Distributed Co-Phasing

Comparative Performance Analysis of Transmission Techniques for Wireless Sensor Networks

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Outline

Wireless Sensor Networks

- Collection of nodes dedicated to perform a specific task
- Can be deployed in hard-to-reach places
- WSNs can be used in a variety of sensing and monitoring applications
    - Environmental and habitat monitoring
    - Healthcare applications
    - Military applications
    - Home automation
    - Traffic monitoring
    - Quality control, inventory management and other commercial applications

Wireless Sensor Networks

Characteristics

- Low-cost, simple devices
- Battery powered and/or energy harvesting, highly power constrained
- Random topology, densely deployed
- Little processing power and communication complexity
- Often: correlated observations

Challenges

- Fading wireless channel: low throughput, prone to link outages
- Nodes prone to failures: distributed processing highly desirable
- Time varying network topology
- Limitations on the processing, storage and communication complexity

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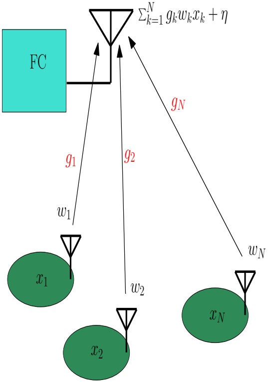
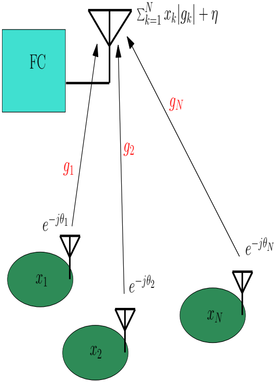
Distributed Beamforming

- Beamforming: Use multiple antennas to transmit a common information such that it adds coherently at the receiver
  - Can reduce the transmit power required/ increased range
  - Increased security and decreased interference
- Distributed Transmit Beamforming: Attempt beamforming with distributed nodes



Distributed Co-Phasing

- Pre-compensate for the channel angle before transmission.
- Reduces to a real-valued channel, if the channel angle perfectly known
- Requires channel angle estimation at the nodes



Technical Challenges for Achieving DCP

- Carrier frequency, phase and timing synchronization across nodes
- Common information across nodes
- Acquisition of channel state information (CSI) at the nodes
  - CSI feedback from destination (FC) is expensive in large scale networks
  - Possible approaches
    - Periodic pilot transmission from FC (works for TDD)
    - Low-rate feedback based tracking
- Nodes are power constrained
  - Energy efficiency and lifetime maximization are major concerns
  - Energy not shared across nodes

Literature Survey

- An overview of the initial results in DTB [Mudumbai et al., 2009]
- Schemes to achieve synchronization [Mudumbai et al., 2007], [Preuss et al., 2010], [Berger et al., 2007]
- A feedback based approach to DTB [Mudumbai et al., 2010], [Che Lin. et al., 2010], [Bucklew et al., 2008]
- Optimum beamforming weights under various constraints [Dong et al., 2009], [Jing et al., 2009], [Havary-Nassab et al., 2008]

Outline

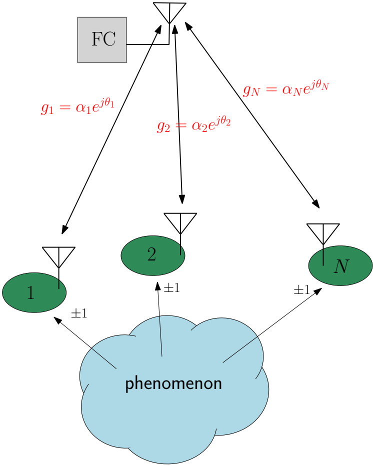
System Model

Sensors pre-compensate for the channel angle before transmission

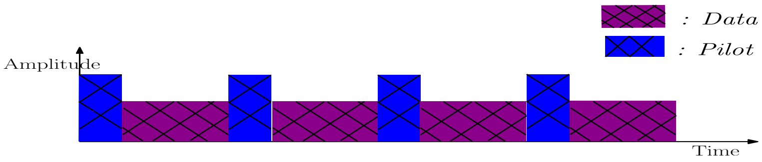
Assumptions

- Time division duplexing model, channel reciprocity
- Perfect carrier frequency and phase synchronization across sensors
- Flat fading channel between the sensors and the fusion center (FC)

We focus on a comparative performance analysis of transmission techniques under the above model



System Description



- Frame structure assumed for communication.
    - Channel assumed constant over one frame
- Each frame consists of
    - N_P pilot symbols broadcast by FC to sensors for channel estimation
    - N_D data symbols transmitted by sensors to FC
- Complex-baseband channel between sensor k and FC is $g_k = \alpha_k e^{j\theta_k}$.

Channel Estimation

- Pilot of power E_P broadcast by FC is received at sensors k as

$$r_k[n] = g_k\sqrt{E_p} + \eta_k[n], \quad n = 1, 2, \ldots, N_P.$$

- ML channel estimation at the sensor k
  - $\widehat{\theta}_k$: Estimated phase at sensor k

$$\widehat{\theta}_k = \tan^{-1}\left(\frac{\Im\left\{\frac{1}{N_P}\sum_{n=1}^{N_P} r_k[n]\right\}}{\Re\left\{\frac{1}{N_P}\sum_{n=1}^{N_P} r_k[n]\right\}}\right)$$

  - $\widehat{\alpha}_k$: Estimated gain at sensor k

$$\widehat{\alpha}_k = \left|\frac{1}{N_P\sqrt{E_P}}\sum_{n=1}^{N_P} r_k[n]\right|$$

Uplink Transmission

- Sensors pre-rotate channel angle before transmission
- They transmit at the same time on a common frequency band
- The received signal at the FC is:

$$r[n] = \sum_{k=1}^{N} x_k[n] e^{-j\widehat{\theta}_k} g_k + \eta[n].$$

- Detection scheme at the FC:
  - Decision statistic at the FC is $\Re\{r[n]\}$.
  - Detection scheme is a simple threshold test on the decision statistic

$$\Re\{r[n]\} \lessgtr 0$$

Four Transmission schemes

- We consider four transmission schemes.
  - Use different amount of CSI
  - Different power allocation strategies at transmitting sensors
- Fixed power allocation schemes:
  - DCP scheme (Baseline DCP scheme)
  - Censoring sensors scheme (CS-C1 and CS-C2 schemes)
- Variable power allocation schemes:
  - Truncated Channel Inversion (TCI) scheme
  - Maximum Ratio Transmission (MRT) scheme

Contributions of This Thesis

- Propose four transmission techniques
- Analyze the characteristics of the phase error due to channel estimation
- Analyze the performance of the transmission techniques in terms of average received SNR at the FC
    - Characterize the received SNR in the presence of channel estimation errors at the transmitting nodes
- Develop an approximate expression for the average BER at the FC for the baseline DCP transmission scheme
- Compare the performance of the transmission techniques
    - Compare DCP based transmission with DSTC based transmission scheme through simulations

Outline

Distributed Co-Phasing

- Received signal at the FC with DCP:

$$r[n] = \sum_{k=1}^{N} x_k[n] e^{-j\widehat{\theta}_k} g_k + \eta[n],$$

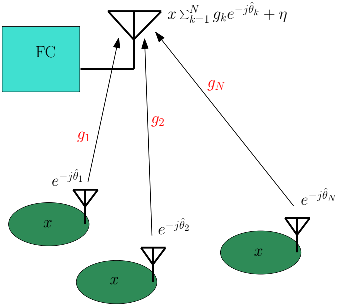
  where $x_k[n] = b_k[n]\sqrt{E_s}$ and $\eta[n] \sim \mathcal{CN}(0, 2\sigma_N^2)$.

- With perfect correlation of sensor observations, the received signal is:

$$r[n] = x[n] \sum_{k=1}^{N} e^{-j\widehat{\theta}_k} g_k + \eta[n]$$

- The decision variable at FC is:

$$r_r[n] = \Re\{r[n]\} = x[n] \sum_{k=1}^{N} \alpha_k \cos\theta_{e,k} + \eta_r[n],$$



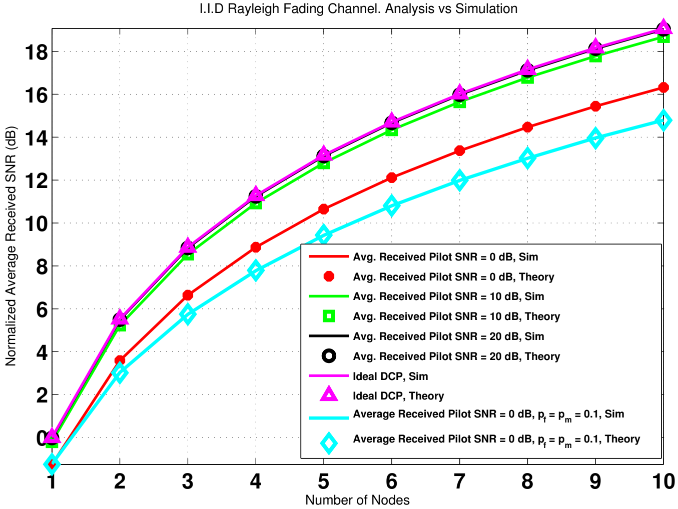
Average Received SNR

- Average received SNR at the FC: (for identical bits at the sensors)

$$\overline{\gamma}_{DCP} = \frac{E_s}{\sigma_n^2} \sum_{k=1}^{N} E\left[\alpha_k^2 \cos^2\left(\theta_{e,k}\right)\right] + \frac{E_s}{\sigma_n^2} \sum_{k=1}^{N} \sum_{j=1, j\neq k}^{N} E\left[\alpha_k \cos\left(\theta_{e,k}\right)\right] E\left[\alpha_j \cos\left(\theta_{e,j}\right)\right]$$

- For non-identical bits at sensors
  - Assume sensors detect an underlying hypothesis with a common false probability P_{FA} and missed detection probability P_M.
  - Assuming conditionally independent decisions, $E[b_i b_j] = (1 - 2P_{FA})^2$ if hypothesis H_0 is true.
  - Here, $x_k[n] = b_k\sqrt{E_s}$ and hence $\bar{\gamma}_{DCP} = \frac{2E_s}{N_0} E\left\{\left|\sum_{k=1}^{N} b_k \alpha_k \cos\theta_{e,k}\right|^2\right\}$.

Average Output SNR Improvement



Rayleigh fading channel

Average BER

- With perfect detection, the decision statistic at FC is:

$$r_r[n] = x[n] \sum_k \cos\theta_{e,k} + \eta_r[n]$$

- Average BER is obtained by averaging the conditional probability of error

$$P_e(\alpha_1, \theta_{e,1}, \ldots, \alpha_N, \theta_{e,N}) = \mathcal{Q}\left(\sqrt{\frac{2E_S}{N_0}} \sum_{k=1}^{N} \alpha_k \cos\theta_{e,k}\right)$$

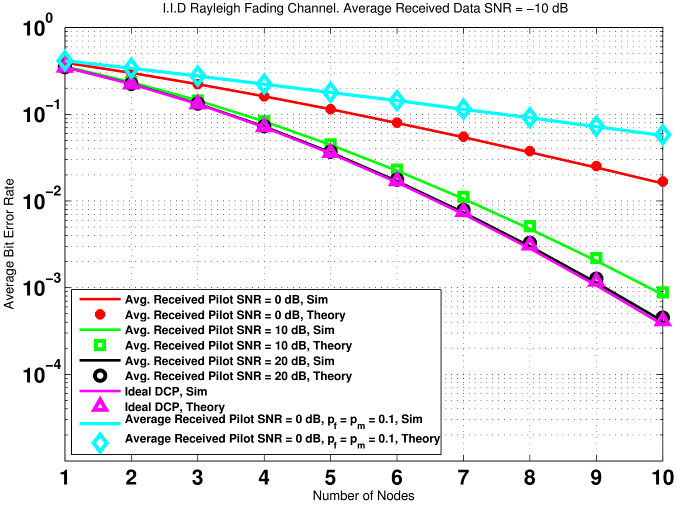
- An approximation, called the Improved Gaussian Approximation [Holtzman, 92] for $\overline{P}_e$ is

$$\begin{aligned}\overline{P}_e \quad \approx \quad & \frac{2}{3}\mathcal{Q}(E[R]) + \frac{1}{6}\mathcal{Q}\left(E[R] + \sqrt{3}\sqrt{E[R^2] - E[R]^2}\right) \\ & + \frac{1}{6}\mathcal{Q}\left(E[R] - \sqrt{3}\sqrt{E[R^2] - E[R]^2}\right),\end{aligned}$$

  where $R \triangleq \sqrt{2E_s N_0} \sum_k \alpha_k \cos\theta_{e,k}$.

- Similar approximation possible for the case of imperfect detection also; average over the distribution of the sensor decisions

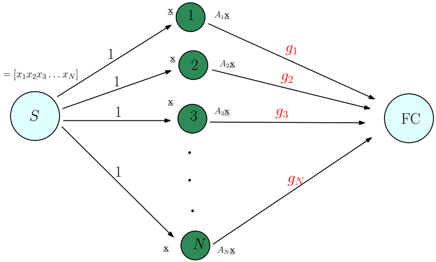
Average BER



Rayleigh fading channel

Comparison With DSTC

System model derived from [Jing et al., 2006][1]



- Source to sensors channels are unit gain and noise-free; this makes system comparable to DCP
- DSTC requires full CSI at the FC
- DCP requires channel angle knowledge at sensors only

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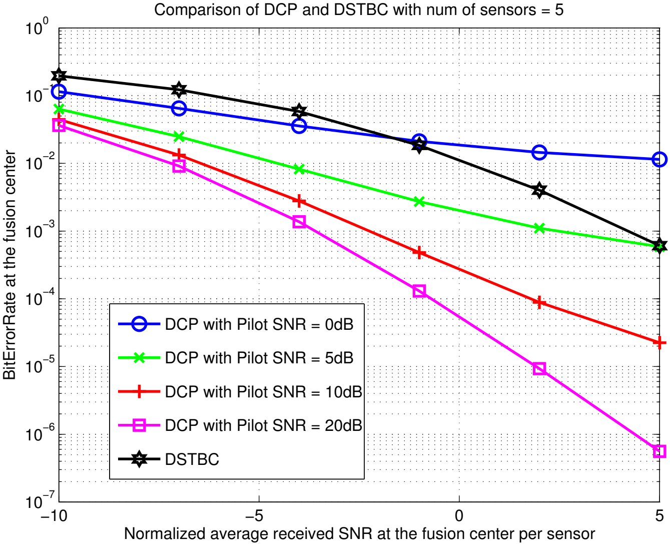
[1] Y. Jing and B. Hassibi, "Distributed Space-Time Coding in Wireless Relay Networks", Trans. on Wirless Communication, 2006

Comparison with DSTC



Outline

Censoring Sensors

- Sensors censor their transmission if the channel gain is below a threshold T
- Save power in bad channel states, as contribution to decision is small
- Two schemes considered here:
  - Case 1, CS-C1: Boost transmit power when channel is good enough
  - Case 2, CS-C2: Vary total number of sensors with threshold
- Fixed power transmission at nodes subject to a long-term power constraint
- Performance in terms of average received SNR at FC is analyzed

System Model

Censoring Sensors based on channel state

- Fixed number of sensors, with power boosting (CS-C1)
    - Received signal at the fusion center:

$$r[n] = \sum_{k=1}^{N} x_k[n] 1_{\{\widehat{\alpha}_k > T\}} e^{-j\widehat{\theta}_k} g_k + \eta[n] \quad n = N_P + 1, \ldots, N_P + N_D$$

where $x_k[n] = b_k \sqrt{\frac{E_s}{p_{T,k}}}$ and $p_{T,k} = \mathrm{Prob}\left\{\widehat{\alpha}_k > T\right\}$

- Fixed power sensors, vary N with threshold T (CS-C2)
    - For N_{eff} sensors to transmit on average:

$$N = \lceil N_{eff}\ \mathrm{Prob}\{\hat{\alpha}_k > T\} \rceil$$

- Probability of transmission:

$$\mathrm{Prob}\{\hat{\alpha}_k > T\} = \exp\left(-\frac{T^2}{\Omega_k + 2\sigma^2}\right)$$

Average Received SNR

- The decision variable at the FC is:

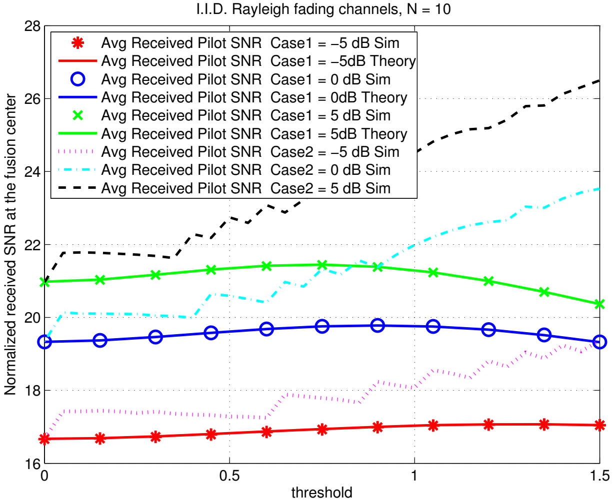
$$z[n] = \sum_{k=1}^{N} x_k[n] \underbrace{1_{\{\hat{\alpha}_k > T\}} \cos\theta_{e,k}\alpha_k}_{\triangleq z_k} + \underbrace{\eta[n]}_{\sim\mathcal{CN}(0,{\sigma_N}^2)} \qquad n = N_P+1,\ldots,N_P+N_D$$

- The average received SNR at the FC:

$$\bar{\gamma}_{CS,1} = \frac{E_s}{{\sigma_N}^2}\left[\sum_{k=1}^{N}\frac{1}{p_{T,k}}E\left[{z_k}^2\right] + \sum_{k=1}^{N}\sum_{j=1,j\neq k}^{N}\frac{1}{\sqrt{p_{T,k}p_{T,j}}}E\left[b_k b_j\right]E\left[z_k\right]E\left[z_j\right]\right]$$

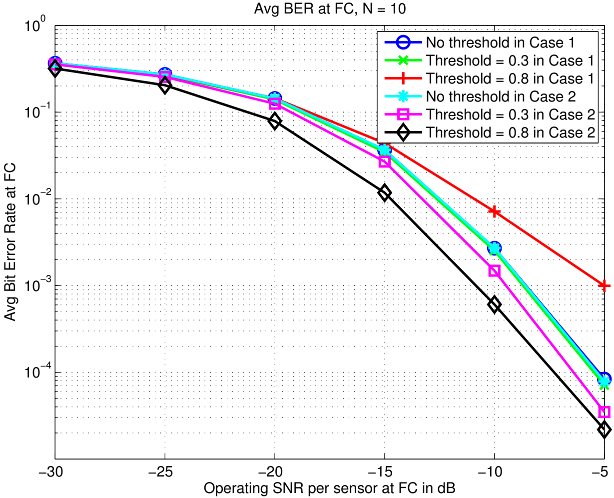
- Expressions are derived for $E[z_k^2]$ and $E[z_k]$ for Rayleigh fading channels.

Average Received SNR Plots



Rayleigh fading channel

Average BER Plots



Rayleigh fading channel

Outline

Variable Power Allocation Schemes

- Sensors allocate a variable power for tranmission based on estimated channel gain
- A long-term average power constraint imposed
- Analyze two schemes:
    - MRT: Power allocated is proportional to square of estimated channel gain
    - TCI: Sensors invert the channel whenever it is above a threshold α_{min}.
- Analyze performance of these schemes in terms of average received SNR at FC

Truncated Channel Inversion

- The received signal at the FC:

$$r[n] = \sum_{k=1}^{N} \sqrt{P(\hat{\alpha}_k)} b_k e^{-j\hat{\theta}_k} g_k + \eta[n]$$

- Choose power allocation scheme at sensor i, subject to an average power constraint as:

$$P(\hat{\alpha}_i) = \begin{cases} \frac{P_0}{\hat{\alpha}_i^2} & \text{if } \hat{\alpha}_i > \alpha_{\text{min}} \\ 0 & \text{else} \end{cases}$$

- To satisfy the average power constraint $\bar{P}$, need

$$\frac{P_0}{\Omega + 2\sigma^2} E_i \left(\frac{\alpha_{\text{min}}^2}{\Omega + 2\sigma^2} \right) = \bar{P}$$

Average Received SNR

- Average received SNR at the FC with TCI:

$$\begin{aligned}\bar{\gamma}_{TCI} = \quad & \sum_{i=1}^{N} E\left[P\left(\hat{\alpha}_i\right)\alpha_i{}^2\cos^2\theta_{e,i}\right] \\ & + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} E\left[\sqrt{P\left(\hat{\alpha}_i\right)}\alpha_i\cos\theta_{e,i}\right] E\left[\sqrt{P\left(\hat{\alpha}_j\right)}\alpha_j\cos\theta_{e,j}\right]\end{aligned}$$

- With i.i.d. channels,

$$\bar{\gamma}_{TCI} = NE\left[P\left(\hat{\alpha}\right)\alpha^2\cos^2\theta_e\right] + N\left(N-1\right)E\left[\sqrt{P\left(\hat{\alpha}\right)}\alpha\cos\theta_e\right].$$

- Closed form expressions for the expectation terms above are derived for Rayleigh fading channels.

Maximum Ratio Transmission Scheme

- The power allocation at the sensors is $P\left(\widehat{\alpha}_k\right) = P_{0,k}\widehat{\alpha}_k^2$.
- Sensors are constrained by an average long-term power constraint $\bar{P}$, i.e., $E\left[\left|\sqrt{P_{0,k}}\widehat{\alpha}_k^2\right|^2\right] = \bar{P}$.
- Sensor power level at the kth sensor, $P_{0,k}$ is:

$$P_{0,k} = \frac{\bar{P}}{\Omega_k + 2\sigma^2}$$

- Decision statistic at the fusion center is given by:

$$y_R = \sum_{k=1}^{N} P_{0,k} x_k \widehat{\alpha}_k \alpha_k \cos\theta_{e,k} + \eta_R$$

Average Received SNR

- Average received SNR at the FC:

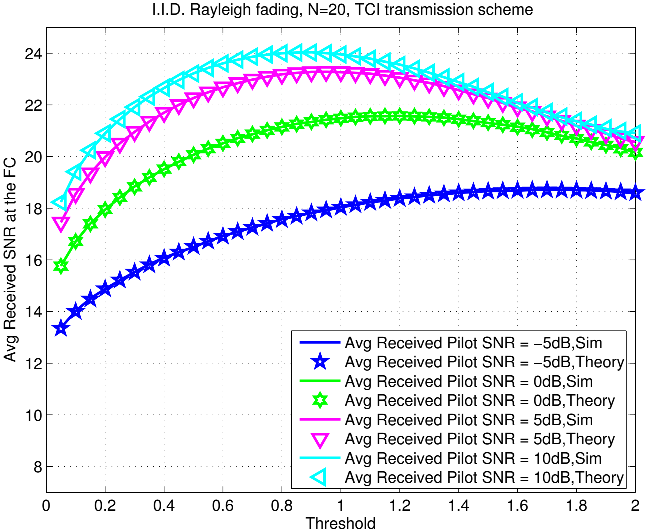
$$\gamma_{MRT} = \frac{1}{\sigma_N^2}\left[\sum_{k=1}^{N} E\left[x_k^2\right] E\left[u_k^2\right] + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} E\left[x_i x_j\right] E\left[u_i\right] E\left[u_j\right]\right],$$

  where $u_k = \alpha_k \widehat{\alpha}_k \cos\theta_{e,k}$.

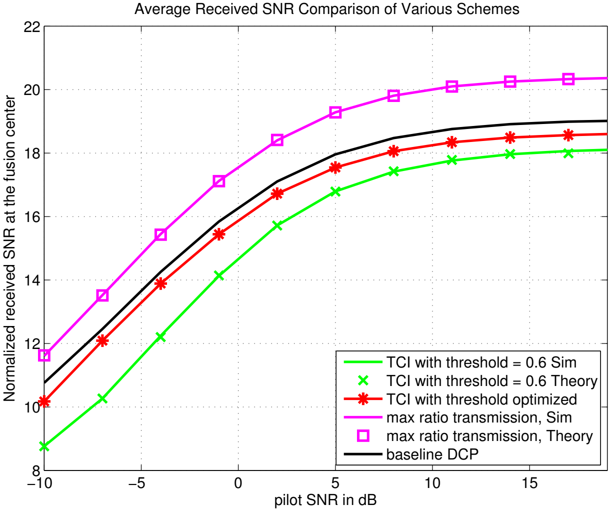
- Closed-form expressions for the expectation terms above can be derived for Rayleigh fading channels as:

$$\begin{aligned} E\left[u_k\right] &= \Omega_k \\ E\left[u_k^2\right] &= \Omega_k^2\left(2 + \frac{\sigma^2}{\Omega_k}\right). \end{aligned}$$

Average Received SNR Plot with TCI

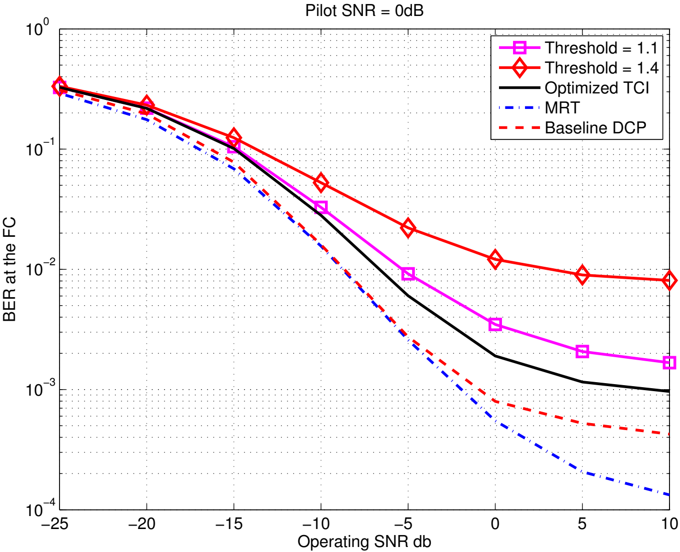


Average Received SNR Comparison



Rayleigh fading channel

Average BER Plot



Outline

Conclusions

- We presented a pilot based DCP system that exploits channel reciprocity for channel estimation
- Analyzed the performance of the system under various transmission scenarious
- Verified the analysis using Monte-Carlo simulations
- Compared the performance of the systems through simulations
- Showed that DCP type transmission is a promising technique for use in WSN type scenario

Possible extensions

- Performance analysis when the FC also estimates the channel
- Extending the DCP transmission to include higher order modulations
- Allowing for partial cooperation among the sensors
- Diversity analysis of the DCP scheme
- Analysis of the influence of synchronization errors on the performance

Publication

- Krishna Chaythanya K. V., Ramesh Annavajjala, Chandra R. Murthy, "Comparative Analysis of Pilot-Assisted Distributed Co-Phasing Approaches in Wireless Sensor Networks" *Submitted to IEEE Trans. on Signal Processing*
    - Revised and resubmitted to IEEE Trans. on Signal Processing.

Thank you!

Backup Slides!

Phase Error Statistics

- CDF of unconditional phase error at sensor k

$$F_{|\theta_{e,k}|}(x) = 1 - \int_0^{\pi - x} \mathcal{L}_{\gamma_k}\left(\frac{\overline{\gamma}_p \sin^2 x}{\sin^2 \beta}\right) \frac{d\beta}{\pi} \quad 0 \leq x < \pi,$$

  where $\gamma_k = \alpha_k^2$, $\mathcal{L}_{\gamma_k}(s)$ is the laplace transform of the p.d.f. of γ_k and $\overline{\gamma}_p$ is the pilot SNR

- Probability of Signal Corruption
  - Effective channel from sensor k to FC after DCP is $\alpha_k \cos\left(\theta_k - \widehat{\theta}_k\right)$
  - Contribution is negative when: $|\theta_{e,k}| > \frac{\pi}{2}$
  - Probability of Signal Corruption

$$\begin{aligned} P_{SC} &= \mathrm{Prob}\left(|\theta_{e,k}| > \pi/2\right) \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_p \Omega_k}{1 + \overline{\gamma}_p \Omega_k}}\right) \quad \text{(Rayleigh fading)}, \end{aligned}$$

  where $\Omega_k = E\left[\alpha_k^2\right]$

Average SNR Plot, Tweaked TCI Scheme