# Resource Allocation in OFDMA Cellular Networks

## An Iterative Re-weighted Minimization Framework

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# Outline

# Goal !

**Resource allocation in OFDMA cellular network:**

As found in the OFDMA literature, resource allocation can be broadly classified into 3 types:

- Utility Adaptive (UA) problem:
  - Maximize a system utility function-maintaining QoS/power constraints, etc.
  - Sum-rate utility: Rate Adaptive (RA) problem.
- Margin Adaptive (MA) problem:
  - Minimize the total transmit power-maintaining QoS/power constraints, etc.
- Energy Adaptive (EA) problem:
  - Maximize the energy efficient utility function- maintaing QoS/power constraints, etc.

## Our Goal

- General framework for resource allocation in OFDMA based cellular networks.

# Single-Cell OFDMA Network

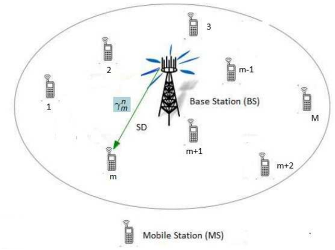



FIGURE – Single-cell multi-user OFDMA network

**Notation:**

$\mathcal{M} \in \{1, 2, \ldots, M\} \Rightarrow$ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\} \Rightarrow$ Subcarrier indices
$\gamma_m^n \Rightarrow$ Subcarrier gain
$p_m^n \Rightarrow$ Allotted power
$y_m^n \Rightarrow$ Binary indicator variable
$R_m^n \Rightarrow$ Rate achieved

**Rate achieved by $m^{th}$ user**

$$R_m^n = \log_2(1 + SNR_m^n); \quad SNR_m^n = \gamma_m^n p_m^n \tag{1}$$

$$\text{Total rate achieved} \Rightarrow R_m = \sum_{n \in \mathcal{N}} \log_2(1 + \gamma_m^n p_m^n); \tag{2}$$

## OFDMA constraint

$$\sum_{m \in \mathcal{M}} y_m^n \leq 1, \quad \forall n \in \mathcal{N} \tag{3}$$

$$y_m^n \in \{0, 1\}, \quad \forall m, n$$

## Maximum power on each subcarrier constraint

$$0 \leq p_m^n \leq y_m^n P^{max}, \quad \forall m, n \tag{4}$$

$$y_m^n = \begin{cases} 1, & 0 \leq p_m^n \leq P^{max} \\ 0, & 0 \leq p_m^n \leq 0 \Rightarrow p_m^n = 0 \end{cases}$$

## Total transmit power constraint

$$\sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{M}} p_m^n \leq P_T \tag{5}$$

# Problem Formulation

## Utility adaptive (UA) optimization problem

$$\begin{aligned} \max_{\{p_m^n, y_m^n\}} \quad & U(R_1, R_2, \ldots, R_M) \\ s.t. \quad c_1 : \quad & \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p_m^n \leq P_T, \\ c_2 : \quad & 0 \leq p_m^n \leq y_m^n P^{\max}, \quad \forall m, n, \\ c_3 : \quad & \sum_{m \in \mathcal{M}} y_m^n \leq 1, \quad \forall n \in \mathcal{N}, \\ c_4 : \quad & y_m^n \in \{0, 1\}, \quad \forall m, n. \end{aligned} \tag{6}$$

## Popular utility functions

- (Weighted)Sum rate utility: $\sum_{m \in \mathcal{M}} (w_m) R_m$
- Harmonic Mean utility: $\frac{1}{\sum_{m \in \mathcal{M}} R_m^{-1}}$
- Min-rate utility: $\min\{R_1, R_2, \ldots, R_M\}$

# Problem Formulation

## Utility adaptive (UA) optimization problem

$$
\begin{aligned}
\max_{\{p_m^n, y_m^n\}} \quad & U(R_1, R_2, \ldots, R_M) \\
\text{s.t.} \quad \mathrm{c}_1: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \le P_T, \\
\mathrm{c}_2: \quad & 0 \le p_m^n \le y_m^n P^{\max}, \quad \forall m, n, \\
\mathrm{c}_3: \quad & \sum_{m\in\mathcal{M}} y_m^n \le 1, \quad \forall n \in \mathcal{N}, \\
\mathrm{c}_4: \quad & y_m^n \in \{0, 1\}, \quad \forall m, n.
\end{aligned}
\tag{7}
$$

## Bad news

- The above problem in its raw form is NP-Hard

# Proposed Algorithm Based on IRM Framework

## Smooth concave utility function

Sum-rate utility function $\Rightarrow \sum_{m\in\mathcal{M}} R_m$

## Original UA problem

$$
\begin{aligned}
\max_{\{p_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \log_2(1 + p_m^n \gamma_m^n) \\
\text{s.t.} \quad & c_1 : \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \leq P_T, \\
& c_2 : 0 \leq p_m^n \leq y_m^n p^{max}, \quad \forall m, n, \\
& c_3 : \sum_{m\in\mathcal{M}} y_m^n \leq 1, \quad \forall n \in \mathcal{N}, \\
& c_4 : y_m^n \in \{0, 1\}, \quad \forall m, n.
\end{aligned}
$$

$\iff$

## Let $x_m^n = p_m^n / P^{max}$

$$
\begin{aligned}
\max_{\{x_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \log_2(1 + P^{max} x_m^n \gamma_m^n) \\
\text{s.t.} \quad & c_1 : \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} P^{max} x_m^n \leq P_T, \\
& c_2 : 0 \leq x_m^n \leq y_m^n, \quad \forall m, n, \\
& c_3 : \sum_{m\in\mathcal{M}} y_m^n = 1, \quad \forall n \in \mathcal{N}, \\
& c_4 : y_m^n \in \{0, 1\}, \quad \forall m, n.
\end{aligned}
$$

The reformulated optimization problem is equivalent to the original optimization problem.

## Optimal solution of $y_m^n$

$$\begin{aligned} \min_{\{y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_m^n + \epsilon)^q \\ s.t. \quad & \sum_{m\in\mathcal{M}} y_m^n = 1, \quad \forall n \in \mathcal{N}, \\ & y_m^n \geq 0, \quad m \in \mathcal{M}, n \in \mathcal{N}. \end{aligned} \tag{8}$$

## Relaxed optimization problem

$$\begin{aligned} \min_{\{x_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} -\log_2(1 + P^{max} x_m^n \gamma_m^n) + \lambda \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} (y_m^n + \epsilon)^q \\ s.t. \quad c_1 : \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \leq P_T, \\ c_2 : \quad & 0 \leq x_m^n \leq y_m^n, \quad \forall m, n, \\ c_3 : \quad & \sum_{m\in\mathcal{M}} y_m^n = 1, \quad \forall n \in \mathcal{N}. \end{aligned} \tag{9}$$

## Relaxed optimization problem

$$\begin{aligned}
\min_{\{x_m^n, y_m^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{-\log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{(y_m^n + \epsilon)^q}_{\text{concave}} \\
s.t. \quad c_1 : \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_m^n \le P_T, \\
c_2 : \quad & 0 \le x_m^n \le y_m^n, \quad \forall m, n, \\
c_3 : \quad & \sum_{m\in\mathcal{M}} y_m^n = 1, \quad \forall n \in \mathcal{N}.
\end{aligned} \tag{10}$$

## Majorization-Minimization (MM) approach

Convex + Concave function

$\Downarrow$

Keep convex part, linearize concave part

## Relaxed optimization problem

$$\min_{\{x_m^n, y_m^n\}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \underbrace{-\log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \underbrace{(y_m^n + \epsilon)^q}_{\text{concave}} \quad (11)$$

$$s.t. \quad \mathrm{c}_1, \mathrm{c}_2 \text{ and } \mathrm{c}_3$$

Can be solved using MM approach

## Keep convex part as it is and linearize concave part

$$\min_{\{x_m^n, y_m^n\}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \underbrace{-\log_2(1 + P^{max} x_m^n \gamma_m^n)}_{\text{convex}} + \lambda q \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \underbrace{(y_m^n(t) + \epsilon)^{q-1} y_m^n}_{\text{convex}} \quad (12)$$

$$s.t. \quad \mathrm{c}_1, \mathrm{c}_2 \text{ and } \mathrm{c}_3$$

# Algorithm 1

**Initialization:** $\lambda = NP^{\max}$, $q \in (0,1)$, $\sigma_1 \in (0,1)$, $\sigma_2 \in (0,1)$, $\delta \in (0,1)$, $\tau > 1$, $w_m^n(1) = 1 \; \forall \; m \in \mathcal{M}$ and $n \in \mathcal{N}$; $\epsilon(1) = 1$.

**while** (1) **do**

  **for** t = 1, 2, . . . , MaxItr **do**

    Solve the following convex sub-problem

$$\min_{\{x_m^n, y_m^n\}} - \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \log_2(1 + P^{max} x_m^n \gamma_m^n) + \lambda q \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} w_m^n(t) y_m^n$$

$$s.t. \quad \tilde{c}_1, \tilde{c}_2 \text{ and } \tilde{c}_3. \tag{13}$$

    **Update:**

$$w_m^n(t+1) = (x_m^n(t) + \epsilon(t))^{q-1}$$

$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(x_m^n(t))\}$$

    **if** $\sum_m \sum_n |x_m^n(t) - x_m^n(t-1)| < \sigma_1$ **then**

      **break** ;

    **end if**

  **end for**

  **if** $f(y_m^n(t)) < \sigma_2$ **then**

    **Stop**.

  **else**

    $\lambda = \tau\lambda$

  **end if**

**end while**

# Single-Cell OFDMA REC Network

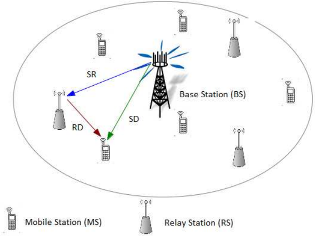
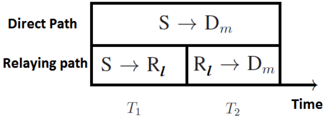



FIGURE – Single-cell multi-user OFDMA REC network

**Notation**

$\mathcal{L} \in \{0, 1, 2, \ldots, L\} \Rightarrow$ DL path indices
$\mathcal{M} \in \{1, 2, \ldots, M\} \Rightarrow$ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\} \Rightarrow$ Subcarrier indices
$\gamma_{\star}^{n} \Rightarrow$ Subcarrier gain
$p_{\star}^{n} \Rightarrow$ Allotted power
$R_{m}^{n} \Rightarrow$ Rate achieved
$\star \in \{SD_m, SR_l, R_l D_m\}$



# Path Selection and Rate Achieved

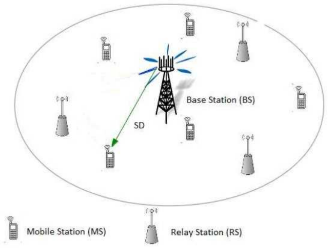



FIGURE – Direct path

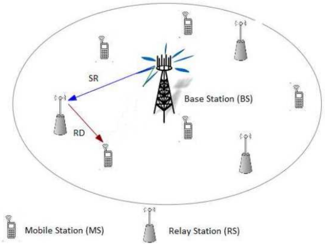



FIGURE – Relay path

## Rate achieved

$$R^n_{0,m} = R^n_{SD_m} = \log_2(1 + \gamma^n_{SD_m} P^n_{SD_m})$$

## Rate achieved

$$R^n_{l,m} = \frac{1}{2} \min\{R^n_{SR_l}, R^n_{R_l D_m}\}$$

## Total achievable rate

$$R_m = \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}} R^n_{l,m}, \quad \forall m \in \mathcal{M}$$

# Problem Formulation

## Utility adaptive (UA) optimization problem

$$\begin{aligned}
&\max_{\{p^n_{SD_m}, p^n_{SR_l}, p^n_{R_lD_m}\}} \quad U(R_1, R_2, \ldots, R_M) \\
s.t. \quad & c_1 : \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \{p^n_{SD_m} + \sum_{l \in \mathcal{L}, l \neq 0} (p^n_{SR_l} + p^n_{R_lD_m})\} \leq P_T, \\
& c_2 : p^n_{SD_m}, p^n_{SR_l}, p^n_{R_lD_m} \geq 0, \quad \forall m, n, \\
& c_3 : \text{OFDMA Constraint} \\
& \qquad \Downarrow \\
& \qquad p^n_{SD_m} p^n_{SR_l} = 0, \quad \forall l, m, n, \\
& \qquad p^n_{SR_l} p^n_{SR_{l}'} = 0, \quad \forall l \neq l', \quad l, l' \in \mathcal{L}, \\
& \qquad p^n_{SD_m} p^n_{R_lD_m} = 0, \quad \forall l, m, n, \\
& \qquad p^n_{R_lD_m} p^n_{R_l' D_m'} = 0, \quad \forall l \neq l', \quad l, l' \in \mathcal{L}.
\end{aligned} \tag{14}$$

- Reformulate the above problem

# Problem Reformulation

- Let us introduce $(L+1)MN$ variables $\{p^n_{l,m}\ \forall l, m, n.\}$

$$p^n_{l,m} = \begin{cases} p^n_{SD_m}, & \text{if } l = 0 \\ (p^n_{SR_l} + p^n_{R_l D_m}), & \text{otherwise.} \end{cases} \tag{15}$$

- Relaying path rate: $R^n_{l,m} = \frac{1}{2}\min\{R^n_{SR_l}, R^n_{R_l D_m}\} \Rightarrow$ is maximized if and only if

$$R^n_{SR_l} = R^n_{R_l D_m}$$
$$\Downarrow$$
$$\gamma^n_{SR_l} p^n_{SR_l} = \gamma^n_{R_l D_m} p^n_{R_l D_m} \tag{16}$$

- Using (15) and (16), we get the achievable rate of user $m$ on $n^{th}$ subcarrier through $l^{th}$ downlink path as [Proof]

$$R^n_{l,m} = \alpha_l \log_2(1 + \beta^n_{l,m} p^n_{l,m}) \tag{17}$$

$$\alpha_l = \begin{cases} 1, & \text{if } l = 0 \\ \frac{1}{2}, & \text{otherwise.} \end{cases} \qquad \beta^n_{l,m} = \begin{cases} \gamma^n_{SD_m}, & \text{if } l = 0 \\ \frac{\gamma^n_{SR_l}\gamma^n_{R_l D_m}}{\gamma^n_{SR_l} + \gamma^n_{R_l D_m}}, & \text{otherwise.} \end{cases}$$

# Reformulated UA optimization problem

## UA problem after reformulation

$$
\begin{aligned}
& \max_{\{p_{l,m}^n\}} && U(R_1, R_2, \ldots, R_M) \\
\text{s.t.} \quad & \mathrm{c}_1: && \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p_{l,m}^n \leq P_T, \\
& \mathrm{c}_2: && 0 \leq p_{l,m}^n \leq P^{\max}, \quad \forall l, m, n, \\
& \mathrm{c}_3: && p_{l,m}^n p_{l',m'}^n = 0, \quad \forall l \neq l', m \neq m'; \\
& && l, l' \in \mathcal{L} \text{ and } n, n' \in \mathcal{N}.
\end{aligned}
\tag{18}
$$

## Introduce binary indicator variable $y_{l,m}^n$

$$\begin{aligned} \max_{\{p_{l,m}^n, y_{l,m}^n\}} \quad & U(R_1, R_2, \ldots, R_M) \\ s.t. \quad \mathrm{c}_1 : \quad & \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p_{l,m}^n \leq P_T, \\ \mathrm{c}_2 : \quad & 0 \leq p_{l,m}^n \leq y_{l,m}^n P^{\max}, \quad \forall l, m, n, \\ \mathrm{c}_3 : \quad & \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} y_{l,m}^n \leq 1, \quad \forall n \in \mathcal{N} \\ \mathrm{c}_4 : \quad & y_{l,m}^n \in \{0, 1\} \end{aligned} \tag{19}$$

## Non-smooth concave utility function

Min-rate utility function $\Rightarrow \min\limits_{1 \leq m \leq M} R_m$

## Reformulated RA problem

$$
\begin{aligned}
&\max_{\{p^n_{l,m}, y^n_{l,m}\}} \; \min_{1 \le m \le M} R_m \\
\text{s.t.} \quad & c_1 : \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} p^n_{l,m} \le P_T, \\
& c_2 : 0 \le p^n_{l,m} \le y^n_{l,m} P^{\max}, \quad \forall l, m, n, \\
& c_3 : \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} y^n_{l,m} \le 1, \quad \forall n \in \mathcal{N} \\
& c_4 : y^n_{l,m} \in \{0, 1\}
\end{aligned} \tag{20}
$$

## Problems to take care of

- Combinatorial nature of the problem
- Non-smoothness in the objective function

# Transformation of Non-smooth Function to Smooth Function

## Introduce a new variable $\phi$

$$
\begin{aligned}
&\max_{\{p_{l,m}^{n}, y_{l,m}^{n}, \phi\}} \quad \phi \\
s.t. \quad & c_1: \sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{l,m}^{n} \le P_T, \\
& c_2: 0 \le p_{l,m}^{n} \le y_{l,m}^{n} P^{\max}, \quad \forall l, m, n, \\
& c_3: \sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}} y_{l,m}^{n} \le 1, \quad \forall n \in \mathcal{N} \\
& c_4: y_{l,m}^{n} \in \{0, 1\}, \quad \forall l, m, n, \\
& c_5: R_m \ge \phi, \quad \forall m.
\end{aligned}
\tag{21}
$$

## Problems solved

- Non-smooth $\Rightarrow$ Smooth
- Combinatorial problem $\Rightarrow$ Same method as used in previous section

## Steps

- Relaxation
- Convex + Concave part ⇒ Solve it using MM approach

## Relaxed optimization problem

$$\min_{\{x_{l,m}^n, y_{l,m}^n, \phi\}} \quad -\phi + \lambda \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} (y_{l,m}^n + \epsilon)^q \qquad (22)$$
$$s.t. \quad c_1, c_2, c_3 \text{ and } c_5.$$

## Keep convex part as it is and linearize concave part

$$\min_{\{x_{l,m}^n, y_{l,m}^n, \phi\}} \quad -\phi + \lambda q \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} (y_{l,m}^n(t) + \epsilon)^{q-1} y_{l,m}^n \qquad (23)$$
$$s.t. \quad c_1, c_2, c_3 \text{ and } c_5.$$

# Algorithm 2

**Initialization:** $\lambda = NP^{\max}, q, \sigma_1, \sigma_2, \delta \in (0,1), \tau > 1, w^n_{l,m}(1) = 1, \epsilon(1) = 1.$

**while** (1) **do**

  **for** t = 1, 2, . . . , MaxItr **do**

    Solve the following convex sub-problem

$$\min_{\{x^n_{l,m}, y^n_{l,m}, \phi\}} -\phi + \lambda q \sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} w^n_{l,m}(t) y^n_{l,m}$$

$$s.t. \quad \tilde{c}_1, \tilde{c}_2, \tilde{c}_3 \text{ and } \tilde{c}_5. \tag{24}$$

    **Update:**

$$w^n_{l,m}(t+1) = (x^n_{l,m}(t) + \epsilon(t))^{q-1}$$

$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(x^n_{l,m}(t))\}$$

    **if** $\sum_l \sum_m \sum_n |x^n_{l,m}(t) - x^n_{l,m}(t-1)| < \sigma_1$ **then**

      **break** ;

    **end if**

  **end for**

  **if** $f(y^n_{l,m}(t)) < \sigma_2$ **then**

    **Stop**.

  **else**

    $\lambda = \tau\lambda$

  **end if**

**end while**

# Simulation Result

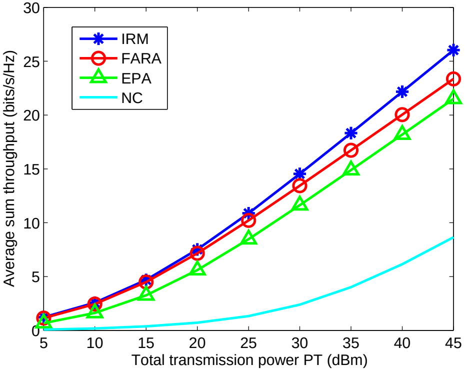



FIGURE – Comparison of the average sum throughput of different algorithms with the max-min resource allocation as the utility function versus the total transmission power. The system parameters are $L = 3$, $M = 8$ and $N = 16$. FARA, EPA. [1]

1. "Fairness Aware Resource Allocation in OFDMA Cooperative Relaying Network", Proc. Int. Conf. Commun.

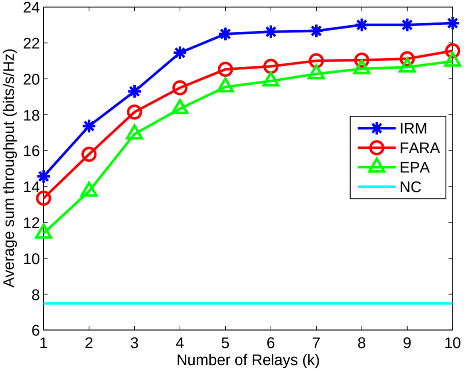



FIGURE – Illustration of the throughput improvement obtainable by using cooperative relaying. The system parameters: $M = 4$, $N = 8$ and $\mathrm{P}_T = 10$ dB.

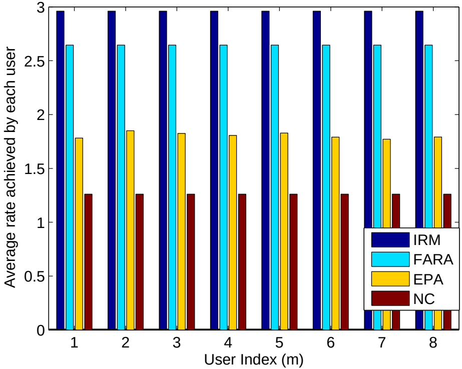



FIGURE – Average rate of each user, with $L = 3$, $M = 8$, $N = 16$ and $P_T = 10$ dB. All the schemes compared are fair resource allocation schemes.

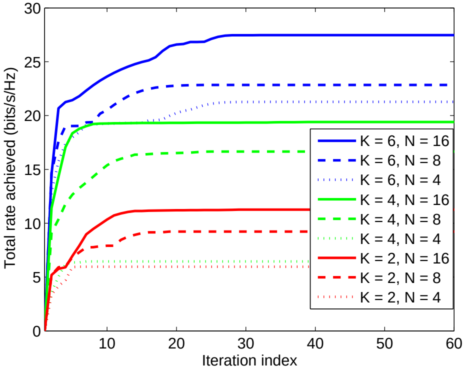



FIGURE – Convergence of IRM algorithm, for different values of $K$ and $N$ with $M = 4$, $P_T = 0$ dB.

TABLE – Comparison between IRM Method and Exhaustive Search: Optimality and time gap with $M = 2$ and $P_T = 0$ dB over 50 channel realizations.

| *K*=1 | | | *K*=2 | | | *K*=3 | | |
|---|---|---|---|---|---|---|---|---|
| $N$ | $Opt_{\text{gap}}$ | $t_{\text{gap}}$ | $N$ | $Opt_{\text{gap}}$ | $t_{\text{gap}}$ | $N$ | $Opt_{\text{gap}}$ | $t_{\text{gap}}$ |
| 2 | 1.207% | 28.53% | 2 | 1.450% | 24.08% | 2 | 0.600% | 18.51% |
| 3 | 3.602% | 8.075% | 3 | 4.290% | 4.159% | 3 | 6.200% | 2.719% |
| 4 | 9.910% | 2.423% | 4 | 7.330% | 1.113% | 4 | 10.66% | 0.067% |

- Optimality gap: $1 - \frac{f_{\text{IRM}}}{f_{\text{Gloabal}}}$
- Time gap: $\frac{t_{\text{IRM}}}{t_{\text{Gloabl}}}$

**Further reduction in optimality gap ?**

- Possible with proper initialization and adaptive update of penalty parameter.

# Multi-Cell OFDMA Network

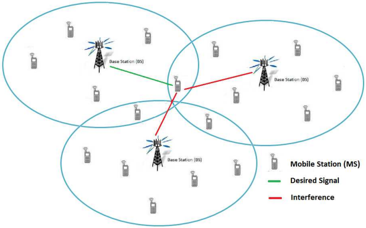



FIGURE – Multi-cell multi-user OFDMA network

## Notation

$\mathcal{K} \in \{1, 2, \ldots, K\}$ ⇒ BS indices
$\mathcal{M} \in \{1, 2, \ldots, M\}$ ⇒ User indices
$\mathcal{N} \in \{1, 2, \ldots, N\}$ ⇒ Subcarrier indices
$\gamma^n_{k,m}, \Gamma^n_{i,m_k}$ ⇒ Subcarrier gain
$p^n_{k,m}$ ⇒ Allotted power
$y^n_{k,m}$ ⇒ Binary indicator variable
$R^n_{k,m}$ ⇒ Rate achieved

## Utility adaptive (UA) optimization problem

$$
\begin{aligned}
\max_{\{p_{k,m}^n, y_{k,m}^n\}} \quad & \sum_{k\in\mathcal{K}} \min\{R_{k,1}, R_{k,2}, \ldots, R_{k,M}\} \\
s.t. \quad \mathrm{c}_1: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{k,m}^n \le P_T, \quad \forall k \in \mathcal{K} \\
\mathrm{c}_2: \quad & 0 \le p_{k,m}^n \le y_{k,m}^n P^{\max}, \quad \forall k, m, n, \\
\mathrm{c}_3: \quad & \sum_{m\in\mathcal{M}} y_{k,m}^n \le 1, \quad \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \\
\mathrm{c}_4: \quad & y_{k,m}^n \in \{0,1\}, \quad \forall k, m, n.
\end{aligned}
\tag{25}
$$

## Epigraph form

$$
\begin{aligned}
\max_{\{p_{k,m}^n, y_{k,m}^n, \phi_k\}} \quad & \sum_{k\in\mathcal{K}} \phi_k \\
s.t. \quad & \mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3, \mathrm{c}_4, \\
& \mathrm{c}_5: \quad R_{k,m} \ge \phi_k, \quad \forall k, m.
\end{aligned}
\tag{26}
$$

## Epigraph form

$$\begin{aligned} \max_{\{p^n_{k,m}, y^n_{k,m}, \phi_k\}} \quad & \sum_{k \in \mathcal{K}} \phi_k \\ s.t. \quad & \mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3, \mathrm{c}_4, \\ & \mathrm{c}_5 : \quad \sum_{n \in \mathcal{N}} \log_2(1 + SINR^n_{k,m}) \geq \phi_k, \quad \forall k, m. \end{aligned} \tag{27}$$

## Signal-to-interference-noise ratio (SINR)

$$SINR^n_{k,m} = \frac{\gamma^n_{k,m} p^n_{k,m}}{\sum_{i \in \mathcal{K}, i \neq k} \sum_{j \in M_i} \Gamma^n_{i,m_k} p^n_{i,j} + N_0}$$

# Solution

## Relaxed optimization problem

$$\min_{\{x^n_{k,m}, y^n_{k,m}, \phi_k\}} \quad -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \underbrace{(y^n_{k,m}+\epsilon)^q}_{\text{h(y):concave}}$$

$$\begin{aligned} s.t. \quad & \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \\ & \tilde{c}_5 : \quad \underbrace{\sum_{n\in\mathcal{N}} \log_2(1+\tilde{SINR}^n_{k,m}) \geq \phi_k}_{\text{non-convex constraint}}, \quad \forall k, m. \end{aligned} \tag{28}$$

- Non-convex constraint ⇒ Decompose it into convex and concave terms

$$\tilde{c}_5 : \quad \sum_{n\in\mathcal{N}} f(x) - g(x) \geq \phi_k$$

where,

$$f(x) = \log_2(N_0 + P^{max}\gamma^n_{k,m}x^n_{k,m} + P^{max} \sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \Gamma^n_{i,m_k} x^n_{i,j})$$

$$g(x) = \log_2(N_0 + P^{max} \sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \Gamma^n_{i,m_k} x^n_{i,j})$$

# Solution Contd.

## $1^{st}$ term $\rightarrow$ f(x)

- f(x) $\Rightarrow$ convex, keep as it is.

## $2^{nd}$ term $\rightarrow$ g(x)

- g(x) $\Rightarrow$ concave, linearize it.

$$g(x^n_{k,m}(t)) + \mathcal{D}g(x^n_{k,m}(t))(x^n_{k,m} - x^n_{k,m}(t)) \quad (29)$$

where $\mathcal{D}g(x^n_{k,m})$ is the derivative matrix of $g(x)$ at $x^n_{k,m}(t)$ and is given by

$$\mathcal{D}g(x^n_{k,m}) = w^n_{1,k,m} = \frac{1}{\sum_{i\in\mathcal{K},i\neq k}\sum_{j\in M_i}\Gamma^n_{i,m_k}P^{max}x^n_{i,j} + N_0}e^n_{i,m}$$

$$\text{Where, } e^n_{i,m} = \begin{cases} 0, & \text{if } i = k, \\ \frac{P^{max}\Gamma^n_{i,m}}{ln2}, & \text{if } i \neq k. \end{cases}$$

## $3^{rd}$ term $\rightarrow$ h(y)

- h(y) $\Rightarrow$ concave, linearize it.

$$h(y^n_{k,m}(t)) + \mathcal{D}h(y^n_{k,m}(t))(y^n_{k,m} - y^n_{k,m}(t)) \tag{30}$$

where $\mathcal{D}h(y^n_{k,m})$ is the derivative matrix of $h(y)$ at $y^n_{k,m}(t)$ and is given by

$$\mathcal{D}h(y^n_{k,m}) = w^n_{2,k,m} = q(y^n_{k,m})^{q-1}$$

## Convex sub-problem

$$\begin{aligned}
\min_{\{x^n_{k,m}, y^n_{k,m}, \phi_k\}} \quad & -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} w^n_{2,k,m}(t) y^n_{k,m} \\
s.t. \quad & \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \\
& \tilde{c}_5 : \quad \sum_{n\in\mathcal{N}} f(x) - w^n_{1,k,m}(t) x^n_{k,m} \geq \phi_k, \quad \forall k, m.
\end{aligned} \tag{31}$$

# Algorithm 3

**Initialization**

**for** t = 1, 2, . . . , MaxItr **do**

Solve the following convex sub-problem

$$
\begin{aligned}
\min_{\{x^n_{k,m}, y^n_{k,m}, \phi_k\}} \quad & -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} w^n_{2,k,m}(t) y^n_{k,m} \\
s.t. \quad & \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \qquad\qquad (32) \\
& \tilde{c}_5 : \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} f(x) - w^n_{1,k,m}(t) x^n_{k,m} \geq \phi_k \quad \forall k, m.
\end{aligned}
$$

**Update:**

$$
\begin{aligned}
w^n_{1,k,m}(t) &= \mathcal{D}g(x^n_{k,m}(t)) \\
w^n_{2,k,m}(t) &= \mathcal{D}h(y^n_{k,m}(t)) \\
\epsilon(t+1) &= \min\{\epsilon(t), \delta f(y^n_{k,m}(t))\}
\end{aligned}
$$

**if** Stopping criterion satisfies **then**

**Stop** ;

**else**

Repeat

**end if**

**end for**

Algorithm 3 – Pseudo-code for solving multi-cell without relay UA problem

# Multi-Cell OFDMA REC network

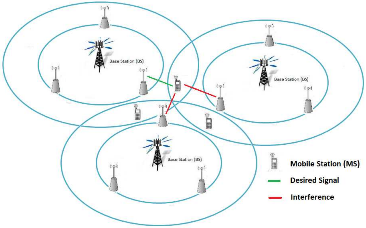
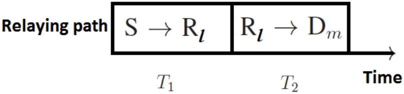



FIGURE – Multi-cell multi-user OFDMA REC network

## Notation

$\mathcal{K} \in \{1,2,\ldots,K\}$ ⇒ BS indices
$\mathcal{L}_k \in \{1,2,\ldots,L\}$ ⇒ RS indices
$\mathcal{M}_k \in \{1,2,\ldots,M\}$ ⇒ User indices
$\mathcal{N} \in \{1,2,\ldots,N\}$ ⇒ Subcarrier indices
$\gamma^n_{k,l,m}$ ⇒ Subcarrier gain
$p^n_{k,l,m}$ ⇒ Allotted power
$y^n_{k,l,m}$ ⇒ Binary indicator variable
$R^n_{k,l,m}$ ⇒ Rate achieved

## Problem Formulation




### Assumptions

- We assume, all BS-RS work in sync at frame level
- We focus on cell-edge users
- Rate achieved by each cell-edge user

$$R^n_{k,l,m} = \frac{1}{2}\min\{R^n_{k,l_k}, R^n_{l_k,m_k}\} \tag{33}$$

- We assume the RS-MS link have dominated influence on the transmission rate

$$R^n_{k,l,m} = \frac{1}{2} R^n_{l_k,m_k} \tag{34}$$

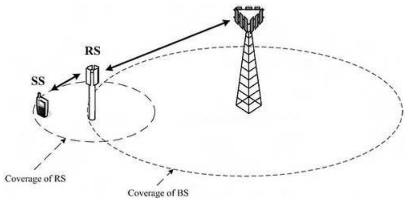
# Reason behind Strong Assumption

**BS-RS link**

- Strong LoS propogation channel
- Interference is negligible

**RS-MS link**

- NLoS communication link
- Interference is dominant




**Total transmission rate**

$$\text{Thus,} \quad R^n_{k,l,m} = \frac{1}{2} R^n_{l_k,m_k} \tag{35}$$

$$\Downarrow$$

Dependent only on $p^n_{l_k,m_k}$ and $y^n_{l_k,m_k}$, Greatly reduces problem complexity !

## Utility adaptive (UA) optimization problem

$$
\begin{aligned}
\max_{\{p^n_{l_k,m_k}, y^n_{l_k,m_k}\}} \quad & \sum_{k \in \mathcal{K}} \min\{R_{l_k,1_k}, R_{l_k,2_k}, \ldots, R_{l_k,M_k}\} \\
s.t. \quad \mathrm{c}_1: \quad & \sum_{l_k \in \mathcal{L}_k} \sum_{m_k \in \mathcal{M}_k} \sum_{n \in \mathcal{N}} p^n_{l_k,m_k} \leq P_{T_{RS_{l_k}}}, \quad \forall l_k \in \mathcal{L}_k \text{ and } k \in \mathcal{K}, \\
\mathrm{c}_2: \quad & 0 \leq p^n_{l_k,m_k} \leq y^n_{l_k,m_k} P^{\max}, \quad \forall k, l_k, m_k, n, \\
\mathrm{c}_3: \quad & \sum_{l_k \in \mathcal{L}_k} \sum_{m_k \in \mathcal{M}_k} y^n_{l_k,m_k} \leq 1, \quad \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \\
\mathrm{c}_4: \quad & y^n_{l_k,m_k} \in \{0,1\}, \quad \forall k, l_k, m_k, n.
\end{aligned} \tag{36}
$$

## Signal-to-interference-noise ratio (SINR)

$$
SINR^n_{l_k,m_k} = \frac{\gamma^n_{l_k,m_k} p^n_{l_k,m_k}}{\sum_{l_i \in \mathcal{L}_i, i \in \mathcal{K}, i \neq k} \sum_{j \in M_i} \sum_{l_i \in \mathcal{L}_i} \Gamma^n_{l_i,m_k} p^n_{l_i,j} + N_0}
$$

# Solution

## Relaxed optimization problem

$$\min_{\{x^n_{l_k,m_k}, y^n_{l_k,m_k}, \phi_k\}} \quad -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{l_k\in\mathcal{L}_k} \sum_{m_k\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} \underbrace{(y^n_{l_k,m_k} + \epsilon)^q}_{\text{h(y): concave}}$$

$$\begin{aligned} s.t. \quad & \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \\ & \tilde{c}_5 : \underbrace{\sum_{l_k\in\mathcal{L}_k} \sum_{n\in\mathcal{N}} \log_2(1 + S\tilde{I}NR^n_{l_k,m_k}) \geq \phi_k,}_{\text{non-convex constraint}} \quad \forall k, m. \end{aligned} \tag{37}$$

- Non-convex constraint ⇒ Decompose it into convex and concave terms

$$\tilde{c}_5 : \sum_{l_k\in\mathcal{L}_k} \sum_{n\in\mathcal{N}} f(x) - g(x) \geq \phi_k$$

where, $f(x) = \log_2(N_0 + P^{max}\gamma^n_{l_k,m_k} x^n_{l_k,m_k} + P^{max} \sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \sum_{l_i\in\mathcal{L}_i} \Gamma^n_{l_i,m_k} x^n_{l_i,j})$

$$g(x) = \log_2(N_0 + P^{max} \sum_{i\in\mathcal{K}, i\neq k} \sum_{j\in M_i} \sum_{l_i\in\mathcal{L}_i} \Gamma^n_{l_i,m_k} x^n_{l_i,j})$$

# Solution Contd.

## $1^{st}$ term → f(x)

- f(x) ⇒ convex, keep as it is.

## $2^{nd}$ term → g(x)

- g(x) ⇒ concave, linearize it.

$$g(x^n_{l_k,m_k}(t)) + \mathcal{D}g(x^n_{l_k,m_k}(t))(x^n_{l_k,m_k} - x^n_{l_k,m_k}(t)) \tag{38}$$

where $\mathcal{D}g(x^n_{l_k,m_k})$ is the derivative matrix of $g(x)$ at $x^n_{l_k,m_k}(t)$ and is given by

$$\mathcal{D}g(x^n_{l_k,m_k}) = w^n_{1,l_k,m_k} = \frac{1}{\sum_{i\in\mathcal{K},i\neq k}\sum_{j\in M_i}\sum_{l_i\in\mathcal{L}_i}\Gamma^n_{l_i,m_k}P^{max}x^n_{i,j} + N_0}e^n_{i,m_k}$$

$$\text{Where, } e^n_{i,m} = \begin{cases} 0, & \text{if } i = k, \\ \frac{P^{max}\Gamma^n_{li,m_k}}{ln2}, & \text{if } i \neq k. \end{cases}$$

## $3^{rd}$ term → h(y)

- h(y) ⇒ concave, linearize it.

$$h(y^n_{l_k,m_k}(t)) + \mathcal{D}h(y^n_{l_k,m_k}(t))(y^n_{l_k,m_k} - y^n_{l_k,m_k}(t)) \quad (39)$$

where $\mathcal{D}h(y^n_{l_k,m_k})$ is the derivative matrix of $h(y)$ at $y^n_{l_k,m_k}(t)$ and is given by

$$\mathcal{D}h(y^n_{l_k,m_k}) = w^n_{2,l_k,m_k} = q(y^n_{l_k,m_k})^{q-1}$$

## Convex sub-problem

$$\begin{aligned} \min_{\{x^n_{k,m}, y^n_{k,m}, \phi_k\}} \quad & -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{l_k\in\mathcal{L}_k} \sum_{m_k\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} w^n_{2,l_k,m_k}(t) y^n_{l_k,m_k} \\ s.t. \quad & \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \\ & \tilde{c}_5 : \sum_{l_k\in\mathcal{L}_k} \sum_{n\in\mathcal{N}} f(x) - w^n_{1,l_k,m_k}(t) x^n_{l_k,m_k} \geq \phi_k, \quad \forall k, m_k. \end{aligned} \quad (40)$$

# Algorithm 4

**Initialization**
**for** t = 1, 2, . . . , MaxItr **do**
Solve the following convex sub-problem

$$\min_{\{x^n_{k,m},y^n_{k,m},\phi_k\}} \quad -\sum_{k\in\mathcal{K}} \phi_k + \lambda \sum_{k\in\mathcal{K}} \sum_{l_k\in\mathcal{L}_k} \sum_{m_k\in\mathcal{M}_k} \sum_{n\in\mathcal{N}} w^n_{2,l_k,m_k}(t) y^n_{l_k,m_k}$$
$$s.t. \quad \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \qquad (41)$$
$$\tilde{c}_5 : \sum_{l_k\in\mathcal{L}_k} \sum_{n\in\mathcal{N}} f(x) - w^n_{1,l_k,m_k}(t) x^n_{l_k,m_k} \geq \phi_k \quad \forall k, m_k.$$

**Update:**
$$w^n_{1,l_k,m_k}(t) = \mathcal{D}g(x^n_{l_k,m_k}(t))$$
$$w^n_{2,l_k,m_k}(t) = \mathcal{D}h(y^n_{l_k,m_k}(t))$$
$$\epsilon(t+1) = \min\{\epsilon(t), \delta f(y^n_{l_k,m_k}(t))\}$$
**if** Stopping criterion satisfies **then**
**Stop** ;
**else**
Repeat
**end if**
**end for**

Algorithm 4 – Pseudo-code for solving UA problem multi-cell with relay

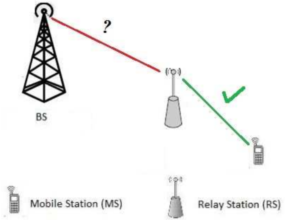
# Resource Allocation/Path Selection during $T_1$ and $T_2$

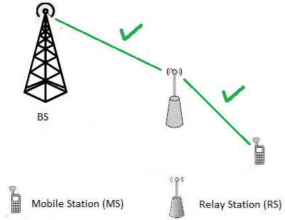
**During $T_2$**

- Near-optimal path/resource allocation for RS-MS links

**During $T_1$**

- What to do during $T_1$ ?




**During $T_1$**

- Same subcarrier allocation strategy
- Solve for the power allocation
  - Solve for $p^n_{k,l_k}$ s.t. $R^n_{k,l_k} = R^n_{l_k,m_k}$


WORK IN PROGRESS

# Summary of our work

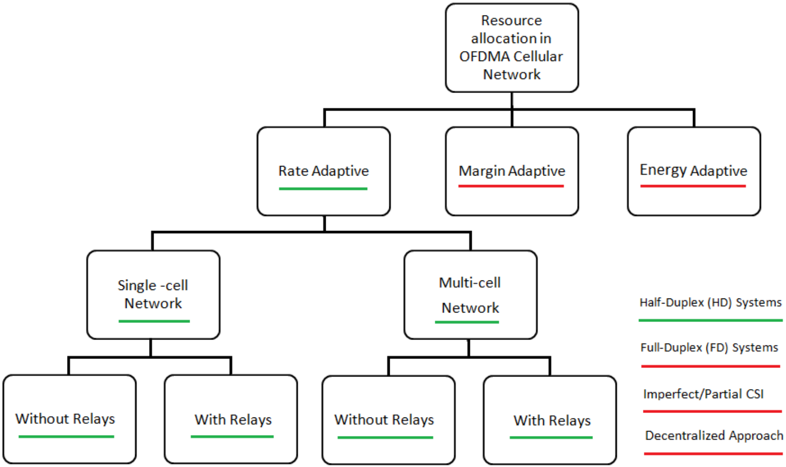
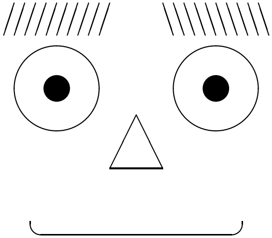



FIGURE – Summary of our work

**Thank You :)**



## Problem Reformulation

- The rate achieved by a MS $m$ on subcarrier $n$ through RS $l$ is the minimum of the first-hop and second-hop link capacities, given by

$$R^n_{l,m} = \frac{1}{2}\min\{R^n_{SR_l}, R^n_{R_lD_m}\}$$

where,

$$R^n_{SR_l} = \log_2(1 + p^n_{SR_l}\gamma^n_{SR_l}),$$
$$R^n_{R_lD_m} = \log_2(1 + p^n_{R_lD_m}\gamma^n_{R_lD_m}).$$

- $R^n_{l,m}$ is maximized if and only if the rate achieved on the first-hop and second-hop link are the same, i.e.,

$$R^n_{SR_l} = R^n_{R_lD_m}$$
$$\Downarrow$$
$$p^n_{SR_l}\gamma^n_{SR_l} = p^n_{R_lD_m}\gamma^n_{R_lD_m}$$
$$\frac{p^n_{SR_l}}{p^n_{R_lD_m}} = \frac{\gamma^n_{R_lD_m}}{\gamma^n_{SR_l}}$$

Add 1 to both L.H.S and R.H.S,

$$\frac{p^n_{SR_l}}{p^n_{R_lD_m}} + 1 = \frac{\gamma^n_{R_lD_m}}{\gamma^n_{SR_l}} + 1$$

# Problem Reformulation Contd.,

$$\frac{p^n_{SR_l} + p^n_{R_lD_m}}{p^n_{R_lD_m}} = \frac{\gamma^n_{R_lD_m} + \gamma^n_{SR_l}}{\gamma^n_{SR_l}}$$

$$p^n_{R_lD_m} = \frac{(p^n_{SR_l} + p^n_{R_lD_m})\gamma_{SR_l}}{\gamma^n_{R_lD_m} + \gamma^n_{SR_l}}$$

- Let us introduce $(L+1)MN$ new variables $\{p^n_{l,m} \forall\, l, m, n\}$, to indicate the power allocated to user $m$ on subcarrier $n$ through path $l$ over a slot duration. We have

$$p^n_{l,m} = \begin{cases} p^n_{SD_m}, & \text{if } l = 0 \\ p^n_{SR_l} + p^n_{R_lD_m}, & \text{otherwise.} \end{cases}$$

- Using the above reformulation, $p^n_{R_l,D_m}$ can be re-written as

$$p^n_{R_lD_m} = \frac{(p^n_{l,m})\gamma_{SR_l}}{\gamma^n_{R_lD_m} + \gamma^n_{SR_l}}, \quad \forall l \neq 0. \tag{42}$$

# Problem Formulation Contd.,

- The rate $R^n_{l,m}$ $(l \neq 0)$ is given by

$$R^n_{l,m} = \frac{1}{2} R^n_{SR_l} = \frac{1}{2} R^n_{R_l D_m} = \frac{1}{2} \log_2(1 + \gamma^n_{R_l D_m} p^n_{R_l D_m})$$

$$R^n_{l,m} = \frac{1}{2} \log_2(1 + \gamma^n_{R_l D_m} p^n_{R_l D_m}) \tag{43}$$

Replacing (42) in the above equation (43), we get

$$R^n_{l,m} = \underbrace{\frac{1}{2}}_{\alpha_l} \log_2(1 + \underbrace{\frac{\gamma^n_{R_l D_m} \gamma^n_{SR_l}}{\gamma^n_{R_l D_m} + \gamma^n_{SR_l}}}_{\beta^n_{l,m}} p^n_{l,m}), \quad \forall l \neq 0, m, n. \tag{44}$$

For $l = 0$, $R^n_{l,m}$ is given by

$$R^n_{l,m} = \underbrace{1}_{\alpha_l} \log_2(1 + \underbrace{\gamma^n_{SD_m}}_{\beta^n_{l,m}} p^n_{l,m}), \quad l = 0, m, n. \tag{45}$$

Thus,

$$\alpha_l = \begin{cases} 1, & \text{if } l = 0 \\ \frac{1}{2}, & \text{otherwise} \end{cases} \qquad \beta^n_{l,m} = \begin{cases} \gamma^n_{SD_m}, & \text{if } l = 0 \\ \frac{\gamma^n_{SR_l} \gamma_{R_l D_m}}{\gamma^n_{SR_l} + \gamma^n_{R_l D_m}}, & \text{otherwise.} \checkmark \end{cases}$$