Journal Watch:

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A Pessimistic Approximation for the Fisher Information Measure

Authors: Manuel S. Stein and Josef A. Nossek

- Suppose $\hat{\theta}$ is an unbiased estimate of θ given N observations $Y = \{y_1, y_2, \ldots, y_N\}$.

$$\mathrm{Var}(\hat{\theta}) \geq \frac{1}{F_Y(\theta)}$$

where $F_Y(\theta) = \int_{\mathcal{Y}^N} \left(\frac{\partial \log p(Y;\theta)}{\partial \theta} \right)^2 p(Y;\theta) dY$ is the Fischer information.

- In many situations, $p(Y;\theta)$ takes a complicated form, and evaluation of Fischer information becomes intractable.

- Thus, we seek a tight lower bound for the Fischer information which can be computed directly from the observations. Preferably, in terms of moments and its derivatives.

- Earlier result: $F(\theta) \geq \frac{1}{\mu_2(\theta)} \left(\frac{\partial \mu_1(\theta)}{\partial \theta} \right)^2$.

- A tighter lower bound is proposed: $F(\theta) \geq \frac{1}{\mu_2(\theta)} \frac{\left(\frac{\partial \mu_1(\theta)}{\partial \theta} + \frac{\beta^\star(\theta)}{\sqrt{\mu_2(\theta)}} \frac{\partial \mu_2(\theta)}{\partial \theta} \right)^2}{1+2\beta^\star(\theta)\bar{\mu}_3(\theta)+\beta^{\star 2}(\theta)(\bar{\mu}_4(\theta)-1)}$

where $\beta^\star(\theta) = \frac{\frac{\partial \mu_1(\theta)}{\partial \theta} \sqrt{\mu_2(\theta)} \bar{\mu}_3(\theta) - \frac{\partial \mu_2(\theta)}{\partial \theta}}{\frac{\partial \mu_2(\theta)}{\partial \theta} \bar{\mu}_3(\theta) - \frac{\partial \mu_1(\theta)}{\partial \theta} \sqrt{\mu_2(\theta)(\bar{\mu}_4(\theta)-1)}}$.

Low Rank Positive Semidefinite Matrix Recovery From Corrupted Rank-One Measurements

Authors: Yuanxin Li, Yue Sun and Yuejie Chi,   Affiliation: Ohio State Univ.

- Estimation of low-rank PSD matrix from a set of rank-one measurments corrupted by arbitrary outliers.
- Recover PSD matrix $\mathbf{X}_0$ given m rank-one measurements $z_1, z_2, \ldots, z_m$.

$$z_i = \langle \boldsymbol{Z}_i, \boldsymbol{X}_0 \rangle = \langle \boldsymbol{a}_i \boldsymbol{a}_i^T, \boldsymbol{X}_0 \rangle = \boldsymbol{a}_i^T \boldsymbol{X}_0 \boldsymbol{a}_i.$$

- Such measurments could arise due to physical limitations, e.g., incapability to capture phases, optical imaging from intensity measurements, noncoherent measurements etc.
- Example: Intensity measurements,

$$z_i = |\langle \boldsymbol{a}_i, \boldsymbol{x}_0 \rangle|^2 = \boldsymbol{a}_i^T \left(\boldsymbol{x}_0 \boldsymbol{x}_0^T \right) \boldsymbol{a}_i = \boldsymbol{a}_i^T \boldsymbol{X}_0 \boldsymbol{a}_i$$

  where $\mathbf{X}_0 = \mathbf{x}_0 \mathbf{x}_0^T$ is the lifted rank-one matrix.

- In many applications, one may have aggregate of measurments:

$$z_i = \frac{1}{L} \sum_{l=1}^{L} |\langle \boldsymbol{a}_i, \boldsymbol{x}_l \rangle|^2 = \boldsymbol{a}_i^T \left(\frac{1}{L} \sum_{l=1}^{L} \boldsymbol{x}_l \boldsymbol{x}_l^T \right) \boldsymbol{a}_i \approx \boldsymbol{a}_i^T \boldsymbol{X}_0 \boldsymbol{a}_i.$$

Low Rank Positive Semidefinite Matrix Recovery From Corrupted Rank-One Measurements

- Recover PSD matrix $\mathbf{X}_0$ given m rank-one measurements $z_1, z_2, \ldots, z_m$.

$$z_i = \langle \boldsymbol{Z}_i, \boldsymbol{X}_0 \rangle = \langle \boldsymbol{a}_i \boldsymbol{a}_i^T, \boldsymbol{X}_0 \rangle = \boldsymbol{a}_i^T \boldsymbol{X}_0 \boldsymbol{a}_i.$$

- Prior art: Phase Lift algorithm

$$\min_{\boldsymbol{X} \succeq 0} \operatorname{Tr}(\boldsymbol{X}) \quad \text{s.t.} \quad \|\boldsymbol{z} - \mathcal{A}(\boldsymbol{X})\|_1 \leq \epsilon.$$

  In presence of outliers, ϵ can be arbitrarily large.
  Robust Phase-Lift algorithm fixes this issue.
  Current work extends it to handle low-rank PSD matrix recovery.

- A parameter free convex relaxation is proposed.

$$\text{(Robust-PhaseLift:)} \quad \hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X} \succeq 0} \|\boldsymbol{z} - \mathcal{A}(\boldsymbol{X})\|_1.$$

- As long as $m \geq c_1 n r^2$, and $s \leq \frac{s_0}{r}$ (outlier sparsity), the solution to above satisfies $||\hat{\mathbf{X}} - \mathbf{X}_0||_F \leq \frac{c_2 r \epsilon}{m}$ with probability exceeding $1 - exp(-\gamma m / r^2)$.

- Proposed optimization is solved via non-convex subgradient descent technique.

A unified framework for low autocorrelation sequence design via majorization-minimization

Authors: Licheng Zhao, Junxiao Song, Prabhu Babu, and Daniel P. Palomar

- Design sequences with low autocorrelation sidelobes.
  Applications: CDMA cellular systems, radar systems, cryptography etc.
- Autocorrelation of a sequence $\mathbf{x} = (x_1, x_2, \ldots, x_n)$.

$$r_k = \sum_{n=1}^{N-k} x_n x_{n+k}^* = r_{-k}^* \quad \text{(aperiodic)}$$

$$r_k = \sum_{n=1}^{N} x_n x_{(n+k)(\text{mod } N)}^* = r_{-k}^* \quad \text{(periodic)}$$

- Metrics for goodness of autocorrelation sidelobes:

$$\text{ISL} = \sum_{k=1}^{N-1} |r_k|^2, \qquad \text{WISL} = \sum_{k=1}^{N-1} w_k |r_k|^2, \qquad \text{PSL} = \max_{k=1,2,\cdots,N-1} \{|r_k|\}.$$

- Proposed unified metric: Weighted Peak or Integrated Sidelobe Level

$$\text{WPISL} = \sum_{k=1}^{N-1} w_k |r_k|^p.$$

A unified framework for low autocorrelation sequence design via majorization-minimization

- Other constraints of concern in addition to sidelobe supression.
  1. Constant Modulus constraint.
  2. Peak-to-Avg-Power (PAPR) constraint
  3. Discrete phase constraint
  4. Similarity constraint

- Proposed problem formulation:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \sum_{k=1}^{N-1} w_k \left| r_k\left(\mathbf{x}\right)\right|^p \\ \text{subject to} & \mathbf{x} \in \mathcal{X}, \end{array}$$

  where $r_k(\mathbf{x}) = \mathbf{x}^H \mathbf{U}_k \mathbf{x}$ for some suitable Toeplitz shift matrix $\mathbf{U}_k$, and

$$\mathcal{X} = \left\{ \mathbf{x} \in \mathbb{C}^N \middle| \, \|\mathbf{x}\|_2^2 = c_e^2 \right\} \cap \left(\cap_i C_i \right).$$

- Above optimization is solved via majorization-minimization procedure (accelerated by SQUARE-EM or local majorization).

- See paper for three useful majorization tricks.

Multidimensional Harmonic Retrieval via Coupled Canonical Polyadic Decomposition - Model and Identifiablity

Authors: Mikael Sorensen and Lieven De Lathauwer

- Multidimensional Harmonic Retrieval (MHR) is a fundamental signal proc. problem with applications in radar, sonar, wireless communication and channel sounding.
- MHR structure can arise due to Doppler affects, structured RX/TX antenna arrays, carrier frequency offsets.
- Question: What are the necessary and sufficient conditions for uniqueness of MHR structure given a set of observations of a noisy MHR signal.
- There exists a link between MHR and Canonical Polyadic Decomposition (CPD).
- Using the CPD interpretation of MHR problem, one can derive necessary and sufficient consitions for a uniqueness of MHR structure.
- Surprisingly, using CPD framework one can show that the necessary and sufficient conditions are the same.

Multidimensional Harmonic Retrieval via Coupled Canonical Polyadic Decomposition - Model and Identifiablity

- Canonical Polyadic Decomposition (CPD):
- Consider a third order tensor $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$.
- A polyadic decomposition is a decomposition of $\mathcal{X}$ into rank-1 terms

$$\mathbb{C}^{I \times J \times K} \ni \mathcal{X} = \sum_{r=1}^{R} \mathbf{a}_r \otimes \mathbf{b}_r \otimes \mathbf{c}_r \,.$$

- The rank of a tensor $\mathcal{X}$ is equal to the minimal number of rank-1 tensors that yield $\mathcal{X}$ in a linear combination.
- Let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be matrices made by stacking $\mathbf{a}_r$, $\mathbf{b}_r$ and $\mathbf{c}_r$ as columns. $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are called factor matrices of $\mathcal{X}$.
- Matrix representation of tensor:

$$\mathbf{X}^{(i\cdot\cdot)} = \sum_{r=1}^{R} a_{ir} \mathbf{b}_r \mathbf{c}_r^T = \mathbf{B} D_i (\mathbf{A}) \mathbf{C}^T,$$

$$\mathbb{C}^{IJ \times K} \ni \mathbf{X} := \begin{bmatrix} \mathbf{X}^{(1\cdot\cdot)} \\ \vdots \\ \mathbf{X}^{(I\cdot\cdot)} \end{bmatrix} = \begin{bmatrix} \mathbf{B} D_1 (\mathbf{A}) \\ \vdots \\ \mathbf{B} D_I (\mathbf{A}) \end{bmatrix} \mathbf{C}^T = (\mathbf{A} \odot \mathbf{B}) \mathbf{C}^T.$$

Other Interesting Papers:

- Massive MIMO Channel Subspace Estimation From Low-Dimensional Projections.
- Joint BS-User Association, Power Allocation, and User-Side Interference Cancellation in Cell-free Heterogeneous Networks.
- Extending Classical Multirate Signal Processing Theory to Graphs â Part I and II.
- Sparse Reconstruction Algorithm for Nonhomogeneous Counting Rate Estimation.