Journal watch
IEEE Transactions on Signal Processing
01 Jan and 15 Feb 2017

Ramu Naidu R

IISc, SPC lab

February 4, 2017

Definition

Let M, k be specified integers with $k \ll M$. Let $\mathbb{G} = \{G_1, G_2, \ldots, G_g\}$ be a partition of $\{1, 2, \ldots, M\}$, such that $|G_i| \le k$ for all i. For $S \subseteq \{1, 2, \ldots, g\}$, define $G_S = \bigcup_{i \in S} G_i$. A subset $\Lambda \subseteq \{1, 2, \ldots, M\}$ is said to be S-**group** k-**sparse** for some $S \subseteq \{1, 2, \ldots, g\}$ if $\Lambda = G_S$ and $|\Lambda| \le k$ and **group** k-**sparse** if it is S-group k-sparse for some set $S \subseteq \{1, 2, \ldots, g\}$. A vector $x \in \mathbb{R}^M$ is said to be **group** k-**sparse** if its support set supp(x) is contained in a group k-sparse set.

- Clearly all notions of group sparsity are with reference to a particular partitioning $\mathbb{G}$

- It is easy to see that if $g = M$ and each set G_i consists of exactly one element, then group k-sparsity reduces to conventional k-sparsity
- From the definition, it is clear that every group k-sparse vector is also k-sparse. However, the converse need not be true
- The authors proposed and analyzed a non convex model and algorithm for group sparse recovery based on regularized least squares with an $l^0(l^2)$ penalty
- They proposed the following optimization problem:

$$\min_{x \in \mathbb{R}^M} \left\{ J_\lambda(x) = \frac{1}{2}\|\Psi x - y\|^2 + \lambda \|x\|_{l^0(l^2)} \right\}, \tag{1}$$

where $\|x\|_{l^0(l^2)} = |\{i : \|x_{G_i}\|_{l^2} \neq 0\}|$, with respect to the given partition $\{G_i\}_{i=1}^{g}$.

- Later they analyzed the model (1) using the concept blockwise mutual coherence (BMC) of the matrix Ψ with respect to the partition $\{G_i\}_{i=1}^g$ defined as follows:

$$\mu = \max_{i \neq j} \mu_{i,j}, \text{ where } \mu_{i,j} = \sup_{u \in \mathcal{N}_i/\{0\}, v \in \mathcal{N}_j/\{0\}} \frac{\langle u, v \rangle}{\|u\| \|v\|}, \quad (2)$$

where $\mathcal{N}_i$ is the subspace spanned by the columns of Ψ_{G_i}.

Theorem

There exists a global minimizer to problem (1).

- By assuming $\mu \in (0, \frac{1}{3g})$, they proposed theoretical results of model (1)

Model-Based Nonuniform Compressive Sampling and Recovery of Natural Images Utilizing a Wavelet-Domain Universal Hidden Markov Model

Behzad Shahrasbi and Nazanin Rahnavard

- One of the central problems in CS is the construction of *sensing matrices* $\Psi \in \mathbb{C}^{m \times M}$ such that an arbitrary s-sparse vector $u \in \mathbb{C}^M$ can be efficiently reconstructed from $y = \Psi u$
- The vector u is said to be s-sparse if it can be decomposed as $u = \Gamma\alpha$, where the unitary matrix $\Gamma \in \mathbb{C}^{M \times M}$ is the sparsifying basis and $\alpha \in \mathbb{C}^M$ has $s-$non zero entries
- The authors proposed a compressive sampling technique for natural images in wavelet domain. Particularly, they constructed sensing matrix Ψ using universal hidden Markov tree (uHMT) from a wavelet decomposition of a natural image
- By modifying the two algorithms, Bayesian CS via belief propagation (CSBP) and approximate message passing (AMP) they solve the image recovery problem

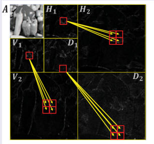

Figure : Wavelet decomposition of an image.

- Here A, H, V , D stand for the approximate coefficients, horizontal, vertical, and diagonal subband coefficients, respectively
- The decomposition has $4^{-j}n, \sum_{i=1}^{J} 34^{i-J-1}n$ approximate and subband coefficients respectively
- Using suband coefficients they constructed the sensing matrix Ψ
- Also they proved that coherence of $\Psi\Gamma$ is in the order of $\sqrt{\log(\frac{n}{\sqrt{\delta}})}$

Perfect Recovery Conditions for Non-negative Sparse Modeling

Yuki Itoh, Marco F. Duarte and Mario Parente

- The authors considered the following linear model with non-negative coefficients:

$$y = \Psi x + e(x \geq 0)$$

- This paper considers the performance of non- negative sparse modeling under a more general scenario, where the observed signals have an unknown arbitrary distortion
- For support recovery the considered the following non-negative Lasso algorithm:

$$\frac{1}{2}\|\Psi x - y\|_2^2 + \lambda\|x\|_1;\ s.t,\ x \geq 0. \tag{3}$$

- The authors derived the model recovered conditions for non-negative Lasso using positive subset coherence (PSC)

- The metric positive subset coherence (PSC) can be defined as follows: For a given $\gamma \subseteq \{1, 2, \ldots, M\}$ with $|\gamma| = J \leq M$ and $i \notin \gamma$,

$$PSC(\gamma; i) = 1 - 1_J^T \Psi_\gamma^\dagger a_i.$$

- Using this metric they proved the following theorem.

Theorem

Let γ be a subset of the column indices of the dictionary matrix Ψ such that $|\gamma| = J \leq m$ and the atoms associated with indices in γ are linearly independent. Let $\hat{x}$ be a solution to NLasso. The support of $\hat{x}$ is equal to γ if the following two conditions hold:

1. *Minimum coefficient condition (MCC):* $\Psi_\gamma^\dagger y \succ \lambda(\Psi_\gamma^T \Psi_\gamma)^{-1} 1_J$.
2. *Non-linearity vs. Subset Coherence Condition (NSCC):* $y^T P_\gamma^\perp a_i < \lambda PSC(\gamma; i); \forall i \in \gamma^c$.

On the Noise Robustness of Simultaneous Orthogonal Matching Pursuit(SOMP)

Jean-Francois Determe, Jerome Louveaux, Laurent Jacques, and Francois Horlin

- The central problem in CS has been extended to finding the sparse solution vectors for multiple measurement vectors (MMV)
- That is, for a sensing matrix Φ of size $m \times M$ with $m \ll M$ and given multiple measurement vectors $y^k, k = 1, \ldots, r$, we look for solution vectors $x^k, k = 1, \ldots, r$ such that

$$y^k = \Phi x^k, k = 1, 2, \ldots, r, \tag{4}$$

  and the vectors $x^k, k = 1, \ldots, r$, are **jointly-sparse** (that is nonzero entries are present at the same locations).
- One can recover the joint sparse vectors by using SOMP algorithm

- The authors have given the upper bound of the probability that SOMP recovers at least one incorrect entry of the joint support during a prescribed number of iterations

Theorem

For a fixed iteration t, let $P \in \mathcal{P}^{(t)}$ and $R = (I - P)Y$. i.e., R is one of the residuals that could be generated by SOMP on the basis of Y at iteration $t-1$ assuming that only correct atoms have been identified. For $g \sim \mathcal{N}(0, I_{K\times K})$ and for all $\alpha > 0$, the probability of SOMP picking an incorrect atom when running one iteration on R is upper bounded by

$$\mathbb{P}\left(f^{(t,p)}_{j_c^{t,p}}(g) \leq \alpha\right) + \sum_{j\in S} \mathbb{P}\left(f_j^{(t,p)}(g) \geq \alpha\right)$$

Other Interesting Papers

- Second-Generation Curvelets on the Sphere by J. Y. H. Chan, et.al.
- Manifold Learning With Contracting Observers for Data-Driven Time-Series Analysis by T. Shnitzer, et.al.
- Coarrays, MUSIC, and the Cram erRao Bound by M. Wang, et.al.
- On the 2D Phase Retrieval Problem by Y. C. Eldar, et.al.

Thank you