Sparse Signal Recovery Based GNSS Signal Acquisition

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Overview

- GNSS Receiver

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- Signal Acquisition

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- Converting into sparse signal problem

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- Future work

GNSS Receiver

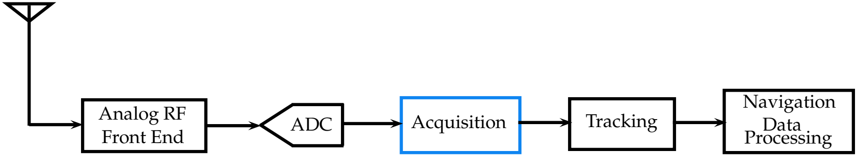
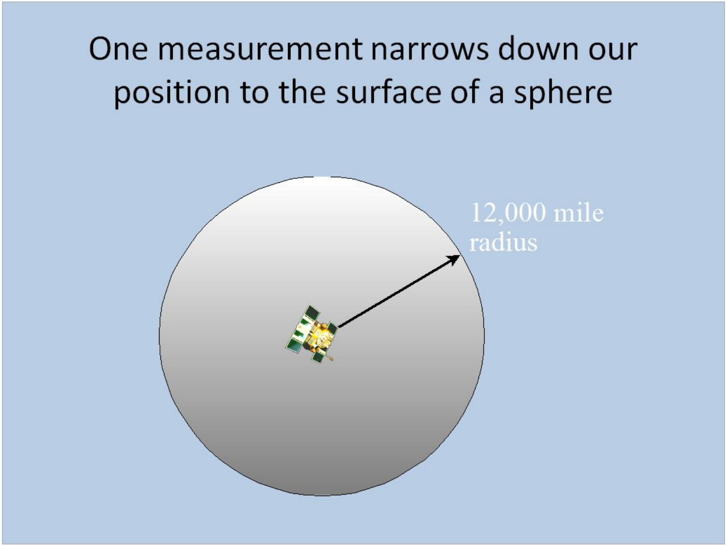


Figure: GNSS Receiver System

GNSS-Global Navigation Satellite System

User Position Determination - Geometric Concept



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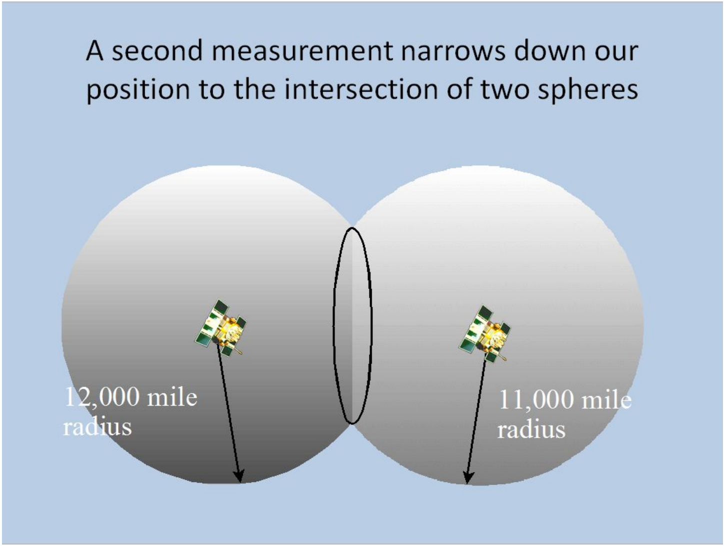

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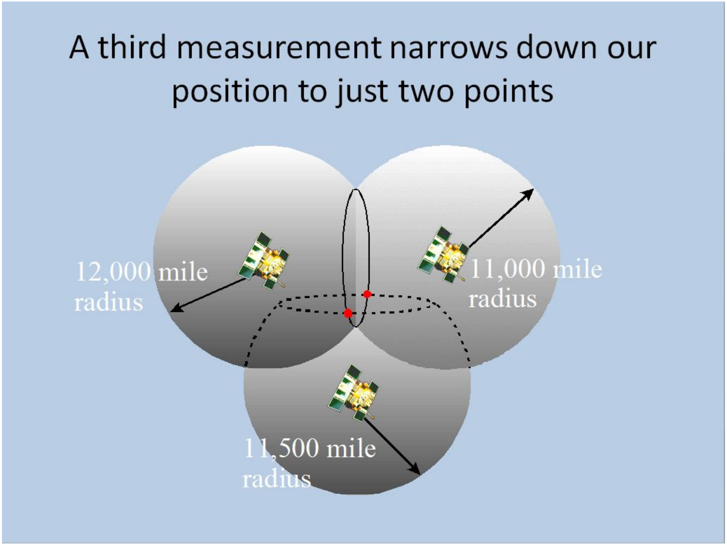

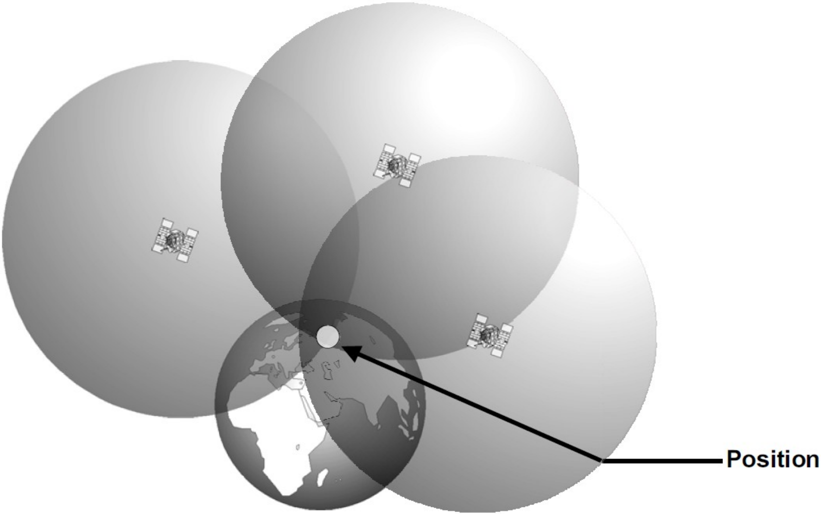

Figure 6: The position is determined at the point where all three spheres intersect

- A two-dimensional position determination requires at only three visible satellites.

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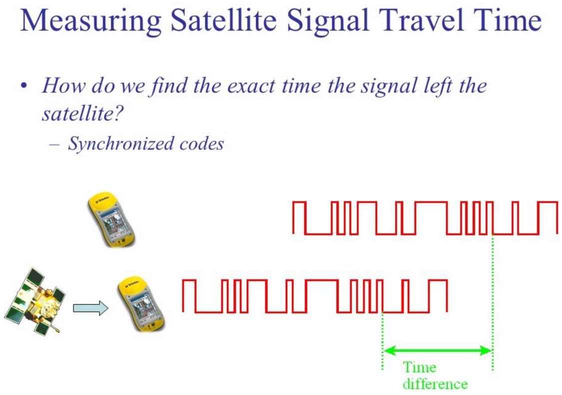
- A three-dimensional position determination requires at least four visible satellites.

- A two-dimensional position determination requires at only three visible satellites.
- A three-dimensional position determination requires at least four visible satellites.
- If the altitude (height) is known, then a two-dimensional position determination can be achieved using at least three visible satellites.

How to find the pseudo range from satellite .. ??

Concept of TOA: Time of Arrival

- The pseudo range, ρ = time of travel $\times$ speed of light, speed of light, $c = 299{,}792{,}458$ m/s.



Satellite Signal

Each signal consists of three components:

- RF carrier frequency.

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Each signal consists of three components:

- RF carrier frequency.
- Pseudorandom noise (PRN) code that serves as a ranging code.
- Navigation message that contains the **ephemeris** and the **almanac** data.

- Solve four set of equations to obtain the user position (U_X, U_Y, U_Z) and Clock Bias (C_B).

$$
\begin{aligned}
(X_1 - U_X)^2 + (Y_1 - U_Y)^2 + (Z_1 - U_z)^2 &= (\rho_1 - C_B)^2 \\
(X_2 - U_X)^2 + (Y_2 - U_Y)^2 + (Z_2 - U_z)^2 &= (\rho_2 - C_B)^2 \\
(X_3 - U_X)^2 + (Y_3 - U_Y)^2 + (Z_3 - U_z)^2 &= (\rho_3 - C_B)^2 \\
(X_4 - U_X)^2 + (Y_4 - U_Y)^2 + (Z_4 - U_z)^2 &= (\rho_4 - C_B)^2
\end{aligned}
$$

where (X_k, Y_k, Z_k) and ρ_k is the position and pseuso range of k^{th} satellite.

How to analyse the received signals from multiple satellites??

Received Signal

$$r[n]$$

- $r[n]$ - output of A/D

Received Signal

$$r[n] = \qquad \sqrt{P_i}$$

- $r[n]$ - output of A/D
- P_i - received signal power

Received Signal

$$r[n] = \quad \sqrt{P_i} c_i[nT_s - \quad]$$

- $r[n]$ - output of A/D
- P_i - received signal power
- $c_i[nT_s - \tau_i]$ - PRN code

Received Signal

$$r[n] = \quad \sqrt{P_i} c_i[nT_s - \quad] b_i[nT_s - \quad]$$

- $r[n]$ - output of A/D
- P_i - received signal power
- $c_i[nT_s - \tau_i]$ - PRN code
- $b_i[nT_s - \tau_i]$ - data bit

Received Signal

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- $r[n]$ - output of A/D
- P_i - received signal power
- $c_i[nT_s - \tau_i]$ - PRN code
- $b_i[nT_s - \tau_i]$ - data bit
- τ_i - code delay

Received Signal

$$r[n] = \quad \sqrt{P_i}c_i[nT_s - \tau_i]b_i[nT_s - \tau_i] \times e^{j(2\pi \quad nT_s + \quad)}$$

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- frequency and phase uncertainity

Received Signal

$$r[n] = \sqrt{P_i} c_i[nT_s - \tau_i] b_i[nT_s - \tau_i] \times e^{j(2\pi f_{d_i} nT_s + \quad)}$$

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- P_i - received signal power
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- frequency and phase uncertainity
  - f_{d_i} - frequency offset

Received Signal

$$r[n] = \sqrt{P_i} c_i[nT_s - \tau_i] b_i[nT_s - \tau_i] \times e^{j(2\pi f_{d_i} nT_s + \theta_i)}$$

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- frequency and phase uncertainity
  - f_{d_i} - frequency offset
  - θ_i - carrier phase

Received Signal

$$r[n] = \sum_{i=1}^{K} \sqrt{P_i} c_i[nT_s - \tau_i] b_i[nT_s - \tau_i] \times e^{j(2\pi f_{d_i} nT_s + \theta_i)}$$

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- $c_i[nT_s - \tau_i]$ - PRN code
- $b_i[nT_s - \tau_i]$ - data bit
- τ_i - code delay
- frequency and phase uncertainity
  - f_{d_i} - frequency offset
  - θ_i - carrier phase
- K - # transmitters

Received Signal

$$r[n] = \sum_{i=1}^{K} \sqrt{P_i} c_i[nT_s - \tau_i] b_i[nT_s - \tau_i] \times e^{j(2\pi f_{d_i} nT_s + \theta_i)} + w[n]$$

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- K - # transmitters
- $w[n] \sim \mathcal{N}(0, \sigma^2)$

Received Signal

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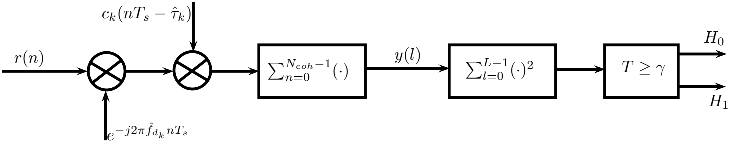
- frequency and phase uncertainity
  - f_{d_i} - frequency offset
  - θ_i - carrier phase
- K - # transmitters
- $w[n] \sim \mathcal{N}(0, \sigma^2)$
- T_s - Sampling time

Signal Acquisition

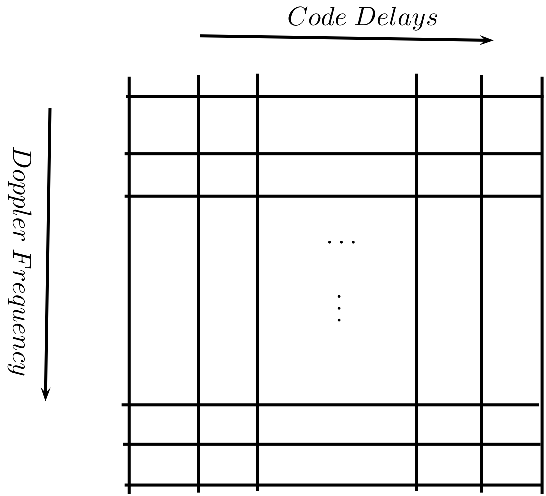


GPS signal acquisition starts with a 2D search process. The goal of the acquisition is to find the visible satellites.

Output of acquisition is

- C/A Code delay, τ and
- The Doppler shift, f_d.

2D -Search



GNSS Signal Structure

Galileo /GPS Signal Characteristics:

GNSS Signal Characteristics

Standard	GPS L_1	Galileo E_1
Modulation	BPSK	BOC(1,1)
Data rate (bps)	50	250
Chip rate (Mcps)	1.023	1.023

Table: GNSS Signal Characteristics

- The GPS coarse/acquisition (C/A) code is a Gold code with a sequence length of 1,023 bits (chips).
- Since the chipping rate of the C/A code is 1.023 MHz, the repetition period of the PRN sequence is $\frac{1{,}023}{1.023\times 10^6 Hz}$ or 1 ms.
- Each data bit has exactly 20 C/A code periods.

Converting into Sparse signal recovery problem

Received signal vector

$$\mathbf{r} = \left[r(0)\ r(1)\ \ldots\ r(N-1) \right]^{\mathbf{T}}$$

$\mathbf{r}$ - $N \times 1$

Code chip matrix

$$\mathbf{C} = \begin{bmatrix} c(1) & c(2) & \ldots & c(N) \\ c(2) & c(3) & \ldots & c(N+1) \\ \ldots & \ldots & \ldots & \ldots \\ c(N) & c(N+1) & \ldots & c(2N) \end{bmatrix}$$

$\mathbf{C}$ - $N \times N$

Converting into Sparse signal recovery problem

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$\mathbf{C}$ - $N \times N$

1. One Satellite, No doppler

$$\mathbf{x} = \mathbf{C} \times \mathbf{r}^{\mathbf{T}} = \begin{bmatrix} 0 \; 0 \; \ldots \; 0 \; 1 \; 0 \end{bmatrix}^{\mathbf{T}}$$

- one sparse vector.

$\mathbf{x}$ - $N \times 1$

Converting into Sparse signal recovery problem

2. One Satellite, P - Doppler frequency bins

$$\tilde{\mathbf{C}} = \left[\mathbf{C} \;\; \mathbf{e}^{\mathbf{j}\mathbf{f}_1}\mathbf{C} \;\; \mathbf{e}^{\mathbf{j}\mathbf{f}_2}\mathbf{C} \;\; \ldots \;\; \mathbf{e}^{\mathbf{j}\mathbf{f}_\mathbf{P}}\mathbf{C}\right]^{\mathbf{T}}$$

$\tilde{\mathbf{C}}$ - $NP \times N$

$$\mathbf{x} = \tilde{\mathbf{C}} \times \mathbf{r}^{\mathbf{T}} = \left[0\;0\;\ldots\;0\;1\;0\;\right]^{\mathbf{T}}$$

- one sparse vector.

$\mathbf{x}$ - $NP \times 1$

Converting into Sparse signal recovery problem

2. One Satellite, P - Doppler frequency bins

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C} & \mathbf{e}^{\mathbf{jf_1}}\mathbf{C} & \mathbf{e}^{\mathbf{jf_2}}\mathbf{C} & \ldots & \mathbf{e}^{\mathbf{jf_P}}\mathbf{C} \end{bmatrix}^{\mathbf{T}}$$

$\tilde{\mathbf{C}}$ - $NP \times N$

$$\mathbf{x} = \tilde{\mathbf{C}} \times \mathbf{r}^{\mathbf{T}} = \begin{bmatrix} 0 \; 0 \; \ldots \; 0 \; 1 \; 0 \end{bmatrix}^{\mathbf{T}}$$

- one sparse vector.

$\mathbf{x}$ - $NP \times 1$

3. K Satellites, P- Doppler frequency bins

$$\mathbf{D} = \begin{bmatrix} \tilde{\mathbf{C}}_1 & \tilde{\mathbf{C}}_2 & \tilde{\mathbf{C}}_3 & \ldots & \tilde{\mathbf{C}}_K \end{bmatrix}^{\mathbf{T}}$$

$\tilde{\mathbf{C}}$ - $NP \times N$

$$\mathbf{x} = \mathbf{D} \times \mathbf{r}^{\mathbf{T}} = \begin{bmatrix} 0 \; 1 \; 0 \; \ldots \; 0 \; 1 \; 0 \; 1 \; 0 \; 1 \; 0 \; 0 \end{bmatrix}^{\mathbf{T}}$$

Converting into Sparse signal recovery problem

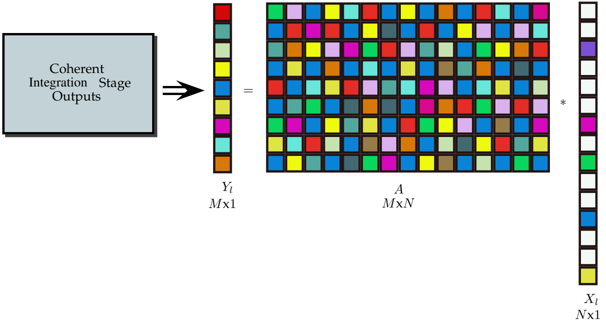
$$\mathbf{Y} = \mathbf{\Phi} \times \underbrace{\mathbf{D} \times \mathbf{r}}_{\mathbf{x}}$$

$$\mathbf{Y} = \mathbf{\Phi} \times \mathbf{x}$$

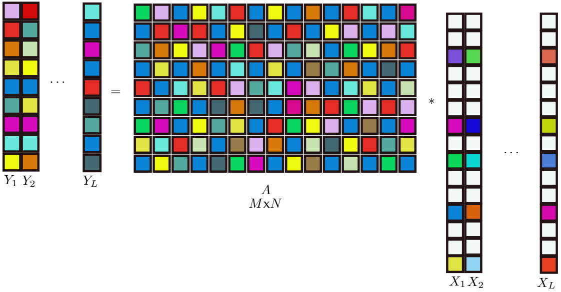
$\mathbf{\Phi}$ - $M \times NPK$ and $\mathbf{x}$ - $NPK \times 1$.

- $\mathbf{x}$ - is a sparse signal, whose sparsity is given by # visible satellites.
- Φ - Dictionary matrix (Assume as Filter bank with corresponding coefficients).
- Y - Measurement matrix at the output of coherent integration stage.

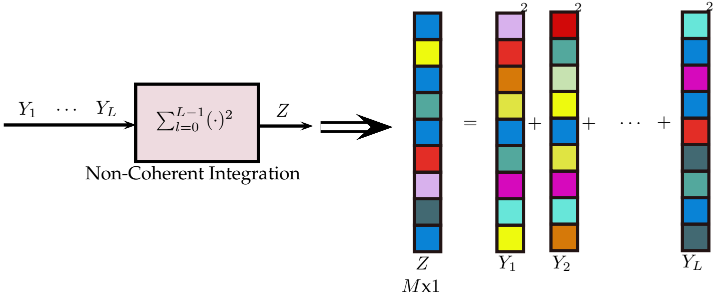
Converting into Sparse signal recovery problem



- Modeling Coherent integration output in CS form where.
- M − # filters used at the receiver and
- N − # satellites × # time bins × # frequency bins.

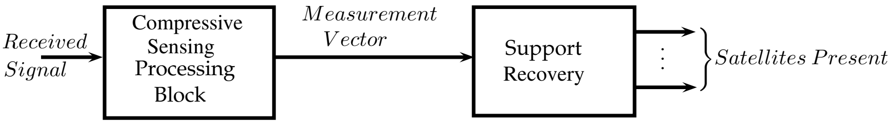


- For L measurements, each $\boldsymbol{X_l}$ has common support, where $l \in [L]$.



- Final measurement vector $\boldsymbol{Z}$ is obtained by noncoherent accumulation over L instants.

CS Processing



Problem Statement

$$\mathbf{Y_l} = \mathbf{AX_l} + \mathbf{w_l}$$

$$\mathbf{Z} = \sum_{l=1}^{L} \mathbf{Y_l}^{.2}$$

where $\mathbf{Y_l} \in \mathbb{R}^N$, $\mathbf{Z} \in \mathbb{R}^M$, $\mathbf{A} \in \mathbb{R}^{M \times N}$, $\mathbf{X_l} \in \mathbb{R}^N$, $l \in (1, 2, ..., L)$ and each $\mathbf{X_l}$ is sparse and have common support.

- Problem of signal detection in GNSS acquistion is equivalent to recovering the support of $\mathbf{X_l}$ from $\mathbf{Z}$ and $\mathbf{A}$.

Problem: Recover supp($\mathbf{X_l}$) from $\mathbf{Z}$ and $\mathbf{A}$.

Prospoed Solution

Problem is solved by converting the power measurements into Rank-one measurements.
We have, each measurement,

$$z_i = \sum_{l=1}^{L} (\boldsymbol{a}_i \boldsymbol{X}_l)^2$$

where $\boldsymbol{a}_i{}^T$ is i^{th} row in $\boldsymbol{A}$, $i \in [M]$ and $\boldsymbol{a}_i{}^T$ is $N \times 1$.
Consider

$$z_i = \frac{1}{L} \sum_{l=1}^{L} (\boldsymbol{a}_i^T \boldsymbol{X}_l)^2$$
$$= \sum_{l=1}^{L} (\boldsymbol{a}_i^T \boldsymbol{X}_l) \times (\boldsymbol{a}_i^T \boldsymbol{X}_l)^T$$

$$
\begin{aligned}
&= \boldsymbol{a}_i^T \left(\frac{1}{L} \sum_{l=1}^{L} \boldsymbol{X}_l \boldsymbol{X_l}^T \right) (\boldsymbol{a}_i^T)^T \\
&\approx \boldsymbol{a}_i^T \boldsymbol{X}_0 (\boldsymbol{a}_i^T)^T
\end{aligned}
$$

where $\boldsymbol{X}_0 = \mathbb{E}[\boldsymbol{X}_l \boldsymbol{X_l}^T]$ corresponds to the covariance matrix of the data when L is sufficiently large.

- The measurements z_i, $i \in [M]$ are quadratic in the sensing vector $\boldsymbol{a_i}$, but linear in $\boldsymbol{X}_0$.

Assumption: Nonzero entries in each $\boldsymbol{X_l}$ is uncorrelated.

$$\boldsymbol{Z} \approx (\boldsymbol{A} \odot \boldsymbol{A})\gamma = \tilde{A}\gamma$$

where $\tilde{A} = \boldsymbol{A} \odot \boldsymbol{A}$, $\gamma = diag(\boldsymbol{X_0}) \in \mathbb{R}^N$ and $\odot$ - Hadamard product.

- Now the goal is to recover γ using $\boldsymbol{Z}$ and $\boldsymbol{A}$, which is similar to well understood signal recovery problem, $\mathbf{y} = \boldsymbol{\Phi}\mathbf{x}$.

Recovery Methods

- Sparse bayesian learning (SBL) algorithm.
- SBL using approximate message passing (AMP-SBL).

Simulation Setup

Numerical experiments are run with the following setup

- $M = 64$, $N = 128$, $L = 1000$ (noncoherent measurements).
- Elements of $\boldsymbol{A}$ are generated according to $\mathcal{N}(0, \frac{1}{M})$.
- Each $\boldsymbol{X_l}$ is sparse and nonzero entries are generated according to $\mathcal{N}(0, 1)$.

- AMP-SBL and SBL perform better for support recovery than other algorithms.

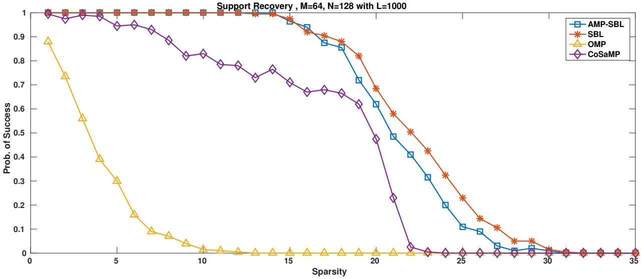


Figure: Success recovery using different algorithms at 25dB SNR

Relation B/W M, N and K

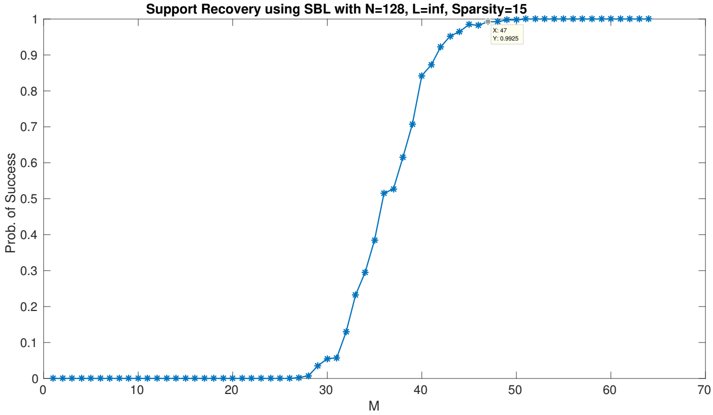


Figure: Success recovery with different M's for fixed sparsity and N.

- $M = \sqrt{2}K\log(N/K)$

Simulations

- Performance of support recovery improves with increasing noncoherent measurements.

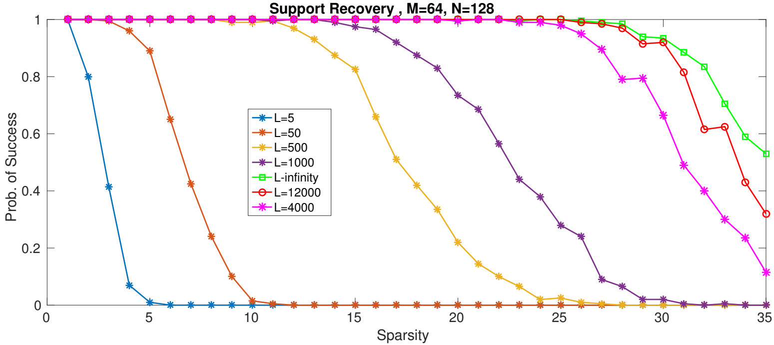


Figure: Success recovery v/s Sparsity with different noncoherent measurements

Summary

- Converted GNSS signl detection problem into support recovery of sparse siganl.
- Proposed a CS based solution for support recovery.
- Simulation results shows that support recovery improves with # non-coherent measurements.

Future Work

- To design algorithms to get the smallest posssible M and L.
- To exploit the structure in the sparse signal vector $\mathbf{x}$.

Thank You

Navigation message:

The navigation message transmitted on the L1 carrier frequency is 25 consecutive 1,500-bit-long frames. Each frame is made up of five subframes. Figure 1.1 shows the structure of a subframe. Each subframe has a duration of 6 seconds, and each consists of 10 words. Each word consists of 30 bits. Subframes 1 to 3 contain the ephemeris and repeat in every frame (i.e., they repeat every 30 sec.), while subframes 4 and 5 contain the almanac and repeat every 25 frames (i.e., they repeat every 12.5 minutes). Each word, in each subframe, contains a 6-bit parity.