \text{subject to} \quad \mathbf{W}_k^H \mathbf{H}_{kk} \mathbf{V}_k = \beta \mathbf{I}_{d_k}$$

where β is selected to obtain trace $\left(\mathbf{V}_k^H \mathbf{V}_k\right) = 1$.

$$\mathbf{R}_k = \sum_{j=1, j \neq k}^{K} [\mathbf{W}_j^H \mathbf{H}_{jk}]^H [\mathbf{W}_j^H \mathbf{H}_{jk}] + \mathbf{I}_{d_k}$$

The optimum solution for $\mathbf{V}_k$ is given by

$$\mathbf{V}_k^{opt} = \beta \mathbf{R}_k^{-1} \mathbf{T}_k^H [\mathbf{T}_k \mathbf{R}_k^{-1} \mathbf{T}_k^H]^{-1}, \quad k = 1, 2, \ldots, K,$$

where $\mathbf{T}_k \triangleq \mathbf{W}_k^H \mathbf{H}_{kk}$.

Precoder Design: Distributed Iterative Algorithm

Step. No.	Action
1	Initialize $\mathbf{V}_k, k = 1, 2, \ldots, K$, arbitrary matrices
2	Compute the matrix $\mathbf{Q}_k$ for $k = 1, 2, \ldots, K$
3	Obtain $\mathbf{W}_k, k = 1, 2, \ldots, K$
4	Compute the matrix $\mathbf{R}_k$
5	Obtain $\mathbf{V}_k, k = 1, 2, \ldots, K$
6	Repeat steps $2 - 5$ until convergence

Table: The iterative precoder design algorithm

Distributed Iterative Algorithm: Convergence

The total interference plus noise power across all receivers

$$P_R = \text{trace}\left(\sum_{k=1}^{K}\left[\sum_{j=1, j\neq k}^{K} \mathbf{W}_k^H[\mathbf{H}_{kj}\mathbf{V}_j][\mathbf{H}_{kj}\mathbf{V}_j]^H\mathbf{W}_k + \mathbf{W}_k^H\mathbf{W}_k\right]\right)$$

$$= \text{trace}\left(\sum_{k=1}^{K}\left[\sum_{j=1}^{K}[\mathbf{W}_k^H\mathbf{H}_{kj}\mathbf{V}_j][\mathbf{W}_k^H\mathbf{H}_{kj}\mathbf{V}_j]^H\right]\right) + K^2 - \alpha K d_k$$

Similarly, the total interference power due to all transmitters

$$P_T = \text{trace}\left(\sum_{k=1}^{K}\left[\sum_{j=1}^{K}[\mathbf{W}_j^H\mathbf{H}_{jk}\mathbf{V}_k]^H[\mathbf{W}_j^H\mathbf{H}_{jk}\mathbf{V}_k]\right]\right) + K^2 - \beta K d_k$$

Distributed Iterative Algorithm: Convergence

- Thus, P_R and P_T represent the same objective function
- The objective function reduces at each iteration and is bounded below by zero
- Hence the alternate iteration on the two objective functions is guaranteed to converge

Discussion on the distributed iterative algorithm

- Need interference covariance matrix at each Rx
- Need intf. cov. matrix of the *virtual reverse channel* at each Tx
- Distributed algorithm
- IA performace better than existing distributed algos
- Desired signal DOF is ensured by constraining the dimension of desired signal subspace

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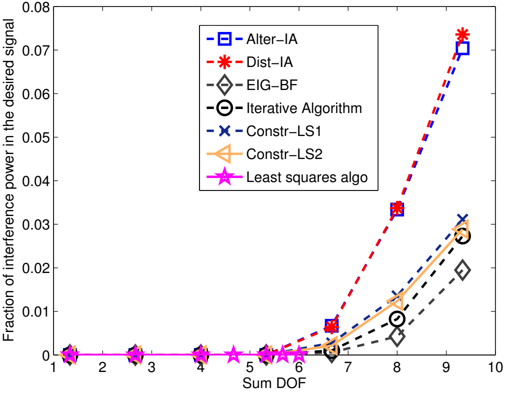
Simulation Results

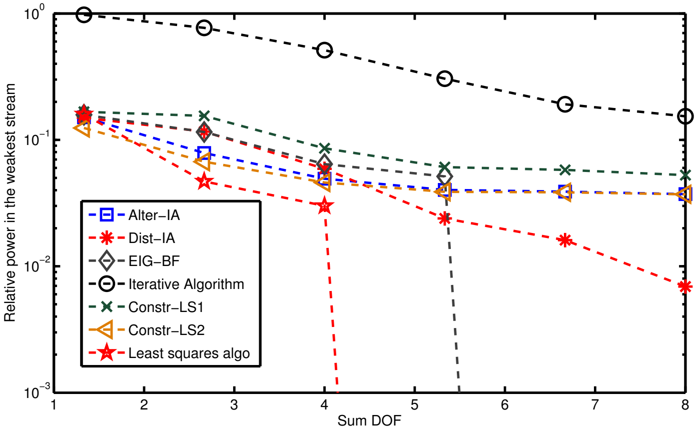
- The performance of the IA algorithms are compared with
  1. Distributed interference alignment algorithm
  2. Alternating minimization algorithm
  3. Least squares algorithm
- The sum DOF obtained $= \sum_{k=1}^{K} \text{rank}\left(\mathbf{H}_{kk}\mathbf{V}_k\right)$
- The simulation results for both the performance metrics proposed earlier are obtained

Simulation Results – 1



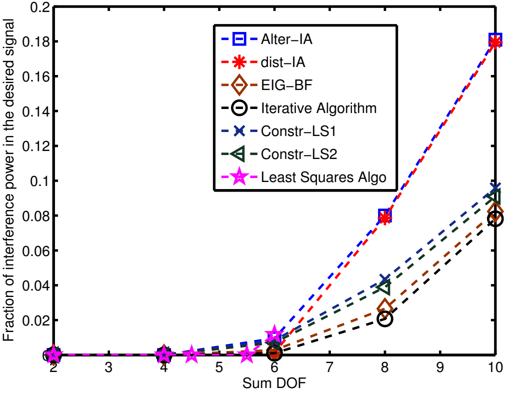
Setup: $K = 4$, $M = 3$, $N = 6$, $S = 3$, DOF outerbound $= 8$

Simulation Results – 2



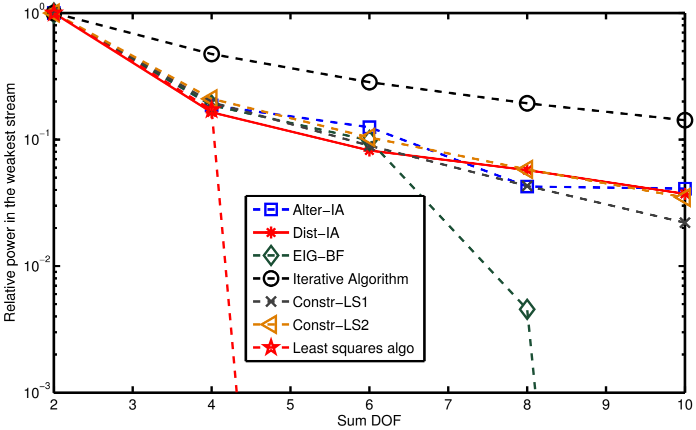
Setup: $K = 4$, $M = 3$, $N = 6$, $S = 3$, DOF outerbound $= 8$

Simulation Results-3



Setup: $K = 4$, $M = 4$, $N = 5$, $S = 2$, DOF outerbound $= 8.88$

Simulation Results-4



Setup: $K = 4$, $M = 4$, $N = 5$, $S = 2$, DOF outerbound $= 8.88$

Summary

- The existence of IA solution and the feasible number of degrees of freedom for a given (K, M, N) system was obtained
- A new outerbound on the DOF was derived, and shown to be tighter than the best existing bounds for certain values of (K, M, N)
- Performance metric that measures the interference suppression and DOF achieved by the algorithm were proposed
- Two numerical algorithms were proposed for designing the interference alignment precoding and receive filtering matrices
- Simulation results confirmed the superior performance of the proposed algorithms compared to existing methods

Future Work

- Interference alignment design with individual user power constraints
- The performance of interference alignment with imperfect CSIT, or with imperfect IA
- A study of the finite-SNR performance of the IA algorithms
- Characterizing the DOF *region* for K user interference channel

Thank You !