Digital Noise Shaping

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Outline

Noise Shaping

- Consider a digital signal $x[n]$, represented with B_{in} bits
- Goal - Quantize $x[n]$ to B_{out} bits ($B_{out} < B_{in}$)
- Constraint - Some of the frequencies are important (should minimize the noise around them)

Sample Time Domain Signal Plot

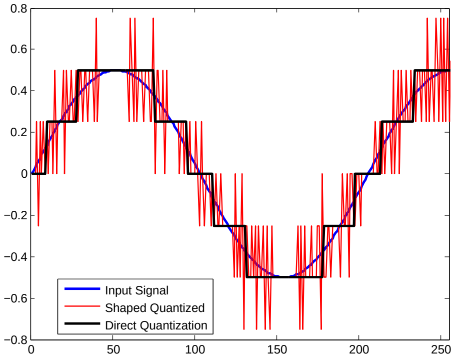


Figure: Time domain plot of noise shaped quantizer

Sample Spectrum

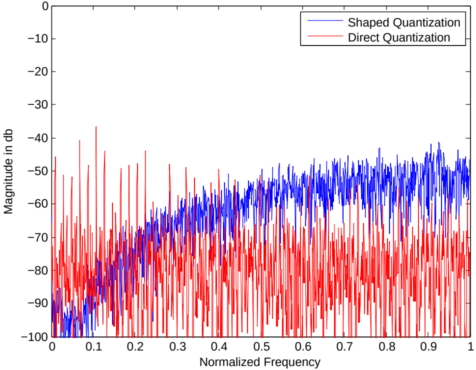


Figure: Spectra of shaped and direct quantization noises

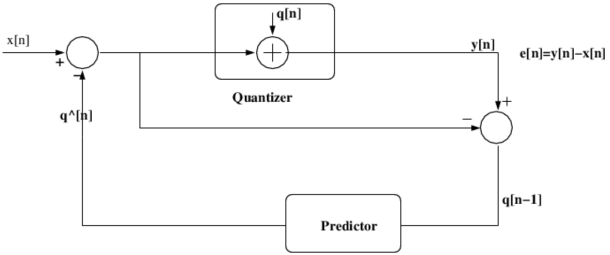
Solution

- Predict the quantization noise (in the band of interest)
- Subtract the predicted noise from the signal and then quantize
- Intutively, the noise in the band of interest should reduce
- The amount of reduction depends on the accuracy of prediction

Example

- Assume that the band of interest is ’dc’
- Then, one possible prediction could be, $\hat{q}[n] = q[n-1]$
- Can show that the quantization noise gets shaped by $H_s(Z) = 1 - z^{-1}$

System Model



Quantizer Model

- Quantizer is non-linear
- Assumption - Adds white noise
- A good assumption as long as quantizer is not overloaded

Notation

- $x[n]$ - input signal
- $q[n]$ - noise added by the quantizer
- $y[n]$ - output signal ($y[n] = x[n] + q[n] - \hat{q}[n]$)
- $e[n]$ - error in the signal ($e[n] = y[n] - x[n]$)

Noise Shaping Transfer Function

- prediction value, $\hat{q}[n] = \sum_{k=0}^{M-1} h(k)q[n-k-1]$
- error signal, $e[n] = q[n] - \hat{q}[n]$
- Z tranform of error signal,

$$E(z) = Q(z)\left[1 - z^{-1}\sum_{k=0}^{M-1} h(k)z^{-k}\right]$$

- $H_s = 1 - z^{-1}\sum_{k=0}^{M-1} h(k)z^{-k}$, denotes the shaping TF

Integrated Noise Power

- PSD of the error signal

$$| E(e^{j\omega}) |^2 = | Q(e^{j\omega}) |^2 | H_s(e^{j\omega}) |^2$$

- $q[n]$ is white noise. Control over H_s only
- Output noise power

$$\sigma_e^2 = \sigma_q^2 \left[1 + \sum_{k=0}^{M-1} h^2(k) \right]$$

- To minimize σ_e^2, $h(k) = 0$, for $k = 0, 1, 2, ...$

Least Squares Problem

- Minimize the integrated error in the band of interest (ω_p)
- Mathematically,

$$\min_{h(k),k=0,1,\ldots,M-1} \int_{\omega_p} \mid H_s(e^{j\omega}) \mid^2 d\omega$$

- No control over noise in the don't care band (ω_s, every thing other than ω_p)

Signal Backoff

- Maximum value of $\hat{q}[n]$ is $q_{max} \sum_{k=0}^{M-1} \mid h(k) \mid$
- Large integrated error (entire band) - Large value of $\hat{q}[n]$
- The maximum signal input at the quantizer is $x_{max} + q_{max} \sum_{k=0}^{M-1} \mid h(k) \mid$
- If maximum value exceeds range of quantizer, system becomes unstable
- Can limit the maximum value by
  - having small x_{max} (achieved by scaling down $x[n]$)
  - limiting the integrated noise

Weighted Least Squares

- Need to have some control over noise in don't care band (due to the instability problem)
- Reformulate the problem as

$$\min_{h(k),k=0,1,\ldots,M-1} \int W(e^{j\omega}) \mid H_s(e^{j\omega}) \mid^2 d\omega$$

- Expand the objective function as

$$F = 1 - 2\sum_{k=0}^{M-1} h(k)\cos\left[\omega(k+1)\right] + \sum_{k=0}^{M-1} h(k)e^{j\omega k}\sum_{m=0}^{M-1} h(m)e^{-j\omega m}$$

- Setting the derivatives w.r.t $h[k]$ equal to zero, leads to following equation

$$\sum_{k=0}^{M-1} A(r,k)h(k) = c(r)$$

- $A(r,k) = \int W(e^{j\omega})\cos\left[\omega(r-k)\right]d\omega$,
  $c(r) = \int W(e^{j\omega})\cos\left[\omega(r+1)\right]d\omega$
- optimal value, $h = A^{-1}c$

Extensions

- Only $q[n-k]$, $k = 1, 2, \dots$ were considered to be the predictor inputs
- We have some other information to use for prediction
- All of the others result in non linear optimization
- In such cases, heuristics are used for design

Example 1

- Consider $B_{in} = 10$, $B_{out} = 4$ and band of interest is $[0.5 - 0.1, 0.5 + 0.1]$

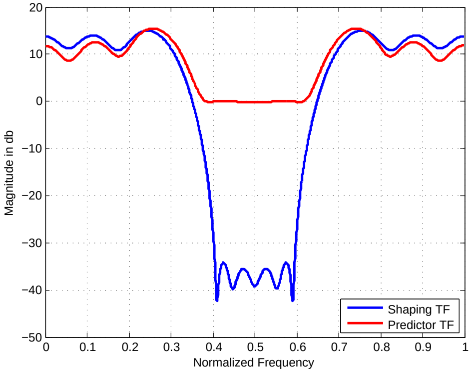


Figure: Shaping TF and predictor TF

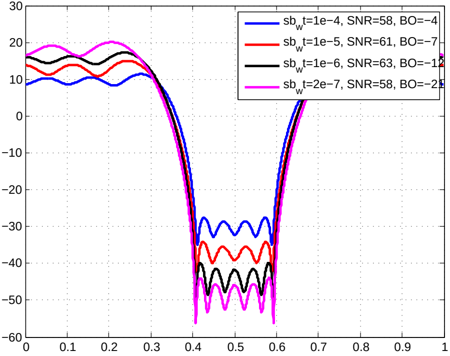


Figure: Shaping TF vs. stop band weight

The End

Thank you very much!
Questions?