On the Fundamental Limits of Space Shift Keying

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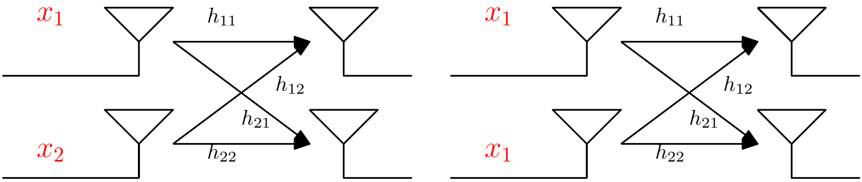
Conventional MIMO

- Multiple RF chains
  - More hardware and cost
- Need to mitigate inter channel interference (ICI)
- Antenna synchronization is required
- Ranges from high spectral efficiency to high diversity orders

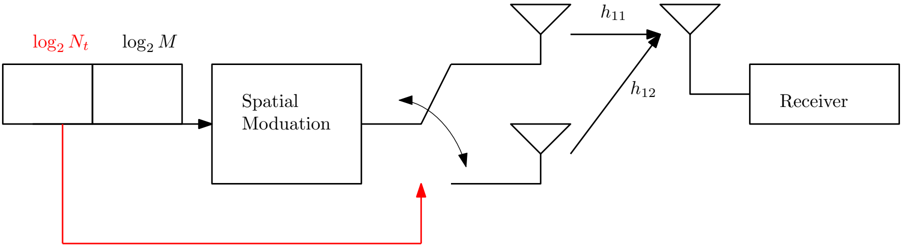


What is Spatial Modulation?

- A new modulation technique
- Open loop
- Single RF chain
- Antenna indices also conveys information, besides the base constellation at the transmitter

Advantages:

- No ICI
- No antenna synchronization is required
- Low cost and hardware



Spatial Modulation

Literature Survey:

Most of the recent work is focused on

- optimal receivers
- bit error probability
- power allocation
  also, it assumes, Gaussian alphabet, which is not used in practice
- precoding design with finite alphabets, but it doesn't consider the transmit power constraint

J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: optimal detection and performance analysis", *IEEE Communication Letters, vol. 12, no. 8, pp. 545547, Aug. 2008*

On the mutual information and precoding for spatial modulation with finite alphabet, *IEEE Wireless Comm. Lett., vol. 2, no. 4, pp. 383386, Aug. 2013.*

Spatial Modulation

Our Contributions:

- Maximizing the lower bound of mutual information for spatial modulation with finite alphabets[1] considering the transmit power constraint
- Minimizing the bit error probability
- Comprehensive comparison between SM and TAS

Assumption: Channel state information (CSI) is available at the transmitter

[1] Signals drawn from constellation, which are discrete and uniformly distributed

Spatial Modulation

System model

$$y = hx_a x_d + z \quad (1)$$

- a and d denotes the antenna index, and data stream radiated from the transmitter
- h is channel of size $1 \times N_t$ whose entities are i.i.d Rayeligh distribution $\mathcal{CN}(0, \sigma^2)$

Mutual Information

Mutual information for SM:

$$I\left(x_a, x_d; y\right) = H(y) - H\left(y|x_a, x_d\right) \tag{2}$$

For the system model described in (1),

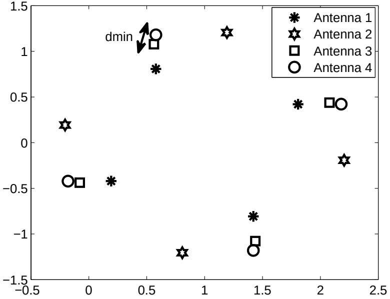
- $H(y|x_a, x_d) = H(z)$
- So, only $H(y)$ in (2) needs to be maximized

$$H(y) = \log_2\left(N_t M\right) - \frac{1}{N_t M}\sum_{k_1=1}^{N_t}\sum_{i_1=1}^{M}\mathbb{E}_z \left[\log_2\left(\sum_{k_2=1}^{N_t}\sum_{i_2=1}^{M}\frac{1}{\pi\sigma^2}\exp\left(-\frac{\|hx_a(k_1)x_d(i_1) - hx_a(k_2)x_d(i_2) + z\|^2}{\sigma^2}\right)\right)\right] \tag{3}$$

Applying Jenson's inequality:

Lower bound

$$H(y) \geq \log_2 (N_t M) - \frac{1}{N_t M} \sum_{k_1=1}^{N_t} \sum_{i_1=1}^{M} \log_2 \left[\sum_{k_2=1}^{N_t} \sum_{i_2=1}^{M} \frac{1}{\pi \sigma^2} \exp \left(- \frac{\| h x_a(k_1) x_d(i_1) - h x_a(k_2) x_d(i_2) \|^2}{2\sigma^2} \right) \right] \quad (4)$$

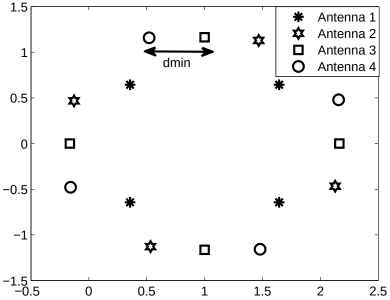


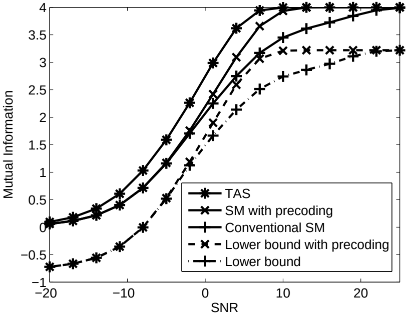
Precoder Design

We maximize the **minimum distance (d_min)** as follows:

- Channel phase compensation
- Constellation rotation

Remark: Excluding any of the above steps i.e., (employing either only channel phase compensation or constellation rotation) will aggravate the performance





Transmit Antenna Selection

- Closed loop
- Single RF
- Low cost and complexity
- No ICI
- No antenna synchronization

Past/Recent Work

- Antenna selection with Alamouti shceme
- Secure transmission using TAS
- Antenna selection using imperfect CSIT

Shihao Yan, Nan Yang, Robert Malaney, and Jinhong Yuan, "Transmit Antenna Selection with Alamouti Scheme in MIMO Wiretap Channels", *available at arXiv:1303.5157v1*

Transmit Antenna Selection

System Model:

$$y_{\text{tas}} = h_{\max} x_{\text{tas}} + z \quad (5)$$

Mutual Information

$$I(x_{\text{tas}}; y_{\text{tas}}) = \log_2(Q) - \frac{1}{Q} \sum_{k_1=1}^{Q}$$
$$\mathbb{E}_z \left[\log_2 \left(\sum_{k_2=1}^{Q} \exp \left(- \frac{\| h_{\max}(x_{\text{tas}}(k_1) - x_{\text{tas}}(k_2)) + z \|^2 - \|z\|^2}{\sigma^2} \right) \right) \right] \quad (6)$$

Comparison of SM with TAS

We compare spatial modulation and transmit antenna selection in terms of following metrics:

- Mutual Information (as discussed)
- Symbol Error Rate

Symbol detection

ML is the optimal receiver, and is given by

$$\hat{x} = \arg\min_{a,d} \left\| y - h x_a x_d \right\|^2 \tag{7}$$

- Outage Probability

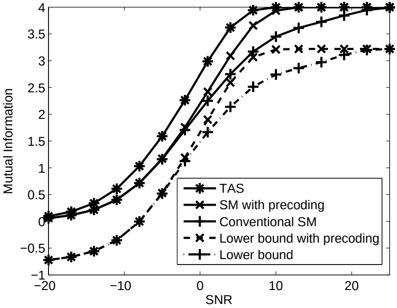
Outage Probability

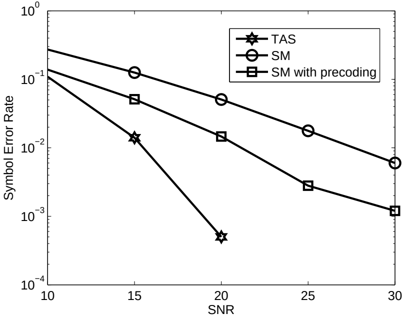
Outage is reported if rate r is less than r_t:

$$P_{\text{out}} = \mathbf{P}(r < r_t \mid \mathbf{C}) \tag{8}$$

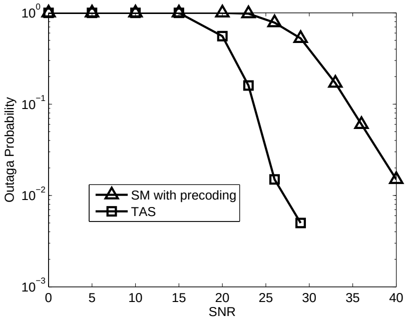
We also investigated the behavior of mutual information with the increase in transmit antennas.

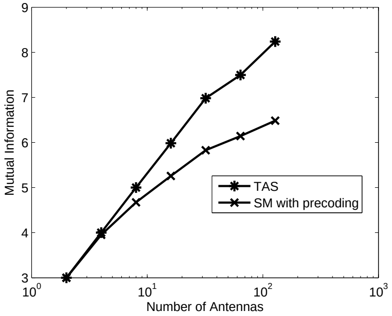
Simulation Results



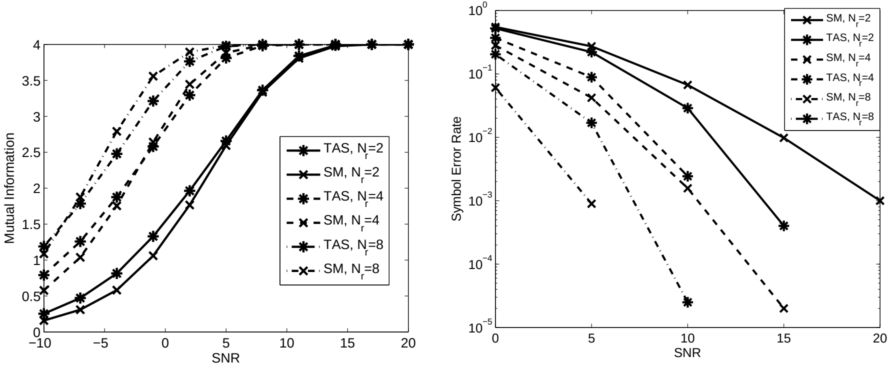


Simulation Results





With more than one receive antenna



This is something interesting !

Space Shift Keying

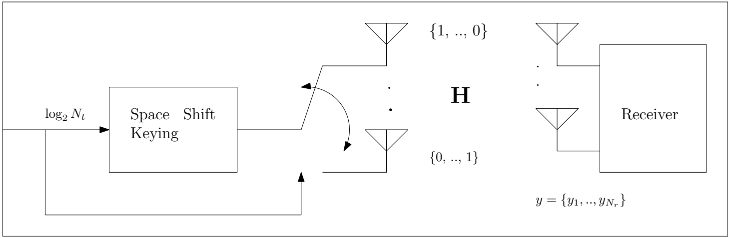


Figure: SSK with N_r receive antennas

- It is like transmitting a Gaussian code word of length N_r
- Now, can fundamental limit for a finite length N_r be computed exactly?

Space Shift Keying

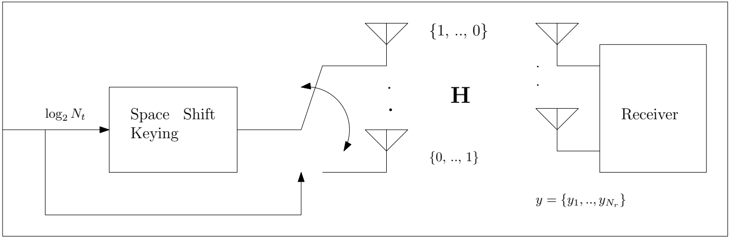


Figure: SSK with N_r receive antennas

- It is like transmitting a Gaussian code word of length N_r
- Now, can fundamental limit for a finite length N_r be computed exactly?

Shannon's result states that

$$\lim_{\epsilon \to 0} \lim_{n \to \infty} \frac{1}{n} \log_2 M^*(n, \epsilon) = C, \tag{9}$$

To achieve a given fraction of capacity with a given error probability, an excellent approximation is

$$M^*(n, \epsilon, P) = nC - \sqrt{nV} Q^{-1}(\epsilon) + O(\log n), \tag{10}$$

V. Strassen, Asymptotische Abschatzungen in Shannons Informationstheorie, *Trans. Third Prague Conf. Information Theory*,Czechoslovak Academy of Sciences, Prague, pp. 689-723

Yury Polyanskiy, H. Vincent Poor, and Sergio Verd, *Channel coding: non-asymptotic fundamental limits*

General Error Bounds

Consider an abstract channel defined by a triple: measurable spaces of inputs **A** and outputs **B** and a conditional probability measure $P_{Y/X} : A \rightarrow B$

- $\{c_1, c_2, ..., c_n\} \subset \mathbf{A}$
- A decoder is a random transformation $P_{Z/Y} : \mathbf{B} \rightarrow \{0, 1, ..., M\}$

$$\epsilon = \max_{m \in \{1,...,M\}} \left[1 - P_{Z/X}(m/c_m)\right] \tag{11}$$

$$i = \log \frac{dP_{Y/X}(Y/X)}{dP_Y(Y)} \tag{12}$$

Binary Hypothesis Testing

Consider a random variable W which takes probability measures P or Q

- A randomized test; $P_{Z/W} : W \to \{0, 1\}$, 0 indicates that the test chooses Q

Best performance achievable among those randomized tests is given by

$$\beta_\alpha(P, Q) = min \sum_{w \in W} Q(w) P_{Z/W}(1/w), \tag{13}$$

where minimum is over all probability distributions

$$P_{Z/W} : \sum_{w \in W} P(w) P_{Z/W}(1/w) \geq \alpha \tag{14}$$

- It gives the minimum under hypothesis Q if the probability of error under hypothesis P is not larger than $1 - \alpha$

Contd.

For any $\gamma > 0$,

$$\alpha \leq \mathbb{P}\left[\frac{dP}{dQ} \geq \gamma\right] + \gamma \beta_\alpha(P, Q) \tag{15}$$

$$\beta_\alpha(P, Q) \leq \frac{1}{\gamma_0} \tag{16}$$

for any γ_0 that satisfies,

$$\mathbb{P}\left[\frac{dP}{dQ} \geq \gamma_0\right] \geq \alpha \tag{17}$$

Each per-codeword cost constraint is defined by subset $\mathbf{F} \subset \mathbf{A}$

A related measure of performance for the composite hypothesis test between Q_Y and the collection $\{P_{Y/X=x}\}_{x\in\mathbf{F}}$:

$$\kappa_\tau(\mathbf{F}, Q_Y) = \inf \sum_{y\in\mathbf{B}} Q_Y(y) P_{Z/Y}(1/y) \tag{18}$$

$$P_{Z/Y:} \inf_{x\in F} \left(P_{Z/X}(1/x) \geq \tau\right) \tag{19}$$

Achievability and Converse Bounds

$\kappa\beta$ bound:

For any $0<\epsilon<1$, there exists an (M,ϵ) code with codewords chosen from $\mathbf{F} \subset \mathbf{A}$, satisfying

$$M \geq \sup_{0<\tau<\epsilon} \sup_{Q_Y} \frac{\kappa_\tau(\mathbf{F}, Q_Y)}{sup_{x\in F}\beta_{1-\epsilon+\tau}\left(P_{Y/X=x}, Q_Y\right)} \tag{20}$$

Theorem 2 (meta-converse):

Consider two different abstract channels $P_{Y/X}$ and $Q_{Y/X}$ defined on the same input and output spaces. For a given code with codewords belonging to $\mathbf{F} \subset \mathbf{A}$, let

$$\epsilon = \text{maximum error probability with} \quad P_{Y/X}$$

$$\epsilon' = \text{maximum error probability with} \quad Q_{Y/X}$$

Then, $\underset{x\in F_{1-\epsilon}}{\beta}(P_{Y/X=x}, Q_{Y/X=x}) \leq 1-\epsilon'$

AWGN Channel

$$Q_{Y^n} = \mathcal{N}(0, (1+P)\mathbf{I_n}) \tag{21}$$

$$\log \frac{dP_{Y^n/X^n}}{dQ_Y^n} = \sum_{j=1}^{n} L_j \tag{22}$$

where L_j are independent random variables distributed as

$$L_j = \frac{1}{2}\log(1+P) + \frac{\log e}{2}\frac{P}{P+1}\left(1 - Z_i^2 + \frac{2}{\sqrt{P}}Z_i\right) \tag{23}$$

Applying Berry-Esseen inequality to 22

Berry-Esseen

Theorem 13 (Berry-Esseen) *Let the* X_k*,* $k = 1, \ldots, n$ *be independent with*

$$\mu_k = \mathbb{E}\left[X_k\right], \; \sigma_k^2 = \mathrm{Var}[X_k], \; \textit{and } t_k = \mathbb{E}\left[|X_k - \mu_k|^3\right]. \tag{2.81}$$

Denote $V = \sum_1^n \sigma_k^2$ *and* $T = \sum_1^n t_k$*. Then*

$$\left| \mathbb{P}\left[\frac{\sum_1^n (X_k - \mu_k)}{\sqrt{V}} \le \lambda \right] - Q(-\lambda) \right| \le 6 \frac{T}{V^{3/2}}, \tag{2.82}$$

where $Q(x)$ *is defined in* (1.18).

Note that for i.i.d. X_k it is known that the factor of 6 in the right hand side can be replaced by 1 or less; see [52]. In this thesis, the exact value of the constant does not affect the results and so we take the conservative value of 6 even in the i.i.d. case.

Regarding the asymptotic behavior of the β_α, the Berry-Esseen inequality implies the following result, proved in Appendix A:

AWGN Channel

Applying Berry-Esseen inequality to 22

$$\log \beta_\alpha^n = -nC_1(P) + \sqrt{nV_1(P)}Q^{-1}(1-\alpha) + O(\log n) \tag{24}$$

To compute $\kappa_\tau(\mathbf{F}, Q_{Y^n})$, the quantity

$$S_n = \sum_{j=1}^{n} |Y_j|^2 \tag{25}$$

is a sufficient statistic for a composite HT problem

The distribution of S_n is the same under all $P_{Y^n/X^n=x^n}$ provided that $x^n \in \mathbf{F_n}$. Then,

$$\kappa_\tau(\mathbf{F_n}, Q_{Y^n}) = \beta_\tau(P_{S_n}, Q_{S_n}) \tag{26}$$

Applying CLT shows that for some constants K_1, K_2 and all $\tau \in [0,1]$ we have

$$\kappa_\tau(F_n, Q_{Y^n}) \geq K_1\tau + O(e^{-K_2 n}) \tag{27}$$

To conclude the proof of achievability we apply Theorem 1 with $\tau_n = \frac{1}{\sqrt{n}}$ and find that

$$\log M_e^*(n,\epsilon) \geq \log \kappa_{\tau_n}(\mathsf{F}_n, Q_{Y^n}) - \log \beta^n_{1-\epsilon+\tau_n} \tag{51}$$

$$\geq nC_1 - \sqrt{nV_1}Q^{-1}(\epsilon - \tau_n) + \log \kappa_{\tau_n} + O(\log n) \tag{52}$$

$$= nC_1 - \sqrt{nV_1}Q^{-1}(\epsilon) + \log \kappa_{1/\sqrt{n}} + O(\log n) \tag{53}$$

$$\geq nC_1 - \sqrt{nV_1}Q^{-1}(\epsilon) + O(\log n), \tag{54}$$

Converse: Take the code satisfying an equal-power constraint with the largest possible cardinality $M_e^*(n, \epsilon)$. We apply Theorem 2 to this code with $Q_{Y^n|X^n} = Q_{Y^n}$, where Q_{Y^n} is defined in (40). Since under the Q-channel the input and output are independent, we easily find

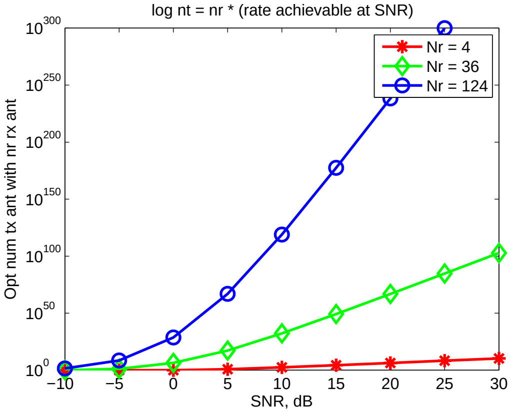
$$1 - \epsilon' \leq \frac{1}{M_e^*(n, \epsilon)} . \tag{55}$$

Putting this into (12) we find

$$\log M_e^*(n, \epsilon) \leq -\log(1 - \epsilon') \leq -\log \beta_{1-\epsilon}^n \tag{56}$$

$$\leq nC_1(P) - \sqrt{nV_1(P)} Q^{-1}(\epsilon) + O(\log n) , \tag{57}$$

Results



log nt = nr * (rate achievable at SNR)
Opt num tx ant with nr rx ant
SNR, dB
Nr = 4
Nr = 36
Nr = 124
10^300
10^250
10^200
10^150
10^100
10^50
10^0
−10
−5
0
5
10
15
20
25
30

Time Varying Channel