Power Controlled Feedback and Training in MIMO Systems

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Outline

Definitions

- Diversity of the MIMO wireless link is defined as [1]

  $d = -\lim_{SNR\to\infty} \frac{\log P_{out}}{\log SNR}$

- Multiplexing gain is defined as [1] $r = \lim_{SNR\to\infty} \frac{R}{\log SNR}$
- CSIR - Perfect Channel state knowledge at the Rx.
- CSIT - Perfect Channel state knowledge at the Tx.
- For a χ^2 random variable H, with $2k$ degrees of freedom
  - $f_H(h) = \frac{1}{(k-1)!} h^{k-1} e^{-h}$
  - $\Pr(h \leq \epsilon) \doteq \epsilon^k$

Definitions(2)

- Given the $N_r \times N_t$ MIMO chnanel matrix $\mathbf{H}$ ($N_r \geq N_t$) with IID Gaussian entries, the joint PDF of the ordered eigenvalues of $\mathbf{H}^H\mathbf{H}$, $\mu_1 \leq \mu_2 \ldots \mu_{N_t}$ is given by [1]
$f(\mu_1, \mu_2, \ldots, \mu_{N_t}) = K_{N_r,N_t}^{-1} \prod_{i=1}^{N_t} \mu_i^{N_r-N_t} \prod_{i<j}(\mu_i - \mu_j)^2 e^{\sum_i \mu_i}$,
where K_{N_r,N_t}^{-1} is a normalization constant.
- Let $\mu_i = SNR^{-\alpha_i}$. The joint PDF $f(\alpha_1, \alpha_2, \ldots, \alpha_{N_t})$ is given by
$K_{N_r,N_t}^{-1}(\log SNR)^{N_t} \prod_{i=1}^{N_t} SNR^{(N_r-N_t+1)\alpha_i} \prod_{i<j}(SNR^{-\alpha_i} - SNR^{-\alpha_j})^2 e^{\sum_i SNR^{-\alpha_i}}$

Related Literature

1. Fundamental DMT with full CSIR - L. Zheng and D. Tse 2003 [1].
2. DMT with Partial CSIT (Noiseless feedback to Tx) - T.T.Kim and M. Skoglund 2007 [2].
3. Exponential Diversity using Spatio-temporal power allocation - V. Sharma et al 2008 [3].
4. Diversity Gains of Power Control with Noisy CSIT - T.T.Kim and G. Caire 2009 [4].
5. Power Controlled Feedback and Training - V. Aggarwal and A. Sabharwal 2010 [5].

SISO - Diversity Trade-off

- Consider a SISO link with Tx power constraint P and Rayleigh fading channel.
- It is known that $P_{out} = \Pr(|h|^2 < \frac{SNR^r - 1}{SNR}) \doteq \frac{1}{SNR^{1-r}}$.
  - Diversity order $d = 1 - r$, where r is the multiplexing gain [6].
  - If SNR^{α} is used as Tx power, then $d = (\alpha - r)$.
- The average Tx power constraint is achieved by using
  - constant Tx power (independent of channel gain).
  - Adapt the Tx power for channel gain (using knowledge from FB path)
  - That is, $\int_{h_0}^{\infty} P(h)dh \leq P$, where h is Rayleigh distributed
- **Note:** Tx power changes the diversity even in SISO channel!

SISO - Average Power Control

- For channel adaptive Tx power control, we need $\int_{h_0}^{\infty} P(h)dh \leq P_{avg}$ [5]
  - e.g., $P(h) = \begin{cases} SNR^2, h < \frac{1}{SNR} \\ SNR, \text{ otherwise} \end{cases}$
  - $P_{avg} = SNR^2 \Pr\left(h < \frac{1}{SNR}\right) + SNR[1 - \Pr\left(h < \frac{1}{SNR}\right)] \leq SNR$
- Thus, with even 1 bit of feedback about h, one can gain improved diversity from 1 to 2.
- That is, one can divide the range of h into 2^B regions and select $P(h)$ such that, average Tx power control is satisfied.

MISO - Diversity Trade-off

- Consider a MISO link with Tx power constraint P_{avg} and Rayleigh fading channel.
- It is known that $P_{out} \doteq \frac{1}{SNR^{N_t}}$.
  - maximum diversity order is $d = N_t$ and DMT trade-off is $N_t(1 - r)$[6].
  - If SNR^{α} is used as Tx power, then $d = N_t(\alpha - r)$.
  - When the CSIT error variance $\sigma_e^2 = SNR^{-d_e}$, then it is shown that $d = N_t(1 + N_t d_e - r)$ [4].

MIMO - Diversity Trade-off

- Consider a MIMO link with Tx power constraint P_{avg} and Rayleigh fading channel.
- It is known that $P_{out} \doteq \frac{1}{SNR^{N_t N_r}}$.
  - Diversity order $d = N_t N_r$ and multiplexing trade-off is $(N_t - r)(N_r - r)$[6].

DMT with full CSIR

- When CSIR is known [2],
  - $P_{out} = \Pr\left(\log det\left[\mathbf{I}_{N_r} + \frac{SNR}{N_t}\mathbf{H}\mathbf{H}^H\right] < r \log SNR\right)$
  - $P_{out} = \Pr\left(\sum_{i=1}^{min(N_r,N_t)} \log(1 + \frac{SNR}{N_t}\lambda_i) < \log SNR^r\right)$
- Let $\lambda_j = SNR^{\alpha_j}$. Then we get
  - $P_{out} \doteq \Pr\left(\prod_{i=1}^{min(N_r,N_t)} SNR^{(1-\alpha_j)^+} < SNR^r\right)$
- Now, using the distribution of singular values $\mathbf{H}\mathbf{H}^H$,
  - $P_{out} \doteq SNR^{-D(r,p)}, D(r,p) =$ $\inf_{\alpha\in\mathcal{A}} \sum_{i=1}^{\min(N_r,N_t)}(2i - 1 + \max(N_r, N_t) - \min(N_r, N_t))\alpha_i$
  - where $\mathcal{A} = \left\{\alpha_i : \alpha_i > 0, \sum_i (p - \alpha_i)^+ < r\right\}$
- This coincides with Zheng-Tse bound when $p = 1$[1].

CSIR vs CSIT

- Zheng and Tse [1] published the well known Diversity and Multiplexing Trade-off (DMT) when CSIR is known.
  - Diversity order $d = N_t N_r$ and multiplexing trade-off is $(N_t - r)(N_r - r)$.
- However, if the Tx has CSIT, then use $\mathbf{P} = \mathbf{H}^\dagger$ as the precoding matrix. That is,

$$\mathbf{Y} = \mathbf{HPX} + \mathbf{N} = \mathbf{X} + \mathbf{N}$$

- We can achieve infinite order diversity!
  - Tx power control is crucial.
- **Note:** Power control over channel instantiations is crucial.

DMT with Perfect Feedback

- Suppose, if the Rx with CSIR feeds back B bits via noiseless feedback channel, the maximum diversity achievable increase exponentially.
  - Using B bits of feedback, SISO link can achieve $K = 2^B$ diversity order.
- With K levels of feedback, the DMT is given by $d_K = D(r, 1 + d_{K-1}(r))$
  - For $r = 0$, $d = (N_r N_t)^K$ and for $r = \min(N_r, N_t)$, $d = 0$.
- As $SNR \to \infty$ and $K \to \infty$, we get infinite diversity order!

DMT with Noisy Feedback

- In SISO, if the Rx with CSIR feeds back B bits via noisy feedback channel using constant Tx power, then the maximum diversity achievable doubles when compared to no CSIT case.
  - The worst case error event in the noisy feedback results in higher Tx power than required. Hence, limit the Tx power to SNR^2 always which results in the diversity limit of 2.
  - Average Tx power for $B = 2$ is,
    $(1 - SNR^{-1})[SNR(1 - p_1 - p_2 - p_3) + SNR^2(p_1 + p_2 + p_3)] + SNR^{-1}[SNR^2(1-p_1-p_2-p_3)+p_1SNR+p_2SNR^2+p_3SNR^2]+\ldots \approx SNR$
- Similarly in MIMO, max. diversity gets limited to $2N_rN_t$.

Power Controlled CSI Feedback (SISO)

- The diversity order gets limited due to constant power used to send the feedback bits though they do not occur with equal probability.
    - That is, the probability of events $(1 - SNR^{-1}), SNR^{-1}, SNR^{-2}, SNR^{-3}$ are unequal.
    - Even if we use higher Tx power for occasional events, the average Tx power does not increase dramatically.

Power Controlled CSI Feedback (SISO)

- If we use exponential Tx power (SNR^{β}) for feedback of rare events, feedback bits are received with more reliability.
  - e.g., for B=2, Average Tx power for sending feedback is $SNR^0(1-SNR^{-1})+SNR^2SNR^{-1}+SNR^3SNR^{-2}+SNR^4SNR^{-3} \doteq SNR$.
  - Diversity is limited by the dominant error event when 1 is transmitted and 2 is received.
  - Prob of rare error event is SNR^{-2}; hence, diversity is limited to 3.

Power Controlled CSI Feedback (MIMO)

- Let $SNR^{q_1}, SNR^{q_2}, \ldots SNR^{q_K}$ are the various power levels used in the Tx.
- Using the MAP receiver, the feedback bits can be received reliably. $\Pr(J_T = 0|J_R = 1) = \Pr\left(tr(\mathbf{H}\mathbf{H}^H)SNR^{q_1} < SNR^{q_0+\epsilon}\right)$
  $\doteq \Pr\left(\sum_{i=1}^{min(N_r,N_t)SNR^{q_1-\alpha_1} < SNR^{q_0+\epsilon}}\right)$
  $\doteq \Pr\left(SNR^{\max(q_1-\min(\alpha_i),0)} < SNR^{q_0+\epsilon}\right)$
  $\doteq \Pr\left(\min(\alpha_i) > q_1 - q_0 - \epsilon\right) = SNR^{-N_rN_t(q_1-q_0)}$

Power Controlled CSI Feedback (MIMO)

- $\Pr(J_T = j | J_R = i) \doteq SNR^{-N_r N_t (q_i - q_j)}$
- The DMT for this case with K levels of feedback is given by $D(r, K) =$ $\max_{q_j \le 1 + D(r,j)} \min_{i=1,\ldots,K-1} (N_r N_t ((q_i^+ - q_{i-1}^+) + D(r,i)))$
- As $K \to \infty$, the maximum diversity order with power controlled feedback is given by $N_r N_t (N_r N_t + 2)$.

Channel Estimation - Constant Training Power

- In real scenarios, CSIR is obtained via training and channel estimation.
- MMSE channel estimates improve with SNR and length of training sequence.
  - $\hat{\mathbf{H}} = \mathbf{H}\frac{1}{1+\frac{N_t}{N\ SNR}} + \text{noise}$.
  - The variance of the channel estimation error is $\doteq 1/SNR$.
- Let $|\hat{\mathbf{H}}|^2 = SNR^{-\hat{\alpha}} \Rightarrow |\tilde{\mathbf{H}}| = |\mathbf{H} - \hat{\mathbf{H}}|^2 = SNR^{-1}SNR^{-\hat{\alpha}}$

Diversity - Constant Training Power

- The outage due to dominant error event is given by
$$\Pr\left(\log\left(1+\frac{|\hat{\mathbf{H}}|^2 SNR^2}{1+SNR^2|\tilde{\mathbf{H}}|^2}\right) < r\log SNR,\right.$$
$$\log\left(1+|\hat{\mathbf{H}}|^2 SNR^2\right) \geq r\log SNR,$$
$$\left.\log\left(1+|\hat{\mathbf{H}}|^2 SNR\right) < r\log SNR\right).$$
- $P_{out} \doteq \Pr((2-\hat{\alpha}-(1-\tilde{\alpha})^+)^+ < r, (2-\hat{\alpha})^+ \geq r, (1-\hat{\alpha})^+ < r)$
- This can be simplified to $P_{out} \geq SNR^{-(1-r)}$(let $\hat{\alpha} = 1-r+\delta$).
- Note: The above calculation assumes noise free feedback bits to Tx.

Diversity - Adaptive Training Power

- **Steps:** (1) Tx sends a training sequence, (2) Rx estimate channel $\hat{\mathbf{H}}_1$ and sends index J_1 to Tx. (3) Tx sends training again with power $P(J_T) = SNR^{1+i}$ followed by data with the same Tx power. The new channel estimate is $\hat{\mathbf{H}}_2$.
- The outage prob. is given by
$$\Pr\left(\log det\left[\mathbf{I} + \frac{P(J_T)}{N_t}\frac{\hat{\mathbf{H}}_2\mathbf{Q}\hat{\mathbf{H}}_2^H}{1+\frac{SNR}{N_rN_t}tr\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\right)}\right] < R\right)$$
- e.g., for $B = 2$, the outage events are $\hat{\mathcal{O}}_i = \left\{\hat{\mathbf{H}} : \log\left(1 + |\hat{\mathbf{H}}|^2 SNR^{1+i}\right) < R\right\}$ where SNR^{1+i} is used for Tx if $\hat{\mathbf{H}} \in \mathcal{O}_i$.

DMT - Adaptive Training Power

- For e.g, Let $B = 2$ and an outage occurs when $J_T = 2$ and $J_R < 2$. That is, $P_{out} \doteq SNR^{-2}$.
- For $K > 1$ and $r < \min(N_r, N_t)$, the DMT that can be achieved with power controlled training is given by $D(r, K) = G(r, 1 + G(r, 1))$
- The maximum diversity for $r = 0$ is $N_r N_t (N_r N_t + 1)$.

DMT - Partial CSIT and CSIR

- **Steps:** (1) Tx uses *SNR* to send training, (2) Rx estimates the channel $\hat{\mathbf{H}}_1$ and sends an index J_R using power controlled feedback, (3) Tx uses received index J_T to determine $P(J_T)$ for training as well as data. New channel estimate is $\hat{\mathbf{H}}_2$.
- An outage occurs when $\Pr\left(\log det\left(\mathbf{I} + P(J_T)\hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H\right) < R\right)$.
- For e.g., for $B = 1$, $\Pr(J_T = 0|J_R = 1) \doteq SNR^{-N_r N_t(1+G(r,1))}$, $\Pr(J_T = 1|J_R = 0) \doteq 0$ and $\Pr(J_R = 0|J = 1) \doteq 0$, where J is 1 if $\log det(\mathbf{I} + \hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H P_0) < R$ and $\log det(\mathbf{I} + \hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H P_1) \geq R$.

DMT - Partial CSIT and CSIR

- Accounting for all possible values of J_R, J_T and J, the outage probability can be computed.
- The DMT can be shown to be $D(r,2) = G(r, 1 + G(r,1))$.
- That is, diversity order achievable with 1 bit of feedback when CSIR and CSIT are imperfect is same as the diversity order achievable with 1 bit of perfect feedback and perfect CSIR.

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