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## esigning Incoherent Frames Through Convex Techniques

## Optimized Compressed Sensing

## Cristian Rusu and Nuria Gonzlez-Prelcic

## Definition

A family of vectors $F=\left\{f_{i}\right\}_{i=1}^{N}$ in $\mathbb{R}^{m}$ is called a frame for $\mathbb{R}^{m}$, if there exist constants $0<A \leq B<\infty$ such that

$$
\begin{equation*}
A\|x\|^{2} \leq \sum_{i=1}^{n}\left|\left\langle x, f_{i}\right\rangle\right|^{2} \leq B\|x\|^{2} \tag{1}
\end{equation*}
$$

- If $A=B$, then $F$ is an $A$-tight frame.
- If $\left\|f_{i}\right\|=1$ for all $i$ and if there exist $\alpha \geq 0$ such that $\left|\left\langle f_{i}, f_{j}\right\rangle\right|=\alpha$ for all $i \neq j$ then $F$ is an Equiangular
- Grassmannian frame is one that minimizes the maximal correlation $\left\langle f_{i}, f_{j}\right\rangle$ among all frames $F=\left\{f_{i}\right\}_{i=1}^{N}$
- Mutual coherence of a frame is largest absolute inner-product between different normalized elements of $F$
- This paper is concerned the problem of designing real frames with low mutual coherence
- One can find the solution for above mentioned problem by solving the following Optimization problem:

$$
\begin{equation*}
\min _{\{F\}} \max _{\{i \neq j\}}\left|\left\langle f_{i}, f_{j}\right\rangle\right| \text { s. } t\left\|f_{i}\right\|_{2}=1 \text { for all } i \tag{2}
\end{equation*}
$$

- The authors have proceed to relax this problem and provide convex optimization formulations to solve it approximately
- The approach taken in this paper is for given a frame $H=\left\{h_{i}\right\}_{i=1}^{N}$ to find a new frame $F=\left\{f_{i}\right\}_{i=1}^{N}$ that is near to the initial one, with smaller mutual coherence by solving the following convex optimization problem: For all $i=1, \ldots, N$

$$
\begin{equation*}
\min _{\left\{f_{i} ;\left\|f_{i}-h_{i}\right\|_{2}^{2} \leq T_{i}\right\}} \max _{\{j ; j \neq i\}}\left|\left\langle h_{j}, f_{i}\right\rangle\right| \tag{3}
\end{equation*}
$$

## eed Sparsity by Non-Separable Regularization

## Ivan W. Selesnick and Iker Bayram

- The authors have developed a convex approach for sparse deconvolution that improves upon $l_{1}$-norm regularization
- They have posed the sparse deconvolution problem named as BISR in the following way:

$$
\begin{equation*}
\tilde{x}=\arg \min _{x \in \mathbb{R}^{2}}\left\{f(x)=\frac{1}{2}\|y-H x\|_{2}^{2}+\lambda \Psi(x)\right\}, \tag{4}
\end{equation*}
$$

where, $\lambda>0$ and $\Psi(x): \mathbb{R}^{2} \rightarrow \mathbb{R}$ is a penalty function.

- This method is based on designing a non-convex penalty function $\Psi(x)$ so that the objective function is convex
- The new penalty overcomes limitations of separable regularization.
- They have given an algorithm for above mentioned problem.

Further,this bivariate problem has been extended to an $N$-point linear inverse problem

## Mariano Tepper and Guillermo Sapiro

- In recent years, Nonnegative Matrix Factorization (NMF) has been frequently used since it provides a good way for modeling many real-life applications
- NMF seeks to represent a nonnegative matrix as the product of two nonnegative matrices
- One can find the solution of NMF by solving the following optimization problem:

$$
\begin{equation*}
\min _{\left\{X \in \mathbb{R}^{m \times r}, Y \in \mathbb{R}^{r \times n}\right\}}\|A-X Y\|_{F}^{2} \text { s. } t X, Y \geq 0 \tag{5}
\end{equation*}
$$

where $r$ is a parameter that controls the size of factors $X$ and $Y$ and, hence, the factorization's accuracy.

- In the general case, NMF is known to be NP-Hard
- However, there are matrices that exhibit a particular structure such that NMF can be solved efficiently
- A nonnegative matrix $A$ is $r$-separable if there exists an index set $\mathcal{K}$ of cardinality $r$ over the columns of $A$ and a nonnegative matrix $Y \in \mathbb{R}^{r \times n}$, such that $A=(A): \mathcal{K} Y$
- When $A$ presents this type of special structure, the NMF problem (now denoted as separable NMF, SNMF) can be simply modeled as

$$
\begin{equation*}
\min _{\left\{\mathcal{K} \subset\{1, \ldots, n\}, Y \in \mathbb{R}^{r \times n}\right\}}\left\|A-(A)_{: \mathcal{K}} Y\right\|_{F}^{2} \text { s. } t|\mathcal{K}|=r, Y \geq 0 \tag{6}
\end{equation*}
$$

- The goal of this paper is to develop algorithms, based on structured random projections, for computing NMF for big data matrices
- The authors have showed that the resulting compressed techniques are faster than their uncompressed variants, vastly reduce memory demands
- Analysis of a Subset Selection Scheme for Wireless Sensor Networks in Time-Varying Fading Channels by S. H. Mousavi, et.al.
- Collaborative Multi-Sensor Classification Via Sparsity-Based Representation by M. Dao, et.al.

Thank you

