Journal Watch:

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Saurabh Khanna,
Signal Processing for Communication, ECE, IISc

Compressive Shift Retreival.

Authors: Henrik Ohlsson, Yonina C. Eldar, Allen Y. Yang, S. Shankar Shastry

Affiliations: Linkoping Univ., Technion and Univ. of Calif., Berkely

Compressive Shift Retreival

- **Problem Statement:**
  - Cyclic shift retrieval problem
  - $\mathbf{y}$ is related to $\mathbf{x}$ via a s- cyclic shift i.e., $\mathbf{y} = \mathbf{D}^s\mathbf{x}$
  - Want to recover unknown cyclic shift s, from compressive measurements $\mathbf{z} = \mathbf{Ay}$ and $\mathbf{v} = \mathbf{Ax}$, with $\mathbf{A} = m \times n$ matrix $(m \leq n)$
- **Main results:**
  - $\mathbf{X}$ = matrix with columns as cyclic shifted versions of $\mathbf{x}$
  - Ideally, to recover s, we would like to solve

$$\min_{\mathbf{q}\in\{0,1\}^n} ||\mathbf{Ay} - \mathbf{AXq}||_2^2 \quad \text{s.t.} \quad ||\mathbf{q}||_0 = 1 \tag{1}$$

  - For noiseless case, if measurement matrix $\mathbf{A}$ satisfies the following conditions:
    1. $\mathbf{A}^H\mathbf{D}^s = \mathbf{D}^s\mathbf{A}^H\mathbf{A}$
    2. $\exists\alpha \in \mathbb{R}, \alpha\mathbf{AA}^H = \mathbf{I}$
    3. all columns of $\mathbf{AX}$ are different

    then $\arg\max_s \; Re < \mathbf{z}, \; \mathbf{AD}^s\mathbf{A}^H\mathbf{v} >$ recovers the true shift. Perfect recovery if $\mathbf{A}^H\mathbf{A}$ and $\mathbf{D}^s$ commute.
  - Partial Fourier matrices satisfy conditions (1), (2) and (3)
  - For noisy case, $\tilde{\mathbf{z}} = \mathbf{z} + \mathbf{e}_z$ and $\tilde{\mathbf{v}} = \mathbf{v} + \mathbf{e}_v$, perfect recovery if ℓ_2 norm difference between any two columns of $\mathbf{A}\tilde{\mathbf{X}}$ is greater than $(||\mathbf{e}_z||_2 + ||\mathbf{e}_v||_2)$

Online Algorithm for Separating Sparse and Low -Dimensional Signal Sequences From Their Sum

Authors: Han Guo, Chenlu Qiu, and Namrata Vasvani

Affiliations: Iowa State University

Online Algorithm for Separating Sparse and Low -Dimensional Signal Sequences From Their Sum

- **Problem Statement:**
  - Online algorithm for recovering sequence of sparse vectors S_t and dense vectors L_t from their sum $M_t = S_t + L_t$
  - Additional structure across time:
    1. For sparse S_t, few new indices are added (few existing ones drop off) at each time epoch.
    2. For dense L_t, we have $L_t = \mathbf{P}_t \mathbf{a}_t$, $\mathbf{P}_t$ is the slow time varying basis matrix with time evolution given by

$$\mathbf{P}_j = [(\mathbf{P}_{j-1}\mathbf{R} \backslash \mathbf{P}_{j,old}), \mathbf{P}_{j,new}]$$

- **Main steps of online algorithm (PRAC-REPRO-CS):**
  1. Perpendicular projection: $\mathbf{y}_t = \Phi_t M_t$, where $\Phi_t = (\mathbf{I} - \mathbf{P}_t \mathbf{P}_t^T)$
  2. Sparse recovery of S_t: assume $\mathbf{y}_t = \Phi_t S_t + \Phi_t L_t$

$$\min_{\mathbf{x}} \lambda ||\mathbf{x}_T||_1 + ||\mathbf{x}_{T^C}||_1 \quad \text{s.t.} \quad ||\mathbf{y}_t - \Phi_t \mathbf{x}||_2 \leq \epsilon, \; T = supp(\hat{S}_{t-1})$$

  3. Recover L_t: $\hat{L}_t = M_t - \hat{S}_t$
  4. Update basis matrix P_t:
     - add new (drop old) basis vectors obtained from incremental SVD of latest batch of estimates $\hat{L}_t$.

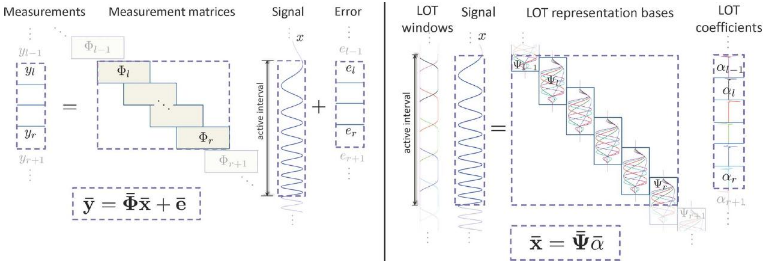
Sparse Recovery of Streaming Signals Using ℓ_1- Homotopy.

Authors: M.Salman Asif and Justin Romberg

Affiliations: Rice University and Georgia Tech

Sparse Recovery of Streaming Signals Using ℓ_1- Homotopy (1/2)

- **Problem Statement:**
  - Considers streaming data i.e, signal changes over time and, it is measured and reconstructed sequenctially over small intervals.
  - Signal measurement and reconstruction windows can be overlapping in time.



  - Two signal models considered
    1. Streaming signal with Lapped Orthogonal Basis (shown above)
    2. Streaming signal with Linear Dynamical Model, i.e., (with non-overlapping measurement window)

$$\mathbf{x}_{t+1} = \mathbf{F}_t \mathbf{x}_t + f_t$$

Sparse Recovery of Streaming Signals Using ℓ_1- Homotopy (2/2)

- Streaming signal with Lapped Orthogonal Basis
  - $$\min_{\alpha} \frac{1}{2}||\bar{\mathbf{y}} - \bar{\Phi}\bar{\Psi}\alpha||_2 + ||\mathbf{W}\alpha||_1$$
    where $\mathbf{w}_i = \frac{\tau}{\beta|\hat{\alpha}_i|+1}$ and, $\hat{\alpha}$ is the signal estimate from previous streaming iteration.

- Streaming signal with Linear Dynamic Model
  - $$\min_{\alpha} \frac{1}{2}||\bar{\mathbf{y}} - \bar{\Phi}\bar{\Psi}\alpha||_2 + ||\mathbf{W}\alpha||_1 + \frac{\lambda}{2}||\bar{\mathbf{F}}_{prev}\bar{\Psi}_{prev}\hat{\alpha} - \bar{\Psi}\alpha||_2^2$$

Likelihood Estimators for Dependent Samples and Thier Application to Order Detection.

Authors: Geng-Shen Fu, Matthew Anderson and Tulay Adah

Affiliations: Univ. of Maryland, Baltimore

Likelihood Estimators for Dependent Samples and Their Application to Order Detection (1/2)

- **Problem Statement:**
  - Estimate the dimension of signal subspace given data $\mathbf{x}(t), t = 0, 1, 2....T$, assuming
$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + (t)$$
  - If $\mathbf{x}(1), \mathbf{x}(2) \ldots \mathbf{x}(T)$ are independent, MDL gives asymptotically efficient estimator of model order.
  - In *Minimum Description Length* (MDL), we try to identify a model $\mathcal{M}$ which minimizes
$$-\log p(\mathbf{x}_1, \ldots \mathbf{x}_T|\mathcal{M}) + C(T) \cdot (\text{ params in } \mathcal{M}) + (\text{ bits encoding extra structure in} \mathcal{M}$$
  - If $\mathbf{x}(1), \mathbf{x}(2) \ldots \mathbf{x}(T)$ are NOT independent, problem becomes non-trivial as joint PDF (Likelihood) is not easy to obtain.

Likelihood Estimators for Dependent Samples and Their Application to Order Detection (2/2)

- **Solution:**
  - First whiten the data $\mathbf{Y} = \mathbf{U}^T\mathbf{X}$
  - Assume uncorrelated $\mathbf{y}_i$ to be stationary and having finite memory of length K_i.
  - Memory length K_i obtaining by finding the minimum downsampling rate K which makes the downsampled sequence i.i.d.
  - Derive approximate log-likelihood $\mathcal{L}(\mathbf{X})$.

$$\mathcal{L}(\mathbf{X}) = -\frac{1}{T}\log P(\mathbf{X}) = -\frac{1}{T}\sum_{i=1}^{N}\log P(\mathbf{y}_i) = \sum_{i=1}^{N}\frac{1}{2}\log\frac{|\hat{C}_{i(K_i)}|}{|\hat{C}_{i(K_i-1)}|}$$

  - $-\frac{1}{T}\log P(\mathbf{y}_i)$ seen as entropy rate h_i of i^{th} component.
  - Proposes generalized version of MDL criterion in terms of entropy rate terms

$$\min_M \left[\sum_{i=1}^{M}\log h_i + (N-M)log\bar{h} + C(T)\cdot(\text{no. of model parameters})\right]$$

Where the model parameters are $\{h_1, \ldots h_M, \bar{h}, \mathbf{U}\}$

Other Interesting Papers:

- Towards the Asymptotic Sum Capacity of the MIMO Cellular Two-Way Relay Channel
- Relabeling and Summarizing Posterior Distributions in Signal Decomposition Problems When the Number of Components is Unknown
- Multitask Diffusion Adaptation Over Networks
- An MGF-Based Unified Framework to Determine the Joint Statistics of Partial Sums of Ordered i.n.d. Random Variables

Thank You !!!