Joint Sparse Channel Estimation and Data Detection in time-varying OFDM Systems using Bayesian Learning

Ranjitha P

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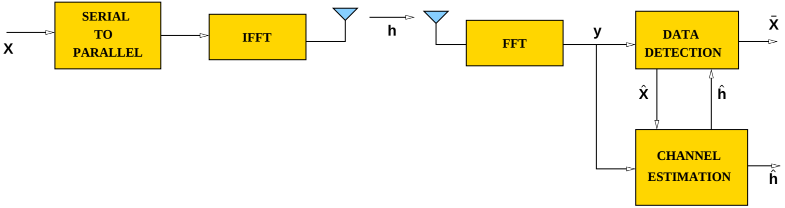
1. Sparse Bayesian Learning in OFDM Systems
   - OFDM System Model, EM and the SBL Algorithm
   - Proposed Algorithm
   - SBL for an OFDM Frame
   - Simulation Results

OFDM System Model and EM Algorithm



- The received signal **y** is given by,

$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h} + \mathbf{v}$$

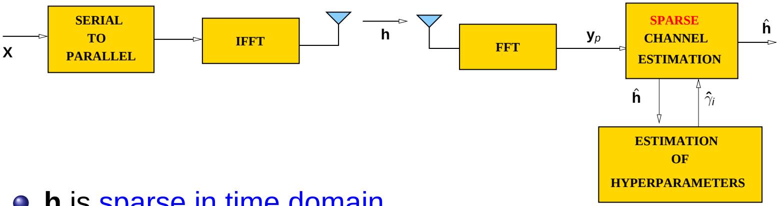
Support Aware EM Algorithm for Channel Estimation and Data Detection

$$\text{E-step}: Q\left(\mathbf{X}/\mathbf{X}^{(p)}\right) = E_{\mathbf{h}/\mathbf{y},\mathbf{X}^{(p)}}\left(\log p(\mathbf{y},\mathbf{h}/\mathbf{X})/\mathbf{y},\mathbf{X}^{(p)}\right)$$

$$\text{M-step}: \mathbf{X}^{(p+1)} = \arg\max_{\mathbf{X}} Q\left(\mathbf{X}/\mathbf{X}^{(p)}\right)$$

- $\log p(\mathbf{y},\mathbf{h}/\mathbf{X}) = \underbrace{\log p(\mathbf{y}/\mathbf{h},\mathbf{X})}_{\text{Log Likelihood, func. of } \mathbf{X}} + \underbrace{\log p(\mathbf{h})}_{\text{not a func. of } \mathbf{X}}$

SBL for Channel Estimation



- **h** is sparse in time domain
- $\mathbf{h}(i) \sim \mathcal{CN}(0, \gamma_i)$, where γ_i is a deterministic but unknown hyperparameter
- The sparsity profile: $\Gamma = \text{diag}(\gamma_1, \gamma_2 \ldots \gamma_L)$, i.e., if the diagonal entries $\gamma_i = 0$, $h_i = 0$

SBL for Basis Selection

$$\text{E step}: Q(\Gamma/\Gamma^{(p)}) = E_{\mathbf{h}/\mathbf{y};\Gamma^{(p)}}(\log p(\mathbf{y},\mathbf{h};\Gamma))$$

$$p(\mathbf{h}/\mathbf{y};\Gamma^{(p)}) = \mathcal{N}(\mu,\Sigma_h), \quad \mu = \sigma^{-2}\Sigma_h\mathbf{A}^H\mathbf{y}$$

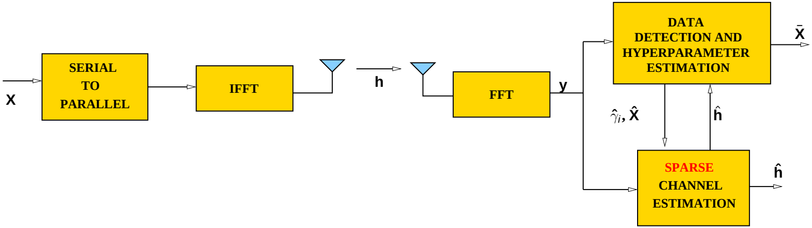
$$\Sigma_h = \left(\sigma^{-2}\mathbf{A}^H\mathbf{A} + \Gamma^{(p)^{-1}}\right)^{-1}, \mathbf{A} \triangleq \mathbf{XF}$$

$$\text{M-step}: \Gamma^{(p+1)} = \arg\max_{\gamma_i>0} Q(\Gamma/\Gamma^{(p)})$$

$$\log p(\mathbf{y},\mathbf{h};\Gamma) = \underbrace{\log p(\mathbf{y}/\mathbf{h})}_{\text{not a func. of } \gamma_i} + \underbrace{\log p(\mathbf{h};\Gamma)}_{\text{func. of } \gamma_i}$$

Upon convergence, many of the γ_i are driven to zero

Proposed Algorithm



- The posterior pdf of **h** is estimated in the E-step
- In the M-step, $\log p(\mathbf{y}/\mathbf{h}, \mathbf{X})$ is used to find the ML estimate of **X** and $\log p(\mathbf{h}; \Gamma)$ is used to find the ML estimate of γ_i

Combined Algorithm

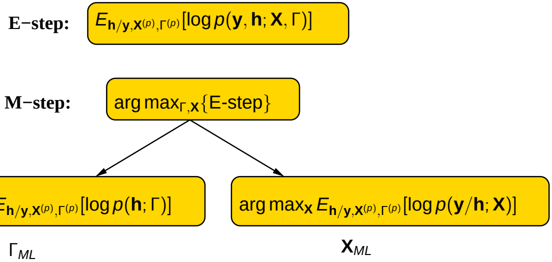


Figure: Proposed Algorithm

OFDM frame

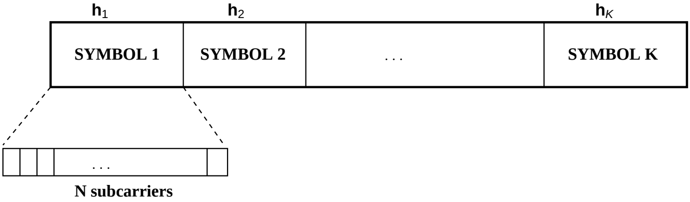


Figure: OFDM frame

- $\mathbf{h}_1 = \ldots = \mathbf{h}_K = \mathbf{h}$: Block fading scenario
- $\mathbf{h}_k = \rho \mathbf{h}_{k-1} + \mathbf{u}_k$: Time-varying scenario

- Joint pdf:

$$p(\mathbf{Y}_{p,K}, \mathbf{h}_1, \ldots, \mathbf{h}_K; \gamma) = \prod_{k=1}^{K} p(\mathbf{y}_{p,k} | \mathbf{h}_k) p(\mathbf{h}_k | \mathbf{h}_{k-1}; \gamma),$$

  where $\mathbf{Y}_{p,K} = [\mathbf{y}_{p,1}, \ldots, \mathbf{y}_{p,K}]$ and $p(\mathbf{h}_1 | \mathbf{h}_0) \triangleq p(\mathbf{h}_1)$.
- Optimization problem:

$$(P3) \qquad \hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_K, \hat{\gamma} = \arg\min_{\mathbf{h}_1, \ldots, \mathbf{h}_K, \gamma} f(\mathbf{h}_1, \ldots, \mathbf{h}_k, \gamma),$$

  where

$$f(\mathbf{h}_1, \ldots, \mathbf{h}_k, \gamma) = \sum_{k=1}^{K} \frac{\|\mathbf{y}_{p,k} - \mathbf{X}_{p,k} \mathbf{F}_p \mathbf{h}_k\|_2^2}{\sigma^2} + K \log |\Gamma| + \mathbf{h}_1^H \Gamma^{-1} \mathbf{h}_1$$

$$+ (K-1) \log(1 - \rho^2) + \sum_{k=2}^{K} \frac{(\mathbf{h}_k - \rho \mathbf{h}_{k-1})^H \Gamma^{-1} (\mathbf{h}_k - \rho \mathbf{h}_{k-1})}{(1 - \rho^2)}$$

Measurement equation and the state equation:

$$\mathbf{y}_{p,k} = \Phi_{p,k}\mathbf{h}_k + \mathbf{v}_{p,k},$$
$$\mathbf{h}_k = \rho\mathbf{h}_{k-1} + \mathbf{u}_k, \quad k = 1, 2, \ldots, K$$

If Γ is known:

$$
\begin{aligned}
\text{Prediction:}\quad \hat{\mathbf{h}}_{k|k-1} &= \rho\hat{\mathbf{h}}_{k-1|k-1} \\
\mathbf{P}_{k|k-1} &= \rho^2\mathbf{P}_{k-1|k-1} + (1-\rho^2)\Gamma \\
\text{Filtering:}\quad \hat{\mathbf{h}}_{k|k} &= \hat{\mathbf{h}}_{k|k-1} + \mathbf{G}_k(\mathbf{y}_{p,k} - \Phi_{p,k}\hat{\mathbf{h}}_{k|k-1}) \\
\mathbf{G}_k &= \mathbf{P}_{k|k-1}\Phi_{p,k}^H\left(\sigma^2\mathbf{I}_N + \Phi_{p,k}\mathbf{P}_{k|k-1}\Phi_{p,k}^H\right)^{-1} \\
\mathbf{P}_{k|k} &= (\mathbf{I}_L - \mathbf{G}_k\Phi_{p,k})\mathbf{P}_{k|k-1}, \\
\text{Smoothing:}\quad \hat{\mathbf{h}}_{k-1|k} &= \hat{\mathbf{h}}_{k-1|k-1} + \mathbf{J}_{k-1}(\hat{\mathbf{h}}_{k|k} - \hat{\mathbf{h}}_{k|k-1}) \\
\mathbf{P}_{k-1|k} &= \mathbf{P}_{k-1|k-1} + \mathbf{J}_{k-1}(\mathbf{P}_{k|k} - \mathbf{P}_{k|k-1})\mathbf{J}_{k-1}^H
\end{aligned}
$$

where $\mathbf{J}_{k-1} = \rho\mathbf{P}_{k-1|k-1}\mathbf{P}_{k|k-1}^{-1}$

If Γ is unknown: use EM. The E and M steps are given by

$$\text{E-step}: Q(\gamma|\gamma^{(p)}) = \mathbb{E}_{\mathbf{h}_1,\ldots,\mathbf{h}_k|\mathbf{Y}_{p,k};\gamma^{(p)}}[\log p(\mathbf{Y}_{p,k}, \mathbf{h}_1, \ldots, \mathbf{h}_k; \gamma)] \tag{1}$$

$$\text{M-step}: \gamma^{(p+1)}(i) = \arg\max_{\gamma(i)} Q(\gamma|\gamma^{(p)}) \quad i = 1, \ldots, L. \tag{2}$$

The M-step given above results in the following optimization problem.

$$Q(\gamma|\gamma^{(p)}) = \mathbb{E}_{\mathbf{h}_k|\mathbf{Y}_{p,k};\gamma^{(p)}}\Bigg[c - \sum_{j=2}^{k} \frac{(\mathbf{h}_j - \rho\mathbf{h}_{j-1})^H \Gamma^{-1} (\mathbf{h}_j - \rho\mathbf{h}_{j-1})}{2(1-\rho^2)} - \frac{k}{2}\log|\Gamma| - \frac{1}{2}\mathbf{h}_1^H \Gamma^{-1}\mathbf{h}_1\Bigg]$$

EM-KSBL

- The function to be maximized in the M-step is given by

$$Q(\gamma, \mathbf{X}_k | \gamma^{(p)}, \mathbf{X}_k^{(p)}) = \mathbb{E}_{\mathbf{h}_k | \mathbf{Y}_k ; \mathbf{X}_k^{(p)}, \gamma^{(p)}} \Bigg[c'' - \sum_{j=1}^{k} \frac{\| \mathbf{y}_j - \mathbf{X}_j \mathbf{F} \mathbf{h}_j \|^2}{2\sigma^2} - \frac{k}{2} \log |\Gamma| - \sum_{j=2}^{k} \frac{(\mathbf{h}_j - \rho \mathbf{h}_{j-1})^H \Gamma^{-1} (\mathbf{h}_j - \rho \mathbf{h}_{j-1})}{2(1-\rho^2)} - \frac{1}{2} \mathbf{h}_1^H \Gamma^{-1} \mathbf{h}_1 \Bigg]$$

- Expression above is a sum of terms: $Q(\gamma | \gamma_k^{(p)})$ and $Q(\mathbf{X}_k | \mathbf{X}_k^{(p)})$ respectively

K-SBL and EM-KSBL Algorithm

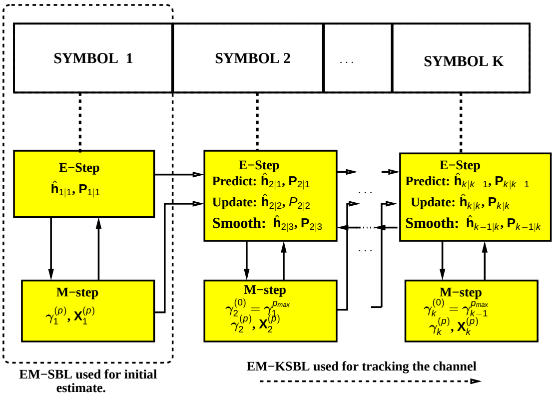


Figure: Block Diagram depicting the EM-KSBL algorithm.

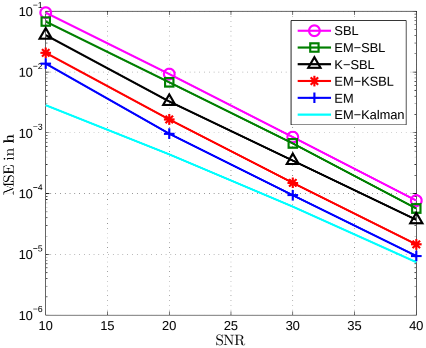


Figure: MSE *vs.* SNR in a time-varying SISO-OFDM system, $f_d T_s = 0.001$.

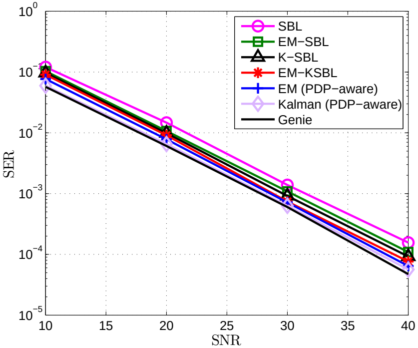


Figure: SER *vs.* SNR in a time-varying SISO-OFDM system, $f_d T_s = 0.001$.

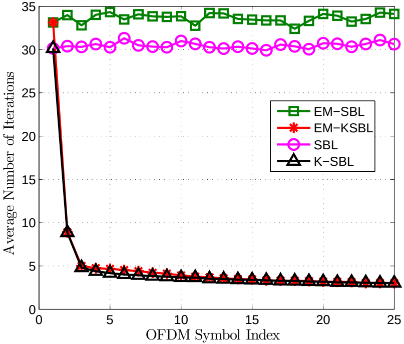


Figure: Number of iterations required for the proposed recursive algorithms, as a function of the OFDM symbol index.

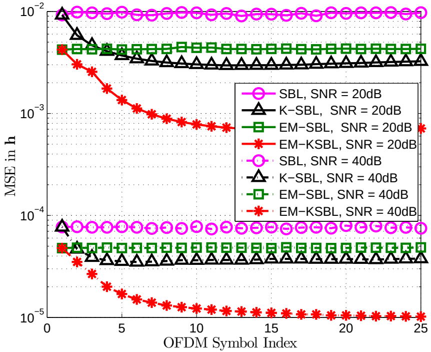


Figure: MSE across an OFDM frame in a time-varying SISO-OFDM system, $f_d T_s = 0.001$.

Thank You