Compressed Sensing Based Schemes for Multiple Transmitter Localization and Communication Footprint Identification

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Outline

- Introduction to Compressive Sensing and Spectrum Cartography
- Problem definition
- Proposed Schemes for Spectrum Cartography
- Design Issues
- Simulation Results

Brief Introduction to Compressive Sensing (CS)

- A set of equations, $y_{M \times 1} = \Phi_{M \times L} s_{L \times 1}$, $M < L$
- For a general s, there are infinitely many solutions
- CS theory: If s is sparse and Φ satisfies Restricted Isometry Property (RIP), then s can be uniquely recoverable (ℓ_1 minimum solution)
- Gaussian and sub-Gaussian matrices like Bernoulli ensemble satisfy RIP

Transmitter Localization and Communication Footprint Construction

- Footprint: All those locations in a given area that receive a power higher than a threshold
- Applications:
  - Spectrum Enforcement: To identify pirate radios
  - Cognitive Radio Networks: White space detection

Spatial spectrum usage map

- Deploy low-cost sensors over the geographical area
- Sensors detect presence/absence of primary and convey 1-bit information to the fusion center
- Straightforward scheme
  - query each sensor in round robin manner, cluster them and construct the map
  - time required is proportional to number of sensors deployed
- Footprint map is a sparse image, time required for reconstruction can be reduced

Problem Definition

- Scenario: T transmitters are located at $l_i = (x_i, y_i)$, with circular radio footprint of r_i, for $i = 1, .., T$.
- Estimate l_i and r_i and construct the circular footprints
- Performance metric:
  - average relative error in area $error_A = \frac{H(I_i, \hat{I}_i)}{N_i}$

    where
    $H(I_i, \hat{I}_i)$ - hamming distance between the images I_i and $\hat{I}_i$,
    I_i - original footprint of the i^{th} transmitter,
    $\hat{I}_i$ - estimated footprint of the i^{th} transmitter,
    N_i - number of grids falling in original footprint area for i^{th} transmitter.
  - MSE in transmitter localization

Proposed Schemes for Transmitter localization and Footprint Reconstruction

Deployment on a small geographical area

- Public buildings like airport terminals, railway stations etc.
- Sensors can transmit directly to the fusion center witout any intermediate relay node over a control channel
- L number of sensors are deployed uniformly at random locations in the geographical area

Sensor to fusion center communication

- Sensors decide on presence of primary in their respective locations
- Alarming sensors synchronously transmit their 1-bit decisions to the fusion center for M times
- Each time they pre-rotate the bit by pseudo-random binary phase shift
- Fusion center knows these binary phase shifts aprori

Mathematical Model of sensors to FC communication

Measurement vector y at FC

$$y = Xh + w \tag{1}$$

$$y = \frac{1}{\sqrt{M}} \begin{bmatrix} x_1 e^{j\theta_{11}} & x_2 e^{j\theta_{12}} & \dots & x_L e^{j\theta_{1L}} \\ x_1 e^{j\theta_{21}} & x_2 e^{j\theta_{22}} & & x_L e^{j\theta_{2L}} \\ \vdots & & \ddots & \vdots \\ x_1 e^{j\theta_{M1}} & x_2 e^{j\theta_{M2}} & \dots & x_L e^{j\theta_{ML}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} + w \tag{2}$$

where
$w \sim \mathcal{CN}(0, \sigma^2)$ (receiver noise),
x_j is decision at j^{th} sensor,
$h_j \sim \mathcal{CN}(0, 1)$ is channel from j^{th} sensor to FC, and

$$\theta_{ij} = \begin{cases} \pi & \text{with probability } 0.5 \\ 0 & \text{with probability } 0.5 \end{cases}$$

Equivalence to CS measurement equation

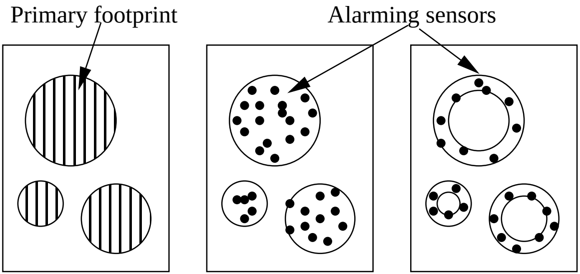
$$y = \frac{1}{\sqrt{M}} \begin{bmatrix} +1 & -1 & \dots & +1 \\ -1 & +1 & \dots & +1 \\ & & \ddots & \\ +1 & +1 & \dots & -1 \end{bmatrix} \begin{bmatrix} x_1 h_1 \\ x_2 h_2 \\ \vdots \\ x_L h_L \end{bmatrix} + w. \qquad (3)$$

CS Measurement equation

$$y = \Phi s + w$$

Schemes for radio map reconstruction

- At fusion center, the sparse vector s is reconstructed from observations y using OMP
- We propose two schemes based on set of alarming sensors transmitting to fusion center



(a) Footprint (b) Scheme 1 (c) Scheme 2

Figure: Primary footprint and reconstruction schemes *Scheme* 1 and *Scheme* 2

Schemes for footprint reconstruction

Scheme 1

- k-means algorithm to cluster the alarming sensors
- Transmitter location - k-means centroid
- Radius - distance of the farthest sensor to cluster center

Schemes for footprint reconstruction

Scheme 2

- Sensors in annulus are alarming sensors
- k-means algorithm to cluster the alarming sensors
- Associate the same power to all sensors in the annulus
- Transmitter location - Average of intersections obtained by triangulation using every pair of alarming sensors
- Radius - distance of the farthest sensor to cluster center

Design Issues

Number of sensors to be deployed - Scheme 1

- Optimization problem:

$$\min_{L,z} Lz \log(1/z) \tag{4}$$

$$\text{subject to } \; 0 < z \leq \kappa, \text{and } L \geq -\frac{a}{\log(1-bz)}$$

where $a = \log(1/p_m)$, $b = \frac{(P_{min}/P_{max})^{2/\eta}}{T_{max}}$ and $z \triangleq \frac{K_{max}}{L}$

- First constraint: due to Sparsity requirements
- Second constraint: to detect the transmitter with at least P_min power with a probability greater than $1 - p_m$
- z is related to the power threshold of the sensors
- $L_{opt} = -\frac{a}{\log(1-b\kappa)}$ and $z_{opt} = \kappa$

- Optimization problem:

$$\begin{aligned} &\min_{L,z} Lz \log(1/z) \\ &\text{subject to} \quad \left(\frac{P_{max}}{P_{min}}\right)^{2/\eta} \left(1 - p_m^{1/L}\right) T_{max} \le z \le \kappa, \end{aligned} \tag{5}$$

- $L_{opt} = \frac{\log p_m}{\log\left(1 - \frac{\kappa}{T_{max}} \left(\frac{P_{min}}{P_{max}}\right)^{2/\eta}\right)}$, $z_{opt} = \kappa$, $\rho = (1+\delta)^{\eta}$

Step 1	Initialize $K = 1$ transmitter.
Step 2	Perform K-means clustering. Fit K circles with the K-means centroids of the clusters as the centers of the circles, (a_i, b_i), and the farthest point in each cluster from the center as radius, r_i, for $i = 1, .., K$.
Step 3	Compute the average distance between the points in each cluster to the closest point in the corresponding circular fit as the metric, i.e., $m = \sum_i \frac{1}{N_i} (\sum_{j=1}^{N_i} \sqrt{(x_{ji} - a_i)^2 + (y_{ji} - b_i)^2} - r_i)$
Step 4	Increment K. Repeat *Step* 2 and *Step* 3 until the *first minimum* of the metric (m) is obtained.
Step 5	Output the K that corresponds to the first minimum.

Simulation Results

- A rectangular geographical area with $N = 4800$ grids and $T = 3$ transmitters
- Footprints cover 23% of total area
- L number of sensors are deployed uniformly at random locations

Comparison of the Proposed method with CH and Hartigan methods

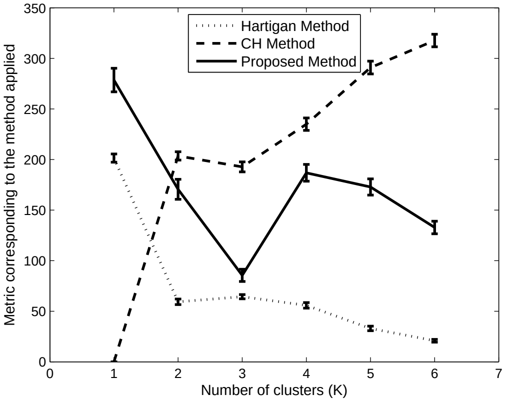


Figure: Identification of number of clusters using CH, Hartigan and proposed methods.

Comparison of three schemes

Table: Footprint Identification Performance of Different Schemes

Schemes	L	$\bar{S}$	M	**Relative error in area**
Scheme 1	960	214	558	0.0236
Scheme 1	480	120	336	0.0352
Scheme 2	**960**	**122**	**336**	**0.0110**
Round − robin	336	-	336	0.0383
Round − robin	558	-	558	0.0302
Round − robin	960	-	960	0.0220

- $Receive\ SNR = 25dB$

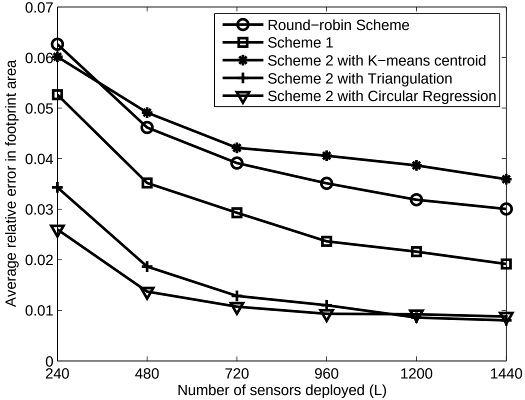


Figure: Comparison of *Scheme* 1 and *Scheme* 2 with respect to number of sensors deployed

SNR study of OMP using Scheme 2 - Success in localization Vs M

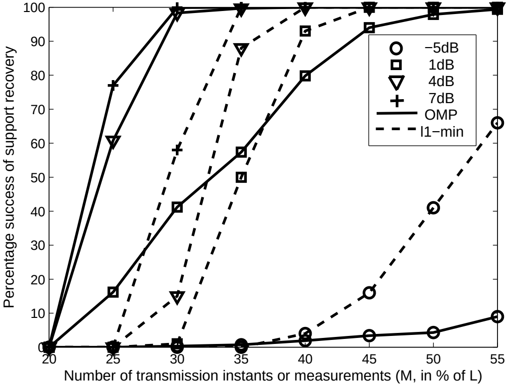


Figure: Percentage of success of localization algorithm for various number of transmissions, M with $L = 20\%$ of $N = 960$and $K = 122$ under different SNR conditions when OMP is used for CS reconstruction.

Results based on Experimental Data

- Wi-Fi AP as transmitter with Transmit power: $24dBm$, Frequency channel: 11^{th} channel of $2.4GHz$ band
- Laptop with WI-Fi card was used as receiver

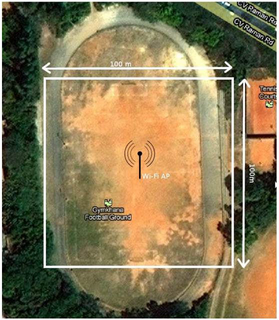
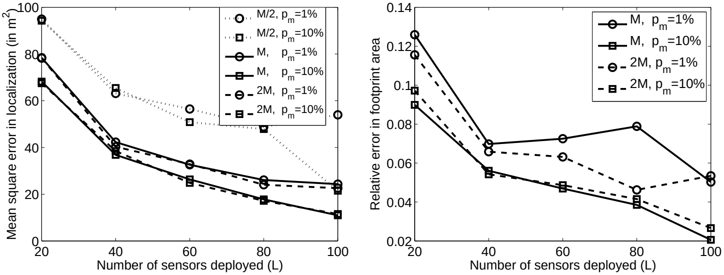
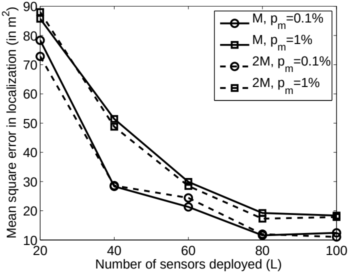


Figure: Football ground showing placement of Wi-Fi AP in the middle of an area of $(100m \times 100m)$. The power measurements were made at randomly chosen 250 locations in the chosen area.

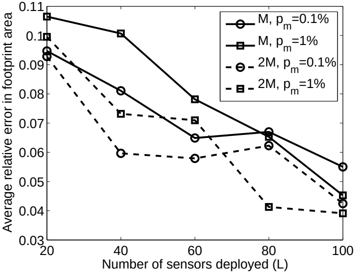


(a) MSE Vs L (b) Relative area error Vs L

Figure: (a) Mean square error in localization, and (b) Relative error in footprint area of Wi-Fi AP Vs Number of sensors deployed L for **Scheme 1**.



(a) MSE Vs L



(b) Relative area error Vs L

Figure: (a) Mean square error in localization, and (b) Relative error in footprint area of Wi-Fi AP Vs Number of sensors deployed L for **Scheme 2**.

Comparison of Power Budget: Numerical Example

- Consider $L = 960$ sensors, Non-coherent On-Off keying receivers at FC
- Round-robin Scheme: A Receive SNR of $14dB$ is required to ensure Prob. of bit error of 10^{-3}
- This requires $14dB \times 120$, i.e. $35dB$ of receive SNR
- Scheme 2 requires $4dB \times 120$, i.e. $25dB$ of receive SNR

Future work

- Shadowing and Rayleigh fading between the transmitter and sensor is not considered in current setup
- Standard deviation of shadowing can range from 4 to 12, that makes circularly boundaries to be highly distorted
- Alternative schemes to handle these

Summary

- Proposed two schemes for constructing the footprint
- Scheme 2 requires lesser number of transmission instants
- Scheme 2 has better error performance compared to the other schemes
- Proposed a method for identifying number of transmitters

Thank you

Table: Number of transmissions (M), sensor threshold (τ_i), and experimental value of probability of missing the transmitter for various values of L for a design with $p_m = 1\%$.

(a) *Scheme* 1

L	20	40	60	80	100
M	13	19	22	25	27
τ_i (*in dBm*)	-53.2	-50.3	-48.7	-47.5	-46.5
p_m (*expt.*) (*in*%)	0.4	0.4	0	0	0.1

(b) *Scheme* 2

L	20	40	60	80	100
M	13	19	22	25	27
τ_i (*in dBm*)	-53.3	-50.6	48.9	-47.7	-46.7
p_m (*expt.*) (*in*%)	1.9	0.76	0.38	0.34	0.32

K-means clustering of alarmed sensors

- K-means algorithm - unsupervised technique for clustering data
- Algorithm for finding K clusters
  - Step 1 - Initialisation - Randomly picks K centroids, and forms K clusters using the data points that are close to each of these centroids
  - Step 2 - Find new centroids corresponding to each of these clusters and clusters the data again
  - Repeat Step 2 till centroids converge

Deployment on a large area

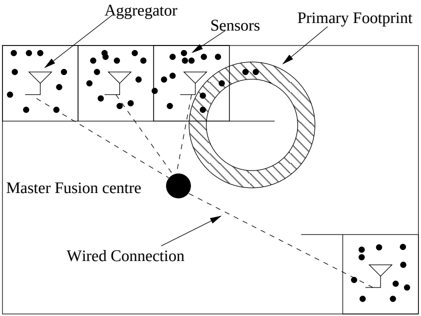


Figure: Depiction of scheme for a large area

- The measurement matrix is block diagonal
- Each block corresponds to psuedo random binary shifts corresponding to a cell

CH method

Hartigan method