

Multiple Transmitter Localization and Communication Footprint Identification using Sparse Reconstruction Techniques

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Outline

- Introduction to Transmitter Localization
- Problem Definition
- Proposed Schemes for Transmitter Localization
- Simulation Results

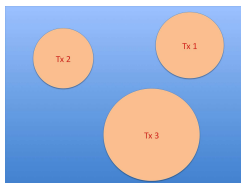
Transmitter Localization and Communication Footprint Construction

- Transmitter Localization: Locating transmitters in a given area
- Communication Footprint: Area around the transmitter where its signal can be received with good fidelity
- Applications
 - Spectrum Enforcement: Identifying pirate radios
 - Cognitive Radio Networks: White space detection

Past Works

- Approach: Using power measurements from sensors deployed around the transmitter
- [Nelson '06, '09], [Nasif '09], [Cevher '08, Feng '09], [Bazerque '10]
- Assumes number of transmitters and their transmit powers to be known
- depend on RSS measurements being sent to a central node
- Disadvantage: Dense deployment of sensors will lead to delay in localization

Our Approach



- Deploy a number of low-cost sensors over the geographical area
- Sensors detect presence/absence of primary at their locations and convey 1-bit information to a Fusion Center (FC) over a control channel
- Construct the spectrum usage map at FC by clustering the alarming sensors

How is Our Approach Different From Past Work?

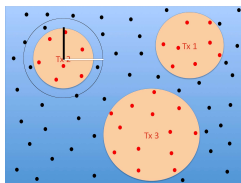
- Number of transmitters and transmit powers are unknown
 - We limit the maximum number of transmitters and transmit power range
- 1-bit transmissions to reduce delay
 - The 1-bit transmissions could be repeated some number of times
 - Supports a dense deployment of sensors
- Communication footprint construction in addition to transmitter localization

Problem Definition

- T transmitters are located at $l_i = (x_i, y_i)$, with radius of **circular** radio footprint of r_i , for $i = 1, \dots, T$
- L sensors deployed uniformly at random locations in the geographical area convey their 1-bit information to the FC
- **Objective:** Estimate T , l_i and r_i and construct the circular footprints at the FC **with minimum delay**
- **Performance metrics:**
 - Relative error in area of reconstructed footprint to original footprint (Hamming distance)
 - MSE in transmitter localization

Assumptions

- Inverse square path loss model between transmitters and sensors
- Channel between Sensors to FC follows multi-path fading
- Neglect sensing errors



Brief Introduction to Compressive Sensing (CS)

- Consider a set of equations

$$y = \Phi s$$

M $(M \times L)$ L $M < L$

- In general, there are infinitely many solutions
- CS theory: If s is sparse and Φ satisfies Restricted Isometry Property (RIP), then s can be uniquely recovered
[\[Donoho '06\]](#) [\[Candes '05, '06\]](#)
- RIP: Every set of s columns of Φ are nearly orthonormal
- Gaussian and Bernoulli ensemble satisfy RIP**

Recovery algorithms for CS

- ℓ_1 minimization
 - $\min_{\hat{s}} \|\hat{s}\|_1$ s.t. $y = \Phi\hat{s}$
 - $M > \mathcal{O}(K \log(L/K))$ [Donoho '06]
- OMP
 - Iterative algorithm
 - Finds Best K columns of Φ which have maximum correlation with y
 - $M > \mathcal{O}(K \log(L))$ [Tropp '07]

Contributions

- Proposing a Sensors to FC Communication protocol which fits into CS framework
- Proposing two schemes for estimating l_i and r_i
- Proposing a method for identifying the number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

Sensor to FC Communication Protocol

- Alarming sensors synchronously transmit a '1' to the FC M times
- Each time sensors transmit, they pre-rotate the bit by a pseudo-random binary phase shift $\{0, \pi\}$
- Fusion center knows these binary phase shifts
- Channel from sensors to FC is assumed to be constant for M transmissions

Mathematical Model of Sensors to FC Communication

Measurement vector y at FC

$$y = X h + w$$

M $(M \times L)$ L

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \frac{1}{\sqrt{M}} \begin{bmatrix} x_1 e^{j\theta_{11}} & x_2 e^{j\theta_{12}} & \dots & x_L e^{j\theta_{1L}} \\ x_1 e^{j\theta_{21}} & x_2 e^{j\theta_{22}} & & x_L e^{j\theta_{2L}} \\ \vdots & & \ddots & \vdots \\ x_1 e^{j\theta_{M1}} & x_2 e^{j\theta_{M2}} & \dots & x_L e^{j\theta_{ML}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}$$

where

$w_i \sim \mathcal{CN}(0, \sigma^2)$ (receiver noise),

$x_j \in \{0, 1\}$ is decision at j^{th} sensor,

h_j is channel from j^{th} sensor to FC, and

$$\theta_{ij} = \begin{cases} \pi & \text{w. p. } 0.5 \\ 0 & \text{w. p. } 0.5 \end{cases}$$

Equivalence to CS Measurement Equation

$$y = \frac{1}{\sqrt{M}} \begin{bmatrix} +1 & -1 & \dots & +1 \\ -1 & +1 & \dots & +1 \\ & & \ddots & \\ +1 & +1 & \dots & -1 \end{bmatrix} \begin{bmatrix} x_1 h_1 \\ x_2 h_2 \\ \vdots \\ x_L h_L \end{bmatrix} + w$$

CS measurement equation

$$y = \Phi \quad s + w$$

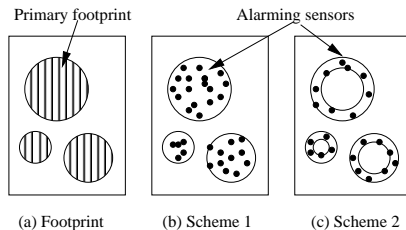
$M \quad (M \times L) \quad L$

- Φ is a Bernoulli ensemble
- s is sparse because $[x_1 x_2 \dots x_L]$ is sparse

Contributions

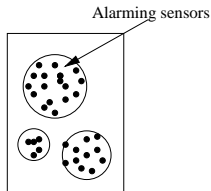
- Sensors to FC Communication protocol to fit into CS framework
- Two schemes for estimating locations l_i and radius r_i at FC
- Method for identifying number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

Schemes for radio map reconstruction



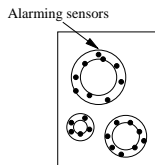
- Proposed schemes based on alarming sensors
 - Scheme 1 - Sensors that are within circular boundaries around transmitters
 - Scheme 2 - Sensors that are within annuli around transmitters

Scheme 1



- K -means algorithm to cluster the alarming sensors
- **Location Estimate** - K -means centroid
- **Radius Estimate** - Distance of the farthest sensor to cluster center

Scheme 2



- Sensors in annulus are alarming sensors
- K -means algorithm to cluster the alarming sensors
- **Trilateration**
 - Associate a representative power to all sensors and draw power contours
 - Location Estimate - Average of intersections obtained by trilateration
- **Circular Regression** - Steepest descent method
- **Radius Estimate** - Distance of the farthest sensor to cluster center

Contributions

- Sensors to FC Communication Protocol
- Two schemes for estimating locations l_i and radius r_i at FC
- **Method for identifying number of transmitters T**
- Design of the number of sensors to be deployed and their power thresholds

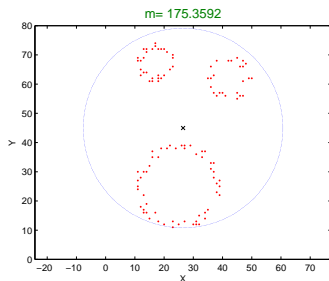
Identifying Number of Transmitters T

- K -means clustering needs number of clusters, K as input
- Calinski and Harbasz (CH) Index [CH '74]
 - Depends on inter cluster and intra cluster distances
- Hartigan Method [Hartigan '75]
 - Depends on intra cluster distances
- Sensitive to size of clusters and distance between clusters

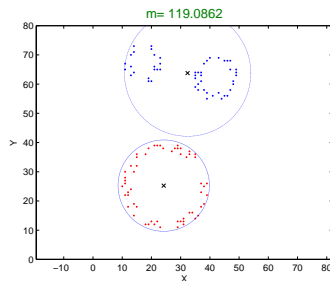
Proposed Metric

- Clusters are annulus shaped or circular shaped
- Fit K circles to clusters
- Average of the deviation of points from the circular fit
- Metric, $m = \frac{1}{N_i} \sum_i \left\{ \sum_{j=1}^{N_i} \left(\sqrt{(x_{ji} - a_i)^2 + (y_{ji} - b_i)^2} - r_i \right)^2 \right\}$
 (a_i, b_i) , and r_i , are center and radius of i^{th} circular fit, N_i -
Number of points in i^{th} cluster.
- **First minimum** of the metric identifies true number of clusters

Example: Estimating Number of Transmitters

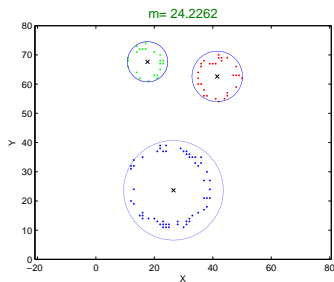


(a) $K = 1$

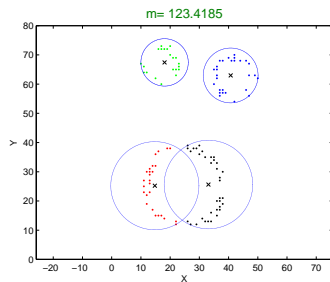


(b) $K = 2$

Example: Estimating Number of Transmitters



(c) $K = 3$



(d) $K = 4$

Algorithm for Estimating the Number of Transmitters

Step 1 Initialize $K = 1$ transmitter.

Step 2 Perform K -means clustering. Fit K circles.

Step 3 Compute the metric

$$m = \frac{1}{N_i} \sum_i \left\{ \sum_{j=1}^{N_i} \left(\sqrt{(x_{ji} - a_i)^2 + (y_{ji} - b_i)^2} - r_i \right)^2 \right\}$$

(a_i, b_i) , and r_i , are center and radius of i^{th} circular fit.

Step 4 Increment K . Repeat *Step 2* and *Step 3* until the **first minimum** of the metric (m) is obtained.

Step 5 Output the K that corresponds to the first minimum.

Contributions

- Sensors to FC Communication Protocol
- Two schemes for estimating locations l_i and radius r_i at FC
- Method for identifying number of transmitters T
- Design of the number of sensors to be deployed and their power thresholds

Number of Sensors to be Deployed - Scheme 1

- Assumptions:
 - T_{max} - Maximum number of transmitters in the area
 - P_{max} and P_{min} - Maximum and minimum powers with which a transmitter can operate
- K_{max} - Maximum number of alarming sensors when T_{max} transmitters operate at P_{max}
- $z \triangleq \frac{K_{max}}{L}$, sparsity constraint - $0 < z \leq \kappa$
- $Pr\{\text{missing a transmitter of } P_{min} \text{ power}\} < p_m$
- To minimize number of transmissions $K_{max} \log(L/K_{max})$

Number of Sensors to be Deployed - Scheme 1

- Optimization problem:

$$\min_{L,z} Lz \log(1/z)$$

$$\text{subject to } 0 < z \leq \kappa, \text{ and } L \geq -\frac{a}{\log(1-bz)}$$

where $a = \log(1/p_m)$, $b = \frac{(P_{min}/P_{max})^{2/\eta}}{T_{max}}$ and $z \triangleq \frac{K_{max}}{L}$

- $L_{opt} = -\frac{a}{\log(1-b\kappa)}$ and $z_{opt} = \kappa$

Number of Sensors to be Deployed - Scheme 2

- Objective: To find number of sensors L , threshold of sensors τ_i and τ_o
- Relative width of the annulus is fixed to δ
- Optimization problem:

$$\min_{L,z} Lz \log(1/z)$$

$$\text{subject to } \left(\frac{P_{max}}{P_{min}}\right)^{2/\eta} \left(1 - \rho_m^{1/L}\right) T_{max} \leq z \leq \kappa,$$

- $L_{opt} = \frac{\log \rho_m}{\log\left(1 - \frac{\kappa}{T_{max}} \left(\frac{P_{min}}{P_{max}}\right)^{2/\eta}\right)}$, $Z_{opt} = \kappa$, $\rho \triangleq \frac{\tau_i}{\tau_o} = (1 + \delta)^\eta$

Existing Schemes

- Round robin scheme - Query each sensor in round-robin manner and cluster the alarming sensors
- Minimum Enclosing circle (MEC) [Shoari '10]
 - Enclose the alarming sensors by the smallest circle after clustering
 - To compare reconstruction accuracy

Simulation Results

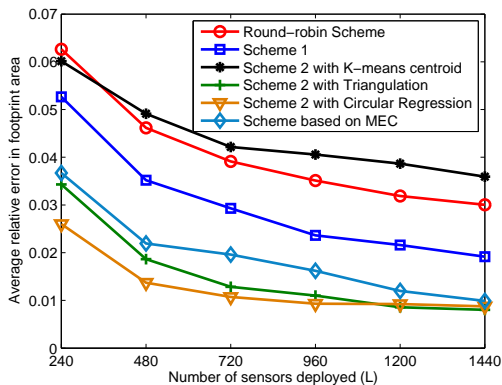
- Deployment of L sensors in a rectangular geographical area with $N = 4800$ grid locations and $T = 3$ transmitters
- Footprints cover 23% of the total area
- Performance measure: Relative error in footprint area (Hamming distance)

Table: Footprint Identification Performance of Different Schemes

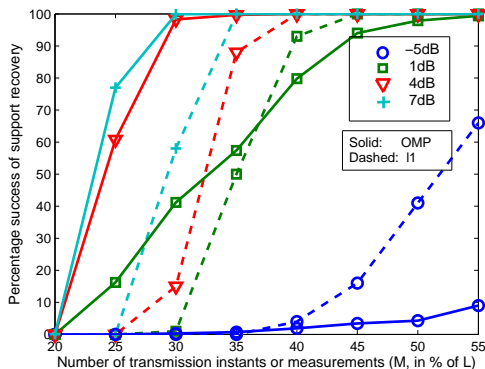
Schemes	L	S	M	Relative error in area
<i>Scheme 1</i>	960	214	558	0.0236
<i>Scheme 1</i>	480	120	336	0.0352
Scheme 2	960	122	336	0.0110
<i>Round – robin</i>	336	-	336	0.0383
<i>Round – robin</i>	558	-	558	0.0302
<i>Round – robin</i>	960	-	960	0.0220

Relative Error in Area Vs Number of Sensors Deployed, L

- *Average Receive SNR = 4dB per sensor*



Success in Localization Vs Number of Transmissions, M



Identification of Number of Clusters

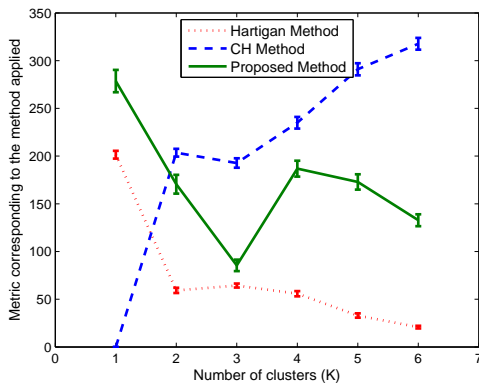


Figure: Comparison of the proposed method with CH and Hartigan methods

Summary

- Alarming sensors employ M consecutive 1 bit transmissions with pseudo random binary phase shifts
- At the FC, these phase-shifts act as elements of a CS measurement matrix which enables the use of sparse recovery methods
- Proposed an iterative method to estimate number of transmitters
- K -means algorithm is used to cluster alarming sensors and thereby locate transmitters
- Scheme 2 performs the best in terms of footprint area error performance among the methods considered

Future Work

- Shadowing and Rayleigh fading between the transmitter and sensor is not considered in current setup
- Standard deviation of shadowing can range from 4 to 12, that makes circular boundaries to be highly distorted
- Need to modify the schemes to handle these

Publications

- Venugopalakrishna Y. R., Chandra R. Murthy, D. Narayana Dutt, and Sneha Latha Kottipalli, "Multiple Transmitter Localization and Communication Footprint Identification Using Sparse Reconstruction Techniques", **accepted for presentation at ICC 2011**, Kyoto, Japan.

Thank You

Comparison of Power Budget: Numerical Example

- Consider $L = 960$ sensors, Non-coherent On-Off keying communication protocol
- Round-robin Scheme: A Receive SNR of $14dB$ is required to ensure prob. of bit error of 10^{-3}
- This requires $14dB \times 120$, i.e. $35dB$ of receive SNR
- Scheme 2 requires $4dB \times 120$, i.e. $25dB$ of receive SNR

MSE in Localization - Scheme 1

(X_s, Y_s) - location of transmitter, (X_i, Y_i) - location of i^{th} sensor

d is radius of footprint in an area of A

m - number of sensors in annulus when L sensors are deployed

Estimate of centroid - $(\sum_{i=1}^m X_i/m, \sum_{i=1}^m Y_i/m)$

$$MSE = \mathbb{E}_m \left\{ \frac{1}{m^2} \mathbb{E}_{X_i, Y_i} \left\{ \sum_{i=1}^m (X_s - X_i)^2 + \sum_{i=1}^m (Y_s - Y_i)^2 \right\} \right\}$$

$X_s - X_i = r \cos \theta$, $Y_s - Y_i = r \sin \theta$, where $\theta \sim \mathcal{U}(0, 2\pi)$, $r = \sqrt{z}d$ where $z \sim \mathcal{U}[0, 1]$

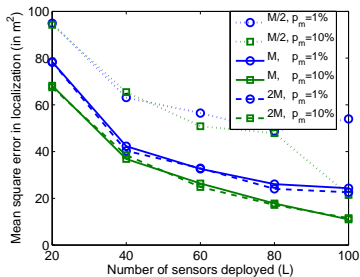
$$\begin{aligned} MSE &= \mathbb{E}_m \left\{ \frac{1}{m^2} \mathbb{E}_r \left\{ mr^2 \right\} \right\}, \\ &= \mathbb{E}_m \left\{ \frac{1}{m} \right\} \frac{d^2}{2}. \\ &\approx \frac{A}{2\pi L} \quad (\text{using } \mathbb{E}_m \{1/m\} \approx 1/\mathbb{E}_m \{m\}). \end{aligned}$$

Evaluation of Proposed Schemes with Experimental Data

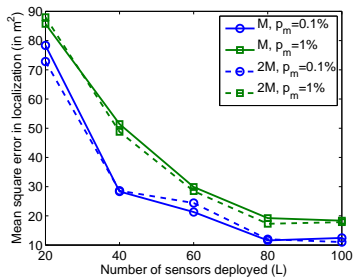
- Wi-Fi AP as transmitter with transmit power: 24dBm , Frequency channel: 11th channel of 2.4GHz band
- Laptop with Wi-Fi card was used as receiver
- Power measurements at randomly chosen 250 locations in ($100\text{m} \times 100\text{m}$) area



MSE Vs Number of Sensors Deployed



(a) Scheme 1



(b) Scheme 2

Figure: MSE Vs L

K-means clustering of alarmed sensors

- K-means algorithm - unsupervised technique for clustering data
- Algorithm for finding K clusters
 - Step 1 - Initialisation - Randomly picks K centroids, and forms K clusters using the data points that are close to each of these centroids
 - Step 2 - Find new centroids corresponding to each of these clusters and clusters the data again
 - Repeat Step 2 till centroids converge