Journal Watch:

IEEE Transanctions on Signal Processing July 1, 2015

Saurabh Khanna,
Signal Processing for Communication, ECE, IISc

Gaussian Mixtures Based IRLS for Sparse Recovery With Quadratic Convergence

Authors: Chiara Ravazzi and Enrico Magli (Polytechnico di Torino, Italy)

- **Problem Statement:**
  - Recover sparse $\mathbf{x} \in \mathbb{R}^n$ from $m(< n)$ measurements $\mathbf{y} \in \mathbb{R}^m$, where

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$$

- **Iterative support detection and estimation approach.**
  - A reweighted constrained optimization is considered

$$\min_{T \in [n], |T| \leq \frac{n}{2}} \quad \min_{\mathbf{x} \in \mathcal{F}(\mathbf{y})} \left\{ \sum_{i \in T} \frac{x_i^2}{\beta} + \sum_{i \notin T} \frac{x_i^2}{\alpha} \right\}$$

    where $\mathcal{F}(\mathbf{y}) = \{\mathbf{x} \text{ s.t. } ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2 \leq \delta\}$
  - Solve for $\mathbf{x}$, T, α and β via Gauss Seidel iterations.

Gaussian Mixtures Based IRLS for Sparse Recovery With Quadratic Convergence

Authors: Chiara Ravazzi and Enrico Magli (Polytechnico di Torino, Italy)

Algorithm 1: Hard support detection and signal reconstruction

Input: Measurements $y \in \mathbb{R}^n$, data matrix $A \in \mathbb{R}^n$

1: Initialization: $\alpha^{(0)} = \alpha_0,\ \beta^{(0)} = \beta_0,\ T^{(0)} = T_0$

2: **for** $t = 0, 1, \ldots, StopIter$ **do**

3: Constrained weighted least square minimization:

$$x^{(t+1)} = \underset{x \in \mathcal{F}(y)}{\arg\min} \left[\sum_{i \in T} \frac{x_i^2}{\beta^{(t)}} + \sum_{i \notin T} \frac{x_i^2}{\alpha^{(t)}} \right]$$

4: Support detection: set threshold $\delta = \delta(\alpha^{(t)}, \beta^{(t)}) > 0$

$$T^{(t+1)} = \left\{ i \in [n] : |x_i^{(t+1)}| > \delta \right\}$$

5: Weights update:

$$\alpha^{(t+1)} = \alpha^{(t+1)}(x^{(t+1)}, T^{(t+1)})$$
$$\beta^{(t+1)} = \beta^{(t+1)}(x^{(t+1)}, T^{(t+1)})$$

6: **end for**

Algorithm 2: Soft support detection and signal reconstruction

Input: Measurements $y \in \mathbb{R}^n$, data matrix $A \in \mathbb{R}^n$

1: Initialization: $\alpha^{(0)} = \alpha_0,\ \beta^{(0)} = \beta_0,\ \pi^{(0)} \in [0,1]^n$

2: **for** $t = 0, 1, \ldots, StopIter$ **do**

3: Constrained weighted least square minimization:

$$x^{(t+1)} = \underset{x \in \mathcal{F}(y)}{\arg\min} \left[\sum_{i \in T} \frac{(1 - \pi_i^{(t)}) x_i^2}{\beta^{(t)}} + \sum_{i \notin T} \frac{\pi_i^{(t)} x_i^2}{\alpha^{(t)}} \right]$$

4: Posterior beliefs of the signal coefficients:

$$\pi^{(t+1)} = \pi(x^{(t+1)}, \alpha^{(t)}, \beta^{(t)})$$

5: Weights update:

$$\alpha^{(t+1)} = \alpha^{(t+1)}(x^{(t+1)}, \pi^{(t+1)})$$
$$\beta^{(t+1)} = \beta^{(t+1)}(x^{(t+1)}, \pi^{(t+1)})$$

6: **end for**

- Above algorithms derived for Bernoulli Gaussian prior
- Quadratic convergence is shown

Online Hyperparameter-Free Sparse Estimation Method

Authors: Dave Zachariah and Petre Stoica (Uppsala University)

- **Problem Statement:**
  - Recover sparse $\theta \in \mathbb{R}^p$ from linear measurements $y_1, y_2 \ldots.$ in an online fashion: $y_t = \mathbf{h}_t^T \theta + w_t, \qquad t = 1, 2 \ldots$
- **Sparse Iterative Covariance Based Estimation (SPICE) as weighted square root LASSO**
  - Say $\theta \sim \mathcal{N}(0, \mathbf{P})$ and $\mathbf{w}_t \sim \mathcal{N}(0, \sigma^2)$
  - For known $\mathbf{P}$ and σ^2, the LMMSE estimator $\hat{\theta} = (\mathbf{H}_n^T \mathbf{H}_n + \sigma^2 \mathbf{P}^{-1})^{-1} \mathbf{H}_n^T \mathbf{y}_n$ is a solution to

$$\arg\min_{\theta} \frac{1}{\sigma^2} ||\mathbf{y}_n - \mathbf{H}_n \theta||_2^2 + ||\theta||_{\mathbf{P}^{-1}}$$

  - Let $\mathbf{P} = \mathrm{diag}(p_1, \ldots p_p)$ and say we choose $\mathbf{P}$ and σ^2 such that $||\mathbf{R}_n^{-\frac{1}{2}} (\mathbf{y}_n \mathbf{y}_n^T - \mathbf{R}_n)||_F$ is minimized, then we are essentially solving the below problem:

$$\arg\min_{\theta \in \mathbb{C}^p} \frac{1}{\sigma^2} ||\mathbf{y}_n - \mathbf{H}_n \theta||_2^2 + ||\theta||_{\mathbf{P}^{-1}} + tr(\mathbf{H}_n \mathbf{P} \mathbf{H}_n^T + \sigma^2 \mathbf{I}_n)$$

  - The equivalent weighted square root LASSO problem

$$\arg\min_{\theta \in \mathbb{C}^p} ||\mathbf{y}_n - \mathbf{H}_n \theta||_2 + ||\mathbf{D}_n \theta||_1$$

where $\mathbf{D}_n = \mathrm{diag}(\sqrt{\frac{||\mathbf{c}_1||_2^2}{n}}, \ldots \sqrt{\frac{||\mathbf{c}_p||_2^2}{n}})$

Online Hyperparameter-Free Sparse Estimation Method

Authors: Dave Zachariah and Petre Stoica (Uppsala University)

- **Online SPICE**
  - Weighted squre root LASSO cost function:

$$\underset{\theta \in \mathbb{C}^p}{\arg\min} \, ||\mathbf{y}_n - \mathbf{H}_n \theta||_2 + ||\mathbf{D}_n \theta||_1$$

  - Decouple the variables θ_i in the cost function using substitution: $\tilde{\mathbf{y}}_i \triangleq \mathbf{y} - \sum_{k \neq 1} \mathbf{c}_k \theta_k$

$$\theta_i^{k+1} = \underset{\theta_i}{\arg\min} \, (||\tilde{\mathbf{y}}_i^k - \mathbf{c}_i \theta_i||_2 + d_{ii} |\theta_i| + const$$

  - Cyclic minimization of θ_i done to obtain optimized θ
  - Closed form recursion for θ_i found by switching to polar coordinates

Robust PCA with Partial Subspace Knowledge

Authors: Jinchun Zhan and Namrata Vaswani

- **Problem Statement:**
  - Given $\mathbf{M}$, decompose as $\mathbf{M} = \mathbf{L} + \mathbf{S}$, where $\mathbf{L}$ is a low rank matrix and $\mathbf{S}$ is a sparse matrix.
  - Partial knowledge of column space of $\mathbf{L}$ is available in form of $\mathbf{G}$
- **Main results:**
  - Modified Principle Component Pursuit (Modified-PCP) is proposed.
  - We can write:
    $\mathbf{M} = (\mathbf{I} - \mathbf{G}\mathbf{G}^T)\mathbf{L} + \mathbf{G}(\mathbf{G}^T\mathbf{L}) + \mathbf{S}$
    or $\mathbf{M} = \mathbf{L}_{\text{new}} + \mathbf{G}\mathbf{X} + \mathbf{S}$.
  - Then, $\mathbf{L}_{\text{new}}, \mathbf{X}, \mathbf{S}$ can be recovered by solving the below optimization:

$$\underset{\mathbf{L}_{\text{new}}, \mathbf{X}, \mathbf{S}}{\text{minimize}} \; ||\mathbf{L}_{\text{new}}||_* + \lambda ||\mathbf{S}||_1$$
$$\text{subject to } \mathbf{L}_{\text{new}} + \mathbf{G}\mathbf{X} + \mathbf{S} = \mathbf{M}$$

  - Conditions given for exact recovery (W.H.P)

Homotopy Based Algorithms for ℓ_0 Regularized Least Squares

Authors: Charles Soussen, Jerome Idier, Junbo Duan and David Brie

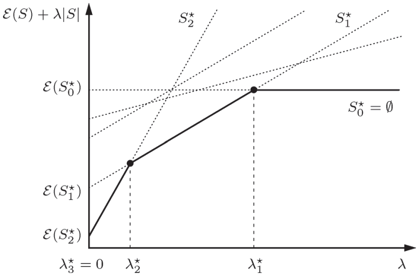
- Recover sparse $\mathbf{x} \in \mathbb{R}^n$ from $m(< n)$ measurements $\mathbf{y} \in \mathbb{R}^m$, where

$$\mathbf{y} = \mathbf{Ax} + \mathbf{w}$$

Unconstrained formulation:

$$\min_{\mathbf{x}} \mathcal{J}(\mathbf{x}, \lambda) = ||\mathbf{y} - \mathbf{Ax}||_2^2 + \lambda||\mathbf{x}||_0$$

- The set of solution to ℓ_0 regularized problem is piecewise constant.
- Crude justification using ℓ_0 curve



Homotopy Based Algorithms for ℓ_0 Regularized Least Squares

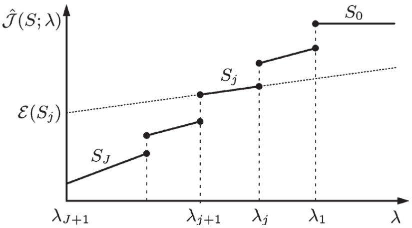
Authors: Charles Soussen, Jerome Idier, Junbo Duan and David Brie

- Another interesting result:

$$\min_S \mathcal{J}(\mathbf{x}, \lambda) = ||\mathbf{y} - \mathbf{A}_S \mathbf{x}_S||_2^2 + \lambda |S| \tag{1}$$

$$\min_S ||\mathbf{y} - \mathbf{A}_S \mathbf{x}_S||_2^2 \quad \text{subject to} \quad |S| \leq k \tag{2}$$

- P1 is more well conditioned than P2, with lesser number of local minimas
- Greedy algorithm proposed for regularization path tracing



Other Interesting Papers:

- Distributed Clustering and Learning Over Networks
- Distributed Diffusion-Based LMS for Node-Specific Adaptive Parameter Estimation
- Quantized Distributed Reception for MIMO Wireless Systems Using Spatial Multiplexing
- Probabilistic SINR Constrained Beamforming Under Channel Uncertainties