Joint Data Detection and Dominant Singular Mode Estimation in Time Varying Reciprocal MIMO Systems

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Abstract

- Aim: Joint data detection and tracking of the dominant singular mode of a time varying channel at the transmitter and receiver of a TDD-MIMO beamforming system
- Proposed algorithm: Modified Expectation Maximization
- Mitigating error propagation: Constraining the estimates within a confidence sphere of the previous estimate.
- Performance: Symbol Error Rate (SER) and the Mean Square Inner Product (MSIP) between the estimated and the true singular vector.

System Model and Transmission Protocol

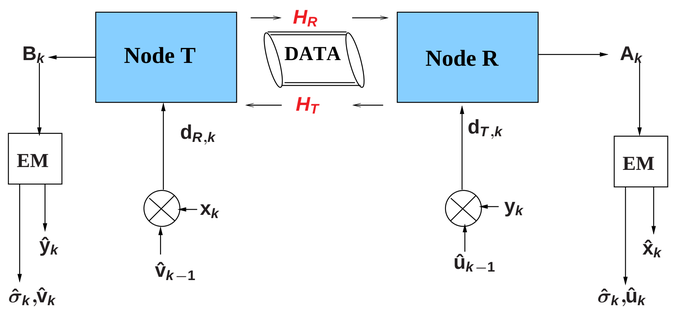


Figure: Pictorial representation of the transmission protocol.

- The input-output equations are given by,

$$\mathbf{A}_k = \mathbf{H}_R \mathbf{d}_{R,k} + \mathbf{W}_{R,k} \qquad \mathbf{B}_k = \mathbf{H}_T \mathbf{d}_{T,k} + \mathbf{W}_{T,k},$$

- Transmission Frame:

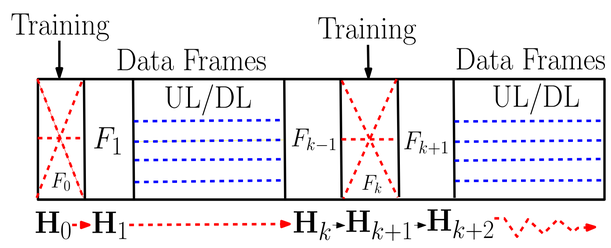


Figure: The channel remains constant within each frame. $F_0, F_1, \ldots, F_k$ are training frames.

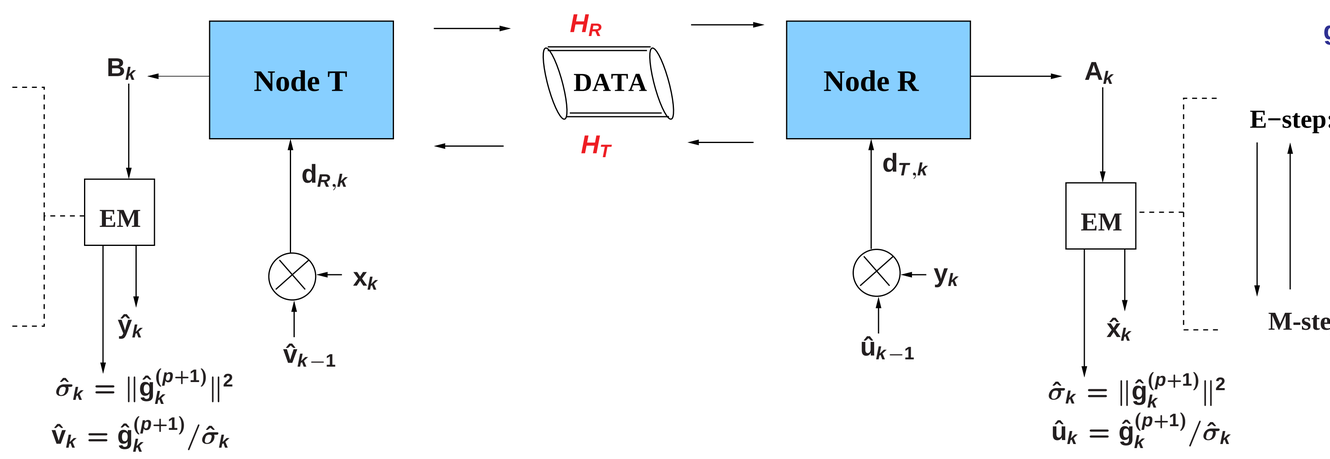
EM Algorithm Over a Single Frame

$\mathbf{g}_k \triangleq \mathbf{H}_R \hat{\mathbf{v}}_{k-1}$

E-step: $\hat{\mathbf{y}}_k \triangleq \mathbb{E}(\mathbf{y}_k) = \dfrac{\sum_{\mathbf{y}_k \in \mathcal{C}^N} \mathbf{y}_k \mathcal{P}(\mathbf{A}_k/\mathbf{y}_k, \hat{\mathbf{g}}_k^{(p)})}{\sum_{\mathbf{y}_k \in \mathcal{C}^N} \mathcal{P}(\mathbf{A}_k/\mathbf{y}_k, \hat{\mathbf{g}}_k^{(p)})}$

$\mathbb{E}(|\mathbf{y}_k|^2) = \dfrac{\sum_{\mathbf{y}_k \in \mathcal{C}^N} |\mathbf{y}_k|^2 \mathcal{P}(\mathbf{A}_k/\mathbf{y}_k, \hat{\mathbf{g}}_k^{(p)})}{\sum_{\mathbf{y}_k \in \mathcal{C}^N} \mathcal{P}(\mathbf{A}_k/\mathbf{y}_k, \hat{\mathbf{g}}_k^{(p)})}$

M-step: $\hat{\mathbf{g}}_k^{(p+1)} = \dfrac{\sum_{n=1}^{N} \mathbb{E}(\mathbf{y}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{y}_k(n)|^2)}$



$\mathbf{g}_k \triangleq \mathbf{H}_T \hat{\mathbf{u}}_{k-1}$

E-step: $\hat{\mathbf{x}}_k \triangleq \mathbb{E}(\mathbf{x}_k) = \dfrac{\sum_{\mathbf{x}_k \in \mathcal{C}^N} \mathbf{x}_k \mathcal{P}(\mathbf{A}_k/\mathbf{x}_k, \hat{\mathbf{g}}_k^{(p)})}{\sum_{\mathbf{x}_k \in \mathcal{C}^N} \mathcal{P}(\mathbf{A}_k/\mathbf{x}_k, \hat{\mathbf{g}}_k^{(p)})}$

$\mathbb{E}(|\mathbf{x}_k|^2) = \dfrac{\sum_{\mathbf{x}_k \in \mathcal{C}^N} |\mathbf{x}_k|^2 \mathcal{P}(\mathbf{A}_k/\mathbf{x}_k, \hat{\mathbf{g}}_k^{(p)})}{\sum_{\mathbf{x}_k \in \mathcal{C}^N} \mathcal{P}(\mathbf{A}_k/\mathbf{x}_k, \hat{\mathbf{g}}_k^{(p)})}$

M-step: $\hat{\mathbf{g}}_k^{(p+1)} = \dfrac{\sum_{n=1}^{N} \mathbb{E}(\mathbf{x}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{x}_k(n)|^2)}$

- E-step : $Q(\mathbf{g}_k/\hat{\mathbf{g}}_k^{(p)}) = E_{\mathbf{x}_k/\mathbf{A}_k, \hat{\mathbf{g}}_k^{(p)}}[\log \mathcal{P}(\mathbf{A}_k, \mathbf{x}_k/\mathbf{g}_k)]$, at Node T or $E_{\mathbf{y}_k/\mathbf{A}_k, \hat{\mathbf{g}}_k^{(p)}}[\log \mathcal{P}(\mathbf{A}_k, \mathbf{y}_k/\mathbf{g}_k)]$, at Node R
- M-step : $\hat{\mathbf{g}}_k^{(p+1)} = \arg\max_{\mathbf{g}_k} Q(\mathbf{g}_k/\hat{\mathbf{g}}_k^{(p)})$

EM Algorithm for Channel Tracking Over Multiple Frames

- Modified M-step : $\hat{\mathbf{g}}_k^{(p+1)} = \arg\max_{\mathbf{g}_k} Q(\mathbf{g}_k/\hat{\mathbf{g}}_k^{(p)}) + \lambda(\epsilon - \|\hat{\mathbf{g}}_{k-1} - \mathbf{g}_k\|^2)\mathcal{I}_{(\|\mathbf{g}_{k-1} - \hat{\mathbf{g}}_k^{(p+1)}\|^2 > \epsilon)}$.

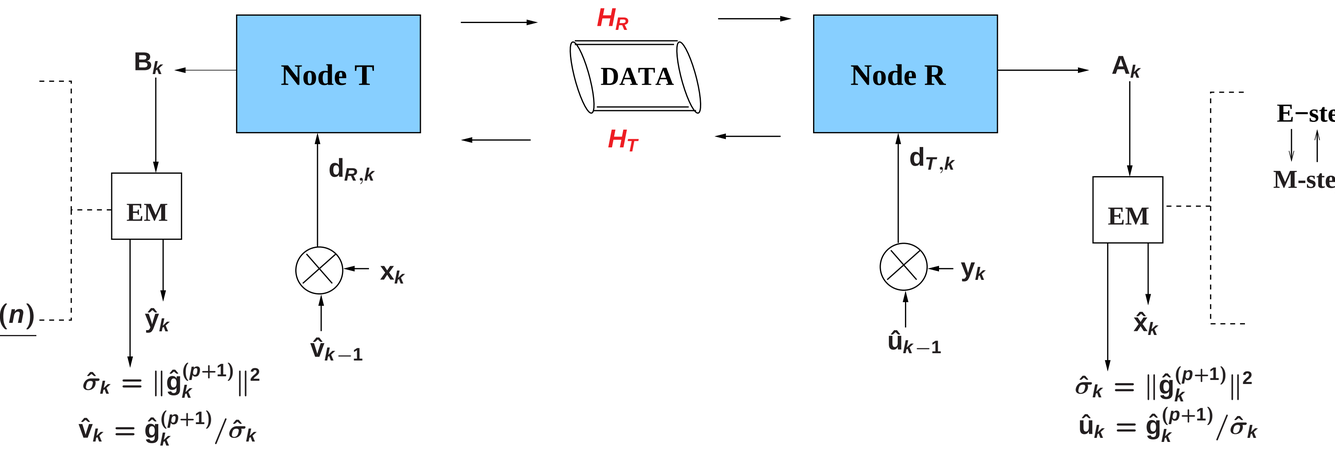
E-step: Compute $\hat{\mathbf{y}}_k \triangleq \mathbb{E}(\mathbf{y}_k)$ and $\mathbb{E}(|\mathbf{y}_k|^2)$
(Same as the single frame case)

M-step: If $\|\hat{\mathbf{g}}_{k-1} - \hat{\mathbf{g}}_k\|^2 \leq \epsilon$,

$$\hat{\mathbf{g}}_k^{(p+1)} = \frac{\sum_{n=1}^{N} \mathbb{E}(\mathbf{y}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{y}_k(n)|^2)}$$

else

$$\hat{\mathbf{g}}_k^{(p+1)} = \frac{\lambda\sigma^2 \hat{\mathbf{g}}_{k-1} + \sum_{n=1}^{N} \mathbb{E}(\mathbf{y}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{y}_k(n)|^2) + \lambda\sigma^2}$$



E-step: Compute $\hat{\mathbf{x}}_k \triangleq \mathbb{E}(\mathbf{x}_k)$ and $\mathbb{E}(|\mathbf{x}_k|^2)$

M-step:

If $\|\hat{\mathbf{g}}_{k-1} - \hat{\mathbf{g}}_k\|^2 \leq \epsilon$,

$$\hat{\mathbf{g}}_k^{(p+1)} = \frac{\sum_{n=1}^{N} \mathbb{E}(\mathbf{x}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{x}_k(n)|^2)}$$

else

$$\hat{\mathbf{g}}_k^{(p+1)} = \frac{\lambda\sigma^2 \hat{\mathbf{g}}_{k-1} + \sum_{n=1}^{N} \mathbb{E}(\mathbf{x}_k(n))^* \mathbf{A}_k(n)}{\sum_{n=1}^{N} \mathbb{E}(|\mathbf{x}_k(n)|^2) + \lambda\sigma^2}$$

Simulation Results: Symbol Error Rate

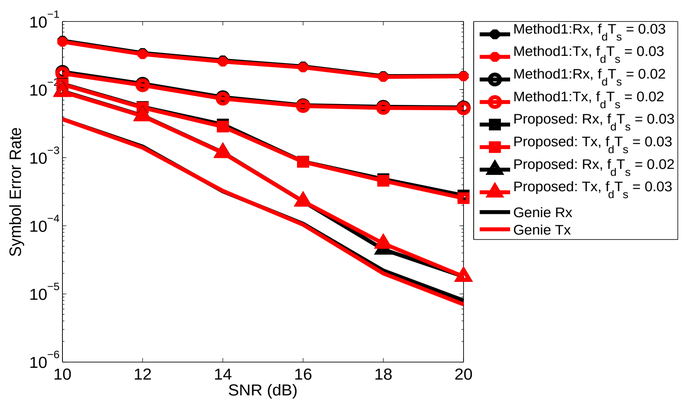


Figure: SER versus SNR in dB for various fade rates.

Simulation Results: MSIP

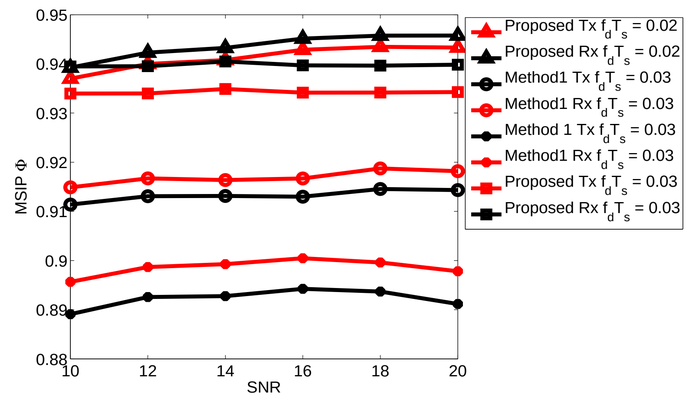


Figure: MSIP versus SNR for various fade rates.

Study of influence of ϵ

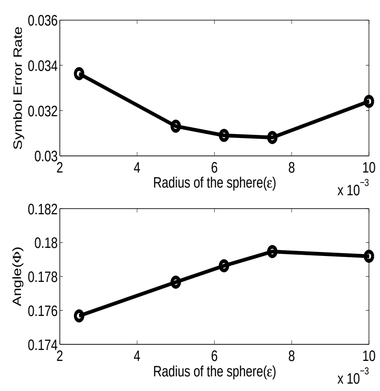


Figure: SER and MSIP versus ϵ, $f_dT_s = 0.01$, $SNR = 0dB$.

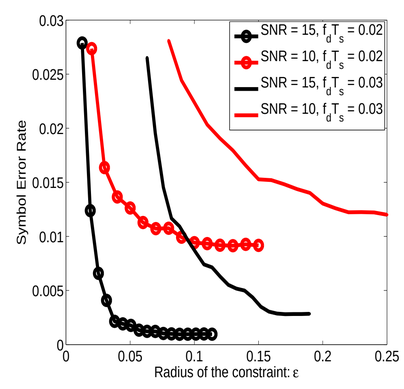


Figure: SER versus ϵ for various fade rates.

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