### Literature Survey on Quantized Compressed sensing and Massive MU-MIMO Systems

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Image: A mathematical states of the state

## Sparse Signal Reconstruction from Quantized Noisy Measurements via GEM Hard Thresholding - Aleksandar Dogandzic, TSP May 2012

- Proposed a generalized EM algorithm for SSR from quantized noisy measurements in which the unquantized measurements are treated as the missing data
  - GEM for approximately computing the maximum likelihood (ML) estimates of the unknown sparse signal and noise variance parameter
  - Convergence under mild conditions
- System Model:  $\mathbf{b} = \mathcal{Q}(\mathbf{y}) = \mathcal{Q}(\mathbf{Hs} + \mathbf{e})$
- E-step:

$$\mathcal{Q}\left(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p)}\right) = \mathbb{E}_{\mathbf{y}|\mathbf{b},\boldsymbol{\theta}}\left[\ln p_{\mathbf{y},\mathbf{b}|\boldsymbol{\theta}}\left(\mathbf{y},\mathbf{b}|\boldsymbol{\theta}\right)|\mathbf{b},\boldsymbol{\theta}^{(p)}\right]$$
(1)

M-step:

$$\min_{\mathbf{s}\in\mathcal{S}_r} \left\| \hat{\mathbf{y}}^{(p)} - \mathbf{Hs} \right\|_2^2 \tag{2}$$

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- M-step requires a combinatorial search which is computationally complex
- Generalized M-step: Increase the expected-data log-likelihood instead of maximizing it

$$\begin{split} \boldsymbol{s}^{(p+1)} = & \mathcal{T}_{r} \left( \boldsymbol{s}^{(p)} + \frac{1}{c^{2}} \boldsymbol{H}^{T} \left( \boldsymbol{\hat{y}}^{(p)} - \boldsymbol{H} \boldsymbol{s}^{(p)} \right) \right) \\ = & \mathcal{T}_{r} \left( \boldsymbol{s}^{(p)} + \frac{\sigma^{(p)}}{c^{2}} \boldsymbol{H}^{T} \boldsymbol{\delta}^{(p)} \right) \tag{11a} \\ \sigma^{2})^{(p+1)} = & \left[ \| \boldsymbol{\hat{y}}^{(p)} - \boldsymbol{H} \boldsymbol{s}^{(p+1)} \|_{2}^{2} + \sum_{i=1}^{N} \operatorname{var}_{y_{i}|b_{i}} \boldsymbol{s} \left( \boldsymbol{y}_{i}|b_{i}, \boldsymbol{\theta}^{(p)} \right) \right] / N \\ = & \| \boldsymbol{\hat{y}}^{(p)} - \boldsymbol{H} \boldsymbol{s}^{(p+1)} \|_{2}^{2} / N + \left( \sigma^{2} \right)^{(p)} \left( 1 - \sum_{i=1}^{N} \boldsymbol{\xi}^{(p)}_{i} / N \right) \tag{11b}$$

### The Use of Unit Norm Tight Measurement Matrices for One-bit Compressed Sensing - Robert Heath Jr et al, ICASSP 2016

- MSE analysis for 1-bit CS schemes based on measurement matrices that correspond to unit norm tight frames (i.e., a frame  $\mathbf{A} \in \mathbb{R}^{m \times N}$  such that  $\mathbf{A}\mathbf{A}^T = Nm^{-1}\mathbf{I}_m$ )
- Motivation:
  - 1-bit adaptive CS (iteratively adapting the threshold of the quantizer based on the MSE of the previous measurements) increases the computational complexity
  - In unquantized CS, unit norm tight frames improves the MSE performance of algorithms like basis pursuit denoising, OMP and the Dantzig selector
- System Model:  $\overline{\mathbf{y}} = \operatorname{sign}(\mathbf{A}\mathbf{x})$
- Relaxed optimization problem:  $\min_{\mathbf{x}} \|\mathbf{x}\|_1$  s.t.  $\operatorname{diag}(\overline{\mathbf{y}}) \mathbf{A} \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^m \overline{\mathbf{y}}_i \mathbf{a}_i^T \mathbf{x} = m$
- New optimization problem with unit norm tight frame measurement matrix:

$$\begin{array}{ll} \underset{\mathbf{x},\mathbf{r},\mathbf{r}\in\mathcal{S}}{\text{minimize}} & \|\mathbf{x}\|_{1} \\ \text{subject to} & \begin{bmatrix} \text{diag}(\bar{\mathbf{y}})\mathbf{A} & \mathbf{I}_{m} \\ & \mathbf{I}_{1\times m}\text{diag}(\bar{\mathbf{y}})\mathbf{A} & \mathbf{0}_{1\times m} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{m\times 1} \\ 1 \\ 1 \end{bmatrix}, \tag{6}$$

where  $\mathbf{r} \in \mathbb{R}^m$  is the slack variable. We will denote

$$\mathbf{B} = \begin{bmatrix} \operatorname{diag}(\bar{\mathbf{y}})\mathbf{A} & \mathbf{I}_m \\ m^{-1}\mathbf{1}_{1\times m}\operatorname{diag}(\bar{\mathbf{y}})\mathbf{A} & \mathbf{0}_{1\times m} \end{bmatrix}.$$
 (7)

$$\mathbf{B}\mathbf{B}^T = \begin{bmatrix} \frac{N+m}{m^2}\mathbf{I}_m & \frac{N}{m^2}\mathbf{1}_{m\times 1} \\ \frac{N}{m^2}\mathbf{1}_{1\times m} & \frac{N}{m^2} \end{bmatrix} \in \mathbb{R}^{(m+1)\times (m+1)},$$

• Effective measurement matrix **B** has properties closely related to those of unit norm tight frames. Hence MSE performance is better than that of other measurement matrices

# Online Dictionary Learning from Large-Scale Binary Data - Giannakis et al, EUSIPCO 2016

- Jointly learn the sparse signal representation and the unknown dictionary
- Existing approach: Stochastic approximation framework used when measurements are of infinite precision
- Ontributions:
  - Batch estimator (Complete data available)
  - Online version that leverages stochastic gradient iterations (Sequential data available)
- System Model:  $\mathbf{y}_{\tau} = \text{sign} \left( \mathbf{D} \mathbf{s}_{\tau} + \mathbf{n}_{\tau} \right)$
- Using the monotonicity of the log likelihood function, the optimization problem to be solved is:  $\hat{\mathbf{D}}, \hat{\mathbf{S}} := \arg\min_{\mathbf{D} \in \mathcal{D}, \mathbf{S} \in \mathcal{S}} \sum_{\tau=1}^{T} - [y_{i\tau} \mathbf{d}_{i}^{T} \mathbf{s}_{\tau} + \lambda \| \mathbf{s}_{\tau} \|_{1}]$ 
  - Block multi-convex w.r.t. D and S
  - Solved using a block coordinate descent iteration
- One-bit online algorithm:  $\min_{\mathbf{s}_t, \mathbf{D}} \mathbb{E} \left\{ \sum_{\tau=1}^T y_{i\tau} \mathbf{d}_i^T \mathbf{s}_\tau + \lambda \left\| \mathbf{s}_\tau \right\|_1 \right\}$ 
  - Expectation w.r.t the unknown probability distribution on  $y_{i,\tau}$  replaced with an instantaneous approximation which discards all past data and leads to computationally affordable updates
  - Alternating minimization approach
  - Sparse vector recovered from the incomplete binary measurements followed by a dictionary refinement step using a single stochastic gradient descent iteration
- Future Scope: Convergence analysis of the iterative algorithm and leveraging kernels for dictionary learning in nonlinear settings

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# Sparse Dictionary Learning from 1-bit data - Sidiropoulos, Giannakis et al, ICASSP 2014

- To estimate *mn* real valued elements of the matrix  $\mathbf{X}^* \in \mathbb{R}^{m \times n}$  from its 1-bit quantized noisy observations
- Question: Can one still obtain a consistent estimate of  $X^*$  when it can be expressed as a product of  $D^* \in \mathbb{R}^{m \times p}$  (dictionary) and  $A^* \in \mathbb{R}^{p \times n}$ ? (Each column in  $A^*$  contains  $k < k_{max} < p$ )
- Approach: Variant of ML estimation
  - Regularize the negative likelihood of each candidate reconstruction with a term that quantifies its complexity
  - More complicated candidates have larger cost
- Main Result:
  - If the candidate reconstructions **X** satisfies certain conditions on the boundedness of its entries, the penalized ML estimation satisfies an upper bound on the estimation error (which depends on the system dimensions and the max value of the individual entry of **X**)
  - If the matrix **X** satisfies the sparsity decomposition mentioned above, then the penalized ML estimate satisfies an upper bound on the estimation error with high probability (which plays a role in the upper bound)
- Note: The framework proposed here may also be applied to analogous tasks of higher-order tensor approximation from 1-bit data (not investigated in this paper. Future research direction)

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# Channel Estimation in Broadband Millimeter Wave MIMO Systems with Few-Bit ADCs: Robert W. Heath Jr. et al, arXiv

- Formulated the problem of estimating BB mmWave channels under few-bit ADC as a noisy, quantized CS problem
- Proposed algorithms based on GAMP and VAMP
- Novel training sequence design that results in low channel estimation error, low complexity, and low PAPR
- Experiment study of design choices like ADC precision, the type and length of the training sequence, and the type of estimation algorithm
- Peformance metrics: MSE, Mutual information, and achievable rate

#### Approach

- mmWave channel is sparse in the angle-delay domain. ISI model for MIMO channel transformed to a quantized CS model
- EM combined with AMP to avoid the need of a prior (GAMP and VAMP)
- Training sequence design using DFT matrices for reducing the computational complexity

#### Take Away:

- EM-VAMP algorithm has a superior performance-complexity tradeoff
- FFT-based implementation facilitates low complexity estimation of channels with large antenna numbers and delay spreads
- 1-bit ADCs incur only small performance losses at low SNR, and 3-4 bit ADCs incur only small losses up to medium SNRs

- Motivation:
  - Reconstruction error when recovering sparse signals from 1-bit measurements is bounded by an exponentially decaying term
  - New adaptive measurement methods to reduce this exponential decay rate exist in the literature. Disadvantage is that, after every new measurement, complete optimization problem needs to be solved
- Contributions
  - Novel adaptive way to compute measurements for 1-bit SSR without the need to solve a full
    optimization problem
  - Main idea: To use the fact that each 1-bit measurement provides information about which side of the measurement subspace the target sparse vector lies in
- System Model:  $\overline{y} = \text{sign} (Ht + n)$
- Reformulated SSR problem:

$$\overline{\mathbf{y}}_{\nu} = \operatorname{sign}\left(\operatorname{vec}\left(\mathbf{U}_{R}\mathbf{H}_{\nu}\mathbf{Z} + \mathbf{N}\right)\right) = \operatorname{sign}\left(\left(\mathbf{Z}^{T} \otimes \mathbf{U}_{R}\right)\mathbf{h}_{\nu} + \mathbf{n}_{\nu}\right)$$
(3)

• Measurement matrix:  $\mathbf{A} = \mathbf{Z}^T \otimes \mathbf{U}_R$ 

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- Drawback of 1-bit CS: After a certain no of measurements, the possible surface of the hypersphere where the signal lies is greatly reduced
- The Adaptive Method:
  - Given m-1 measurements, design measurement m such that it provides a large reduction in the possible area where the solution is located on the hypersphere
  - Main idea: Find the CoG of the convex set defined by the current set of measurements and apply a
    measurement that passes through this point. Its a convex maximization problem which is hard to
    solve. Need to find an approximate position
  - CoG approximation using (This part not very clear)
    - Chebyshev center maximum radius Euclidean ball
    - Maximum volume ellipsoid method
    - Analytic CoG
  - New measurement  $\mathbf{a}_m$  chosen such that  $\mathbf{a}_m^T \mathbf{c}$  is minimized (with constraints on the new measurement)
  - Once new measurement is designed, a fast solution based on ellipsoid method (used to solve linear programs) is proposed
- Performance comparison between the proposed method and a fixed measurement model based method. Performance is found to be superior

# Millimeter Wave MIMO Channel Estimation Based on Adaptive Compressed Sensing - Rappaport et al, ICC 2017 (NOT A 1-bit CS PAPER)

- Formulated the problem of estimating mmWave channels as a noisy, CS problem
- Contributions:
  - Novel approach of constructing beamforming dictionary matrices for sparse channel estimation using the continuous basis pursuit (CBP) concept (instead of grid based methods)
  - Based on CBP dictionary, two low complexity algorithms to exploit channel sparsity for adaptively
    estimating multipath channel parameters in mmWave channels are proposed
- System Model:

 $\mathbf{Y} = \mathbf{W}^H \mathbf{HFS} + \mathbf{Q}$ 

• CS reformulation:

$$\begin{split} \mathbf{y} &= \sqrt{P} \mathrm{vec}(\mathbf{W}^H \mathbf{HF}) + \mathrm{vec}(\mathbf{Q}) \\ &= \sqrt{P}(\mathbf{F}^T \otimes \mathbf{W}^H) \mathrm{vec}(\mathbf{H}) + \mathbf{n}_{\mathrm{Q}} \\ &= \sqrt{P}(\mathbf{F}^T \otimes \mathbf{W}^H) (\mathbf{A}_{\mathrm{BS,D}}^* \ast \mathbf{A}_{\mathrm{MS,D}}) \mathbf{z} + \mathbf{n}_{\mathrm{Q}} \\ &= \sqrt{P}(\mathbf{F}^T \mathbf{A}_{\mathrm{BS,D}}^* \otimes \mathbf{W}^H \mathbf{A}_{\mathrm{MS,D}}) \mathbf{z} + \mathbf{n}_{\mathrm{Q}} \\ &= \sqrt{P} \mathbf{F}^T \mathbf{A}_{\mathrm{BS,D}}^* \mathbf{z}_{\mathrm{BS}} \otimes \mathbf{W}^H \mathbf{A}_{\mathrm{MS,D}} \mathbf{z}_{\mathrm{MS}} + \mathbf{n}_{\mathrm{Q}} \end{split}$$

 Conventionally, angle based dictionary is used, but the true continuous domain AoDs and AoAs may lie off the center of the grid bins. Finer discretization leads to increase in complexity • CBP Approach: Antenna array factor is a continuous and smooth function, which can be approximated by a first-order Taylor expansion (Beam pattern smoother than a grid based approach)

$$\widetilde{\mathbf{A}}_{\mathrm{BS},\mathrm{D}} = \mathbf{B}_{\mathrm{BS}} \mathbf{t}_{\mathrm{BS}} = [\mathbf{a}_{\mathrm{BS}}(\phi_1), \cdots, \mathbf{a}_{\mathrm{BS}}(\phi_N), \\ \Delta \phi \mathbf{b}_{\mathrm{BS}}(\phi_1), \cdots, \Delta \phi \mathbf{b}_{\mathrm{BS}}(\phi_N)]$$

- Hierarchical Codebook: S levels. Each level contains multiple subsets. Each subset contains a set of beamforming vectors. Each beamforming vector covers a certain beamwidth
- Both beamforming and combining dictionaries are designed in a similar way
- Smoother beam patterns when compared to grid based dictionary approach
- Adaptive CS Approach for channel estimation: Training symbols precoded using vectors from the first level, followed by training based on the received strength along the strongest beam
- Two algorithms proposed based on the adaptive CS approach
- Future extensions:
  - To extend the multipath estimation algorithms to the case where the number of dominant paths is unknown
  - To implement the proposed dictionary matrices and algorithms to other types of antenna arrays

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### Goal

• To estimate the parameters of a noisy quantized signal

#### Previous relevant work

• Frequency and phase estimation for temporal and spatial sinusoidal signals using only sign comparisons to zero, including derivations of CRB

#### Contributions

- More general sampling structure which uses time varying thresholds, as well as using a generic deterministic signal parameterized by a vector
- Derivation of Fisher information matrix when the noise is i.i.d Gaussian with known or unknown variance
- As a special case, single sinusoidal parameter estimation and the CRB for amplitute, frequency and phase estimators are computed
- ML estimator for the sinusoidal signal parameters is proposed and its performance compared with the CRB as a function of the number of observations

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- System Model:  $y(t) = \text{sign}(s(t, \beta) + e(t) h(t))$  where e(t) and h(t) are the noise and the comparator respectively
- Likelihood functions are derived and the Fisher information matrix (FIM) is computed
- CRB is computed by computing the inverse of the FIM
- ML estimators for sinusoidal parameters are derived and the MSE performance compared to the CRB
- For each time instant, n-bits are used to encode the thresholds (2<sup>n</sup> thresholds). Thresholds generated based on a uniform distribution

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### Methods for quantized compressed sensing - Deanna Needell et al, ITA 2016

- Compare and catalog the performance of various greedy quantized CS algorithms
- Quantized Basis Pursuit

$$\min_{\mathbf{x}} \|\mathbf{x}\|_{1} \quad \text{s.t} \mathbf{b}_{1} \le \mathbf{\Phi} \mathbf{x} \le \mathbf{b}_{2} \tag{4}$$

- Interior point, simplex methods to solve this linear program
- Current Greedy approaches
  - Quantized Subspace pursuit (QSP): Iterative algorithm which updates the support and the residual during each iteration till convergence

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      Algorithm 1 Quantized Subspace Pursuit (QSP)

      Input: sparsity level K, measurement matrix Φ ∈ ℝ<sup>M×N</sup>, compressed quantized signal y ∈ ℝ<sup>M</sup>

      Initialize: T<sup>0</sup> = supp<sub>K</sub>(Φ*y), y<sub>t</sub><sup>0</sup> = resid(y,Φ<sub>T0</sub>)

      repeat

      T<sup>1</sup> = T<sup>1-1</sup> ∪ supp<sub>K</sub>(Φ*y<sub>t</sub><sup>r-1</sup>).

      x<sub>p</sub> = pcoeff(y,Φ<sub>p1</sub>) and T<sup>1</sup> = supp<sub>K</sub>(x<sub>p</sub>)

      y<sub>t</sub><sup>i</sup> = resid(y,Φ<sub>T1</sub>)

      until |y<sub>t</sub><sup>i</sup>|| ≥ ||y<sub>t</sub><sup>i</sup><sup>r-1</sup>||<sub>2</sub>

      T<sup>1</sup> = T<sup>1-1</sup> ∪ supp<sub>K</sub>(x<sub>p</sub>)

      y<sub>t</sub><sup>i</sup> = resid(y,Φ<sub>T1</sub>)

      until |y<sub>t</sub><sup>i</sup>|| ≥ ||y<sub>t</sub><sup>i</sup><sup>r-1</sup>||<sub>2</sub>

      T<sup>i</sup> = T<sup>i-1</sup> ∪

      Output: \hat{x}/||\hat{x}||_2 where \hat{x}_{[1,...,N]-T^i} = 0 and x_{T^i} = Φ_{T^i}^{\dagger}y
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Quantized Iterative Hard Thresholding (QIHT):

$$\mathbf{x}^{l+1} = \eta_{K} \left( \mathbf{x}^{l} + \mu \mathbf{\Phi}^{*} \left( \mathbf{y} - \operatorname{sign} \left( \mathbf{\Phi} \mathbf{x}^{l} \right) \right) \right)$$
(5)

where  $\mu$  is a scalar that controls the gradient step size

This solves the problem

$$\min_{\boldsymbol{\epsilon} \in S^{N-1}} \mu \left\| \left[ \boldsymbol{y} \odot (\boldsymbol{\Phi} \mathbf{x}) \right]_{-} \right\|_{1} \quad \text{s.t.} \quad \| \mathbf{x} \|_{0} \leq K, \quad \boldsymbol{\epsilon} \geq \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} > \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} < \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} \geq \boldsymbol{\epsilon} > \boldsymbol{\epsilon} > \boldsymbol{\epsilon} \geq \boldsymbol{\epsilon} \leq \boldsymbol{\epsilon} > \boldsymbol{\epsilon} >$$

(Sai Subramanyam Thoota, ECE, IISc)

- Two new greedy approaches for sparse signal recovery
  - Quantized compressed sampling matching pursuit (QCoSaMP): Project y onto the quantization region and then apply LS (difference between CoSaMP and QCoSaMP)

Algorithm 3 Quantized Compressive Sampling Matching Pursuit (OCoSaMP)

Input: measurement matrix  $\mathbf{\Phi} \in \mathbb{R}^{M \times N}$ , quantized compressed signal  $y = \Phi x$ , sparsity level K > 0, maximum number of iterations I Initialize:  $a^0 = 0$ , v = v, k = 0while k < I do Set  $\boldsymbol{u} = \boldsymbol{\Phi} \boldsymbol{v}, \ \boldsymbol{\Omega} = \operatorname{supp}_{2V}(\boldsymbol{u})$ Merge:  $T = \Omega \cup \text{supp}(a^{k-1})$ Projection:  $\mathbf{x} = \operatorname{argmin}_{u \in \mathcal{R}_{\mathbf{y}}} \|\mathbf{y} - \mathbf{u}\|_2$  $\boldsymbol{b}_T = \operatorname{argmin}_{\boldsymbol{v}} \| \boldsymbol{u} - \boldsymbol{\Phi}_T \boldsymbol{x} \|_2$  and  $\boldsymbol{b}_{T^c} = 0$ Set  $a^k = b$ Update:  $\boldsymbol{v} = \boldsymbol{y} - f_O(\boldsymbol{\Phi}\boldsymbol{a}^k)$ k = k + 1. end while Output:  $\hat{x} = a^k / ||a^k||_2$ 

- Adaptive Outlier Pursuit for Quantized Iterative Hard Thresholding (AOP-QIHT):
  - Noise: Prequantization (modeled as iid Gaussian) and post-guantization noise (modeled as a sparse vector)

$$\min_{\boldsymbol{x}\in S^{N-1}, \boldsymbol{n}} \sum_{k=1}^{M} \phi\left((\boldsymbol{\Phi}\boldsymbol{x}+\boldsymbol{n})_{k}, y_{k}\right) \quad \text{s.t.} \quad \|\boldsymbol{n}\|_{0} \leq L, \ \|\boldsymbol{x}\|_{0} \leq K.$$

$$\phi(x, y) = \sum_{j=2}^{2^Q} \left| [(x - \tau_j)(y - \tau_j)]_- \right|,$$

- Ideas of both AOP and QIHT for 1 bit CS combined
- Auxiliary variable defined for sparsity constraint and alternating minimization used to solve the problem

14 / 19

- Performance for a given bit-depth, sparsity and noise level
- 1-bit QIHT and AOP-QIHT algorithms tend to perform best for low-noise cases while OCoSoMP and OSP perform better for higher poice October 21, 2017 Literature Survey

(Sai Subramanyam Thoota, ECE, IISc)

- Characterized the minimal distortion in quantized sparse signal recovery as a function of the sampling ratio (m/n), sparsity rate, noise intensity and the total no. of bits in the quantized representation
- MMSE analysis using REPLICA method (from the literature). Large system analysis with a finite sampling ratio. Relies on various assumptions, which are satisfied (but yet to be proven) for CS problems
- Characterized the MMSE in CS under quantization with a "Single-letter" expression, which is a function of the system parameters
- Indirect distortion rate function MMSE distortion equivalent to the MMSE distortion in estimating the source from any rate R encoded version of its observation through a scalar Gaussian channel
- Achievability and converse theorems to show that under the asymptotic decoupling and posterior distribution assumptions, there exists a quantization scheme that attains an MMSE as close as desired to the replica posterior iDRF
- Main problem solved: Minimize the average distortion over all encoders  $g:\mathbb{R}^m o \{0,1\}^{\lfloor nR \rfloor}$

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- Theoretical results for noisy 1-bit CS and sparse binomial regression
- s-sparse signal in ℝ<sup>n</sup> can be accurately estimated from m = O (s log(n/s)) 1 bit measurements using a simple convex program
- First results that tie together the theory of sparse logistic regression to 1-bit compressed sensing
- Results apply to general signal structures aside from sparsity; Only the size of the set *K* where signals reside needs to be known
- Main Results:
  - Convex program to solve the estimation problem from 1-bit measurements
  - Theorem to upper bound the estimation error (which depends on the number of measurements and a free parameter which decides the probability of successful recovery)
  - Minimum number of measurements needed under the assumptions of the above mentioned theorem
  - Theorem to lower bound the number of measurements when the signal is corrupted by noise (bit flips)
  - Results are specialized for
    - 1-bit CS
    - Sparse logistic regression
    - Low-rank matrix recovery

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- Ochannel Estimation and Precoder Design for Millimeter-Wave Communications: The Sparse Way
- **Q** Channel Estimation in Millimeter Wave MIMO Systems with One-Bit Quantization
- **③** Sparse Channel Estimation for Massive MIMO with 1-bit Feedback per Dimension

#### Capacity Analysis

- Q Capacity Analysis of One-Bit Quantized MIMO Systems With Transmitter Channel State Information
- Quantized Massive MU-MIMO-OFDM Uplink
- **③** High SNR Capacity of Millimeter Wave MIMO Systems with One-Bit Quantization
- Uplink Achievable Rate for Massive MIMO Systems with Low-Resolution ADC
- Hybrid Architectures with Few-Bit ADC Receivers: Achievable Rates and Energy-Rate Tradeoffs

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- 1-Bit Compressive Sensing
- Oistributed Variable-Rate Quantized Compressed Sensing in Wireless Sensor Networks
- A Distributed 1-bit Compressed Sensing Algorithm Robust to Impulsive Noise
- Sample Complexity for 1-bit Compressed Sensing and Sparse Classification
- Noisy Matrix Completion Under Sparse Factor Models
- Variational Bayesian Algorithm for Quantized Compressed Sensing
- **③** An RIP-Based Approach to  $\Sigma\Delta$  Quantization for Compressed Sensing
- A One-Bit Reweighted Iterative Algorithm for Sparse Signal Recovery

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