Transmission Strategies for Sequential Binary Hypothesis Testing in Sensor Networks

Nirmal Shende

Indian Institute of Science, Bangalore

August 20, 2011

System Model

- N sensors binary hypothesis testing problem, where we are trying to decide between hypothesis $\mathcal{H}_0$ and hypothesis $\mathcal{H}_1$

$$\begin{aligned} \mathcal{H}_0 &: X_j^i \sim \mathcal{N}(0, 1), \\ \mathcal{H}_1 &: X_j^i \sim \mathcal{N}(\mu, 1). \end{aligned}$$

- Sensors take the samples and forward the LLR to the Fusion Center (FC)
- The FC runs a sequential test on received LLRs
- The observations of sensors are conditionally independent and identical

System Model

- Finite time required for a sensor to acquire and process a sample and to compute the LLR (T_s)
- Finite time required for a sensor to transmit to the FC (T_t)
- Sensors have power constraint, hence scale the LLR before transmitting
- Channel Between the sensors and the FC is AWGN
- The FC receives scaled and corrupted version of the LLR

Sequential Probability Ratio Test

- If L_j is LLR at time j, compute the cumulative sum W_j at time j, where,

$$W_j = W_{j-1} + L_j, \quad j \geq 0, \quad W_0 = 0.$$

- If $W_j \geq b$, declare in favor of $\mathcal{H}_1$
- If $W_j \leq a$, declare in favor of $\mathcal{H}_0$
- Else continue computing the cumulative sum
- a and b are the lower and upper threshold respectively

SPRT Performance

- The average detection delays are

$$DD_0 = \frac{1}{\Psi_0^F}\left[(1-\alpha)\log\frac{\gamma}{1-\alpha} + \gamma\log\frac{\alpha}{1-\gamma}\right]$$
$$DD_1 = \frac{1}{\Psi_1^F}\left[\gamma\log\frac{\gamma}{1-\alpha} + (1-\gamma)\log\frac{1-\gamma}{\alpha}\right]$$

- $\Psi_l^F = \mathbb{E}[L_j|\mathcal{H}_l], \quad l = 0, 1$
- Denote by
  $f_1(\alpha,\gamma) = \gamma\log\frac{\gamma}{1-\alpha} + (1-\gamma)\log\frac{1-\gamma}{\alpha}$
  $f_0(\alpha,\gamma) = (1-\alpha)\log\frac{\gamma}{1-\alpha} + \gamma\log\frac{\alpha}{1-\gamma}$

Transmission Strategy

- N sensors are divided into n groups, each having $P = N/n$ sensors
- Sensors belonging to the same group transmit the LLRs simultaneously to the FC, where the LLRs get added coherently
- Sensors which do not belong the transmitting group, take the observations.

LLR at Sensors

- Let X_j^i be the observation at sensor j at the i^{th} time instant
- The sensor transmits LLR based on K samples

$$T_i^j = a \sum_{t=0}^{K-1} \frac{(2\mu X_{i-t}^j - \mu^2)}{2}$$

- a is the scaling factor

Fusion Center

- The FC revives

$$Z_i = \sum_{k=1}^{P} T_i^k + N_i; \qquad N_i \sim \mathcal{N}(0, \sigma_f^2)$$

- The FC treats Z_i as the observation, and runs the SPRT by computing T_i^F, which is

$$T_i^F = \log \left(\frac{Z_i|\mathcal{H}_1}{Z_i|\mathcal{H}_0} \right)$$

- Since Z_i is the sum of Gaussian random variables, it is Gaussian. Thus

$$T_i^F = \frac{a\mu^2 KP}{a^2\mu^2 KP + \sigma_f^2} Z_i$$

Fusion Center

- Ψ_1^F can be calculated as follows

$$\begin{aligned}\Psi_1^F &= \mathbb{E}[T_i^F|\mathcal{H}_1] \\ &= \frac{a\mu^2 KP}{a^2\mu^2 KP + \sigma_f^2}\mathbb{E}[Z_i|\mathcal{H}_1] \\ &= \frac{a^2\mu^4 (KP)^2}{2(a^2\mu^2 KP + \sigma_f^2)}.\end{aligned}$$

- $\frac{T_t}{T_s}$ is an integer grater than 1

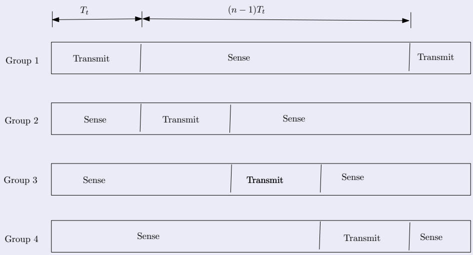

Figure: Sensing and Transmission for Four Groups of Sensors

- The FC receives new LLR every T_t seconds.

Case 1: $T_t/T_s \geq 1$

The average detection delay is given by

$$DD_1 = \frac{f_1(\alpha, \gamma)}{\Psi_1} T_t + K T_s$$

- When the sensors are grouped into n groups, each sensor transmits once every nT_t seconds. Hence the power scaling factor a is chosen such that

$$\mathbb{E}[T_i^k | \mathcal{H}_l]^2 = nP_s, \quad l = 0, 1$$

- Hence

$$a = \sqrt{\frac{nP_s}{\mu^2 K (1 + \frac{\mu^2 K}{4})}}.$$

- Minimizing the detection delay is equivalent to minimizing $\frac{1}{\Psi_1}$

$$\begin{aligned}\frac{1}{\Psi_1} &= \frac{2}{\mu^2 KP} + \frac{2\sigma_f^2}{a^2\mu^4 K^2 P^2} \\ &= \frac{2T_s}{\mu^2 T_t N}\left(1 + \frac{\sigma_f^2}{NP_s}\right)\frac{n}{n-1}\end{aligned}$$

Hence to minimize above expression, n should be set equal to N, which corresponds to the Round Robin scheme.

Case 2

- $\frac{(N-1)T_t}{T_s}$ is an less than 1

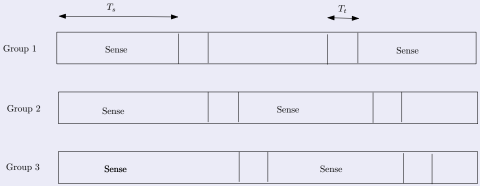

Figure: Sensing and Transmission for Three Groups of Sensors

- In this case the sensors have to wait extra time to collect one sample to transmit
- The FC receives the LLR on an average once every $(T_t + T_s)/n$ seconds

Case 2: $(N-1)T_t/T_s < 1$

- The average detection delay is given by

$$DD_1 = \frac{f_1(\alpha,\gamma)}{n\Psi_1}(T_t + T_s) + T_t + T_s$$

- Each sensor transmits once every $T_t + T_s$ seconds, the power scaling factor is given by

$$a = \sqrt{\frac{(T_t + T_s)P_s}{T_t\mu^2(1+\frac{\mu^2}{4})}} \quad (1)$$

Case 2:

Hence minimizing the detection delay amounts to maximizing $n\Psi_1$.

$$\begin{aligned} n\Psi_1 &= \frac{na^2\mu^4P^2}{2(a^2\mu^2P + \sigma_f^2)} \\ &= \frac{a^2\mu^4N^2}{2(a^2\mu^2N + n\sigma_f^2)} \end{aligned}$$

Which is maximized by $n = 1$, which is MAC with physical layer fusion.

Case 3

- $\exists$ an integer R, $1 \leq R \leq N$, such that,

$$\frac{(R-1)T_t}{T_s} < 1 \quad \text{and} \quad \frac{RT_t}{T_s} \geq 1.$$

- For $1 \leq n \leq R-1$, the average detection delay is minimized by setting $n = 1$
- For $R \leq n \leq N$, the average detection delay is minimized by setting $n = N$
- Hence minimum average detection delay is given by either the PLF or the RR

Optimal Scheme

- The detection delays for the PLF and the RR are given by

$$DD_{\mathrm{RR}} \approx \frac{2T_s}{\mu^2 N}\left(1 + \frac{\sigma_f^2}{NP_s}\right)\frac{N}{N-1} f_1(\alpha,\gamma).$$

$$DD_{\mathrm{PLF}} \approx \frac{2(T_t + T_s)}{\mu^2 N}\left(1 + \frac{\sigma_f^2}{NP_s}\frac{T_t}{T_t + T_s}\right) f_1(\alpha,\gamma)$$

- A sufficient condition for the RR to give less average detection delay than the PLF is

$$\frac{T_t}{T_s} \geq \frac{N}{N-1}.$$

Simulation Results

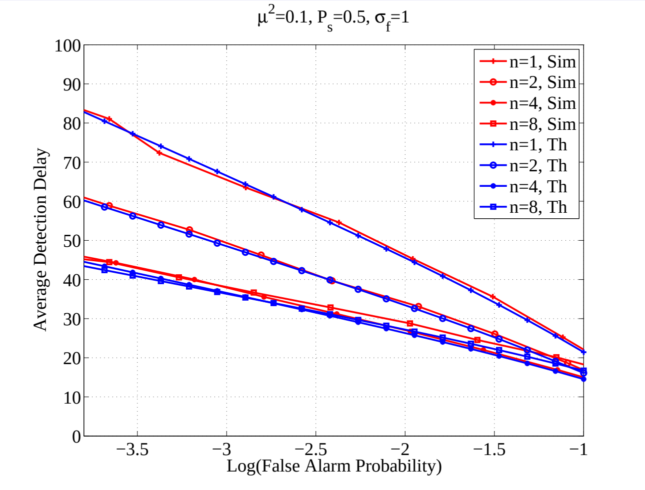

Figure: DD vs FA for N=8, T_t=2, T_s=1.

Simulation Results

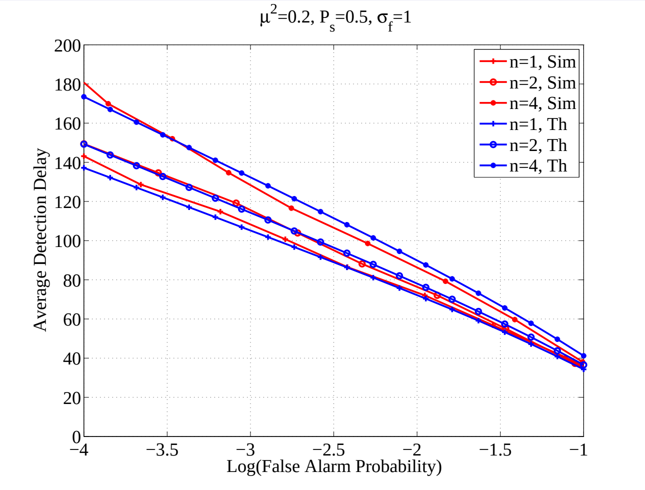

Figure: DD vs FA for N=4, T_t=1, T_s=4.

Optimal Scheme

- $\frac{T_t}{T_s} \geq \frac{N}{N-1}$: Optimal Scheme is the RR
- $\frac{T_t}{T_s} \leq \frac{1}{N-1}$: Optimal Scheme is the RR
- For intermediate value, the sensor power decides the optimal scheme
- For moderate to high secondary SNR, optimal scheme is the RR

Fading Channel

System Model

- The channel between the sensors and the FC is Nakagami-m fading
- The fade coefficient is known at the FC but not at the sensors
- Channel remains flat during one transmission and varies independently across different transmission instances for a particular sensor and also across different sensors
- Focus on the Round Robin transmission, since in the physical layer fusion, the LLRs may be added destructively

- The FC now receives

$$Z_i = h_i^k T_i^k + N_i;$$

- The FC runs SPRT by computing the LLR

$$\tilde{T}_i^F(h_i^k) = \frac{a\mu^2 K}{a^2\mu^2 K + \sigma_f^2/|h_i^k|^2}\tilde{Z}_i$$

where $\tilde{Z}_i = \Re\left((h_i^k)^* Z_i/|h_i^k|^2\right)$

Performance Analysis

- The expected receive LLR and detection delay as function of h is

$$\begin{aligned} \Psi_1(h) &= \frac{a^2\mu^4K^2}{2(a^2\mu^2K + \sigma_f^2/|h|^2)} \\ DD_1(h) &= \frac{f_1(\alpha,\gamma)}{\Psi_1(h)}(T_t) + (N-1)T_t + T_s \end{aligned} \quad (2)$$

- The average detection delay is

$$DD_1 = \mathbb{E}_h[DD_1(h)]$$

$$DD_1 = \left(\frac{2}{\mu^2K} + \frac{2\sigma_f^2}{a^2\mu^4K^2}\frac{m}{m-1}\right) f_1(\alpha,\gamma)T_t + NT_t.$$

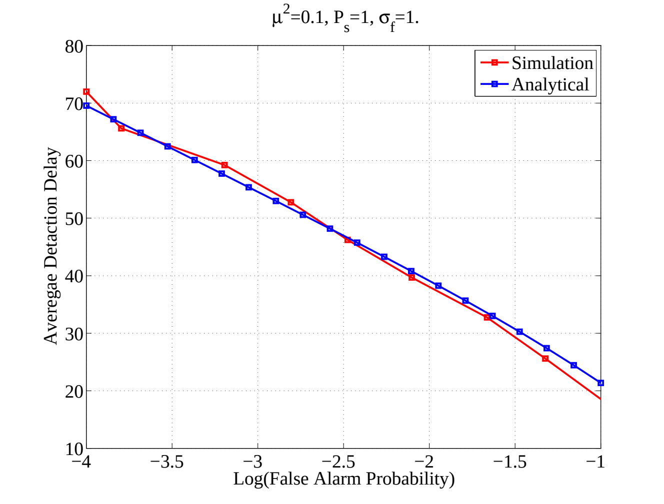

Figure: DD vs FA for Nakagami-m fading with m=4, N=5, T_t=1, T_s=1.

Rayleigh Fading Diversity at FC

- The FC has two receive antennas, and the channel fades are independent across these two antennas.
- Then by using maximal ratio combining, the AWGN RR performance can be recovered for Rayleigh fading channel.

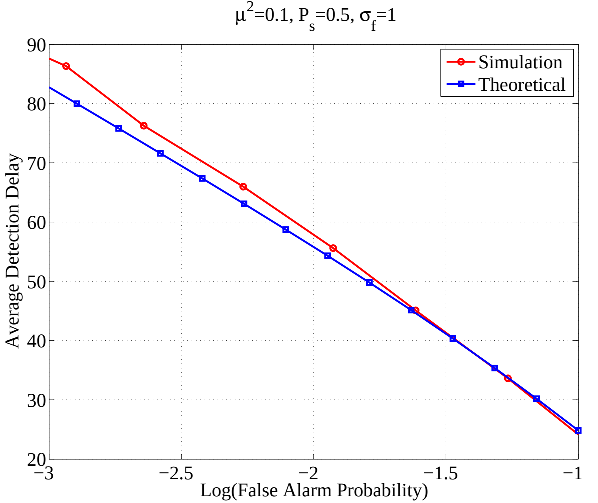

Figure: DD vs FA for Rayleigh fading with two antennas at the FC, $N=5$, $T_t=1$, $T_s=1$.

Thank You

Questions?