Journal Watch:

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Saurabh Khanna,
Signal Processing for Communication, ECE, IISc

An Empirical Bayes Approach to Recovering Linearly Constrained Non-Negative Sparse Signals.

Authors: Jeremy P. Vila and Philip Schniter

Affiliations: Ohio State University

An Empirical Bayes Approach to Recovering Linearly Constrained Non-Negative Sparse Signals

- **Problem Statement:**
  - Non-Negative linearly constrained sparse signal recovery: Recover sparse $\mathbf{x}$ from $\mathbf{y}$, where
$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$$
    and, $\mathbf{x}_i \geq 0$, and $\mathbf{x}$ statisfies $\mathbf{B}\mathbf{x} = \mathbf{c}$
  - Conventional approach: ℓ_1 penalized constrained NN least squares problem:
$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \succeq 0} \frac{1}{2}||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 + \lambda||\mathbf{x}||_1 \;\; \text{s.t.} \;\; \mathbf{B}\mathbf{x} = \mathbf{c}$$
- **Main results:**
  - Proposes three variants of Generalized Approximate Message Passing Algorithms (GAMP) based algorithms
  - Augmented measurement model
$$[\mathbf{y}; \mathbf{c}] = [\mathbf{A}; \mathbf{B}]\mathbf{x} + [\mathbf{w}; 0]$$
$$\begin{aligned} f_{\bar{\mathbf{y}}|\mathbf{x}}(\bar{\mathbf{y}}_m|\mathbf{x}) \quad = \quad & \mathcal{N}(\mathbf{A}_m^T\mathbf{x}, \sigma_w^2\mathbf{I}) \quad m = 1 \ldots M \\ & \delta(\mathbf{y}_m - \mathbf{B}_m^T\mathbf{x}) \quad m = M+1 \ldots M+P \end{aligned}$$

An Empirical Bayes Approach to Recovering Linearly Constrained Non-Negative Sparse Signals

- **Main results: (contd..)**

1. NN Least Squares GAMP
   Improper non negative prior: $f_{\mathbf{x}}(\mathbf{x}) = 1, \mathbf{x} \geq 0, \quad 0 \; o.w.$
   Equivalent unconstrained optimization:
   $$\arg\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 - \log \mathbb{I}_{\mathbf{B}\mathbf{x}=\mathbf{c}} - \sum_{n=1}^{N} \log \mathbb{I}_{\mathbf{x}_n \geq 0}$$

2. NN LASSO GAMP
   Non negative prior: $f_{\mathbf{x}}(\mathbf{x}) = \gamma exp(-\gamma x), x \geq 0, \quad 0 \; o.w.$
   Equivalent constrained optimization:
   $$\arg\min_{\mathbf{x} \geq 0} \frac{1}{2\psi}||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 + \gamma||\mathbf{x}||_1 \text{ s.t. } \mathbf{B}\mathbf{x} = \mathbf{c}$$
3. NN Gaussian Mixture GAMP
   Non negative prior: $f_{\mathbf{x}}(\mathbf{x}) = (1 - \tau)\delta(x) + \tau \sum_{l=1}^{L} w_l \mathcal{N}_+(x; \theta_l, \phi_l), \quad 0 \; o.w.$

A Measurement Rate-MSE Tradeoff for Compressive Sensing Through Partial Support Recovery.

Authors: Ricardo Blasco-Serrano, Dave Zachariah, Dennis Sundman, Ragnar Thobaben and Mikael Skoglund

Affiliations: Uppsala university, KTH Sweden

A Measurement Rate-MSE Tradeoff for Compressive Sensing Through Partial Support Recovery

- **Problem Statement:**
  - Find relation between MSE and measurement rate for $k-$sparse signal recovery problem
  - Measurement rate $r = \liminf_{n \to \infty} \frac{m_n}{\log n}$
  - Asymptotic setting: $n, m \to \infty$, but k is kept fixed
  - Partial support recovery: (γ support set S_γ)

$$\sum_{i \in S_\gamma} \mathbf{x}_i^2 \geq \gamma ||\mathbf{x}||_2^2 \quad (1)$$

- **Main results:**
  1. Partial support set recovery: For given noise variance, γ and the non-zero coefficients $\mathbf{w} \in \mathbb{R}^k$, an expression for measurement rate r is found, such that $P_{err}(support\ recovery) \leq o(1/m)$
  2. MSE: Given some $T \subseteq S$, it is possible to acheive

$$MSE(T) = ||\mathbf{x}_{T^C}||_2^2 + O(\frac{1}{m})$$

  3. Measurement Rate vs MSE tradeoff: Staircase like acheivable region found (has staircase like shape)

A Variational Bayes Framework for Sparse Adaptive Estimation.

Authors: Konstantinos E Themelis, Athanasios A. Rontogiannis and Konstantinos D. Koutroumbas

Affiliations: Inistitute of Astronomy, National Observatory of Athens

A Variational Bayes Framework for Sparse Adaptive Estimation

- **Problem Statement:**
  - Estimate and track $\mathbf{w}(n) \in \mathbb{R}^N$ in time by observing a stream of sequential data $y(n)$

    $$y(n) = \mathbf{x}^T(n)\mathbf{w}(n) + \epsilon(n)$$

    where, $\mathbf{x}(n)$ is a known $N \times 1$ regression vectors
  - $\mathbf{w}(n)$ is known to be sparse
  - Seeking Bayesian motivated recursive least squares type solution
- **Main results:**
- Impose prior on $\mathbf{w}(n) \sim \mathcal{N}(0, \beta^{-1}\mathbf{A})$ where $\mathbf{A} = \text{diag}(\alpha_1, \alpha_2 \ldots \alpha_N)$
- *Generalized Inverse Gaussian* prior assumed on α_i
- Mean field variational Bayesian inference used to obtain update equations for $\mathbf{w}, \alpha, \beta$
- For batch processing, derives update equation when effective prior on $\mathbf{w}(n)$ is taken to be *Student-t* and *Laplacian*
- For recursive processing, at each iteration n, a regularized LS cost function $\tau_{LS-R}(n)$ is minimized

$$\tau_{LS-R}(n) = ||\Lambda^{\frac{1}{2}}(n)\left(\mathbf{y}(n) - \mathbf{X}(n)\hat{\mathbf{w}}(n)\right)||_2^2 + \hat{\mathbf{w}}^T(n)\mathbf{A}(n-1)\hat{\mathbf{w}}(n)$$

- Nice trick to obtain iterative scheme for weight update...(discuss)

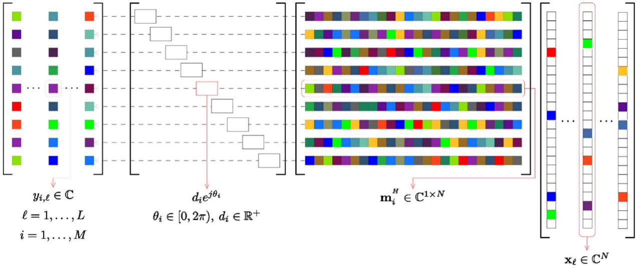
Convex Optimization Approaches for Blind Sensor Calibration Using Sparsity.

Authors: Cagdas Bilen, Remi Gribonval, Laurent Daudet

Affiliations: INRIA, EPFL and University of Paris, Diderot

Convex Optimization Approaches for Blind Sensor Calibration Using Sparsity

- **Problem Statement:**



  - Measurement model at sensor l

$$y_{i,l} = d_i e^{j\theta_i} \mathbf{m}_i^T \mathbf{x}_l \qquad i = 1 \dots M,\ d_i \geq 0,\ \theta_i \in [0, 2\pi)$$

  - Three scenarios considered
    1. Amplitude calibration
    2. Phase calibration
    3. Complete calibration

Convex Optimization Approaches for Blind Sensor Calibration Using Sparsity

- Amplitude calibration
  - Rearrange gain terms to rewrite meas model as:

$$y_{i,l}\tau i = \mathbf{m}_i^T \mathbf{x}_l$$

  - Assume $\sum_i^M \tau i = 1$ to remove gain ambiguity between signal and distortion term
- Phase calibration
  - Note that $g_{i,k,l} = y_{i,k} y_{i,l}^* = \mathbf{m}_i^T \mathbf{x}_k \mathbf{x}_l^H \mathbf{m}_i$
  - Try to recover joint matrix $\mathbf{X} = \mathbf{x}\mathbf{x}^H$, where $\mathbf{x} = (\mathbf{x}_1^H \ldots \mathbf{x}_L^H)^H$
  - A convex optimization problem is formulated to find $\mathbf{X}$

$$\begin{aligned} &\arg\min_{\mathbf{Z}} ||\mathbf{Z}||_1 \\ &\text{subject to } \mathbf{Z} \succeq 0 \\ &g_{i,k,l} = y_{i,k} y_{i,l}^* = \mathbf{m}_i^T \mathbf{x}_k \mathbf{x}_l^H \mathbf{m}_i \end{aligned}$$

  - Recover $\mathbf{x}$ from $\mathbf{X}$ (nice trick here!)
- Complete calibration
  - A combination of above two approaches

Other Interesting Papers:

- Hierarchical Interference Mitigation for Massive MIMO Cellular Networks
- Alternating Projections and Douglas-Rachford for Sparse Affine Feasibility