Power Management Algorithms for a Network of Energy Harvesting Sensors

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Outline

Motivation

- Energy harvesting sensors (EHS) operate using the energy harvested from their surroundings
- No need of battery replacement, maintenance etc
- Theoretically infinite life time!
- Wireless Communication - can deploy EHS at places that are hard to reach
- Applications
  - Monitoring (Habitat, Structural Health, Industrial Processes)
  - Body Area Networks (BAN)

Need for Power Management

- Energy sources - Solar, Wind, Vibration, Thermal, RF
- The amount and nature (sporadicity) of the energy harvested is dependent on the source
  - High and predictable energy of solar Vs small and unpredictable energy of vibration
- Energy available is ∞. But, power is limited/sporadic
- Power management algorithms schedule the utilization of the harvested power in order to get maximum benefit

System Model

- Single hop EHS network with K nodes, powered destination
- Single MCS transmission by each node
  - Transmission based on channel measurement
  - Leads to TCI
- Duty-cycling across nodes to prevent collisions

Goal

Find the optimum power allocation and duty-cycle to maximize the sum throughput subject to energy neutrality at each individual node.

Block Diagram of Each EHS Node

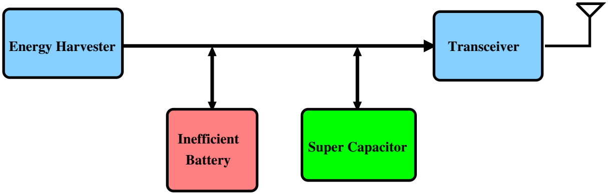

Figure: Power Flow Diagram

- The remaining circuitry (sensor) is assumed to consume a fixed power and hence not shown

Timing Diagram

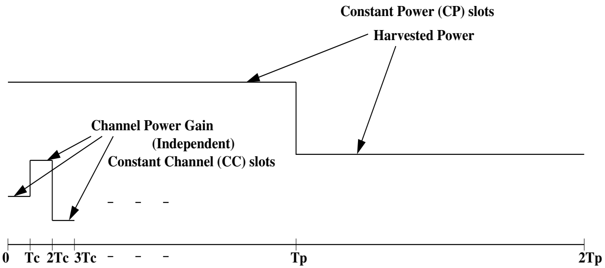

Figure: Constant channel and constant power slots

Role of Supercapacitor

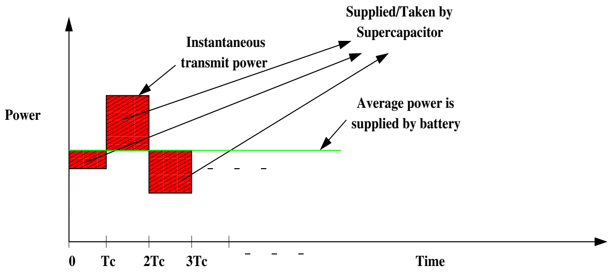

Figure: Truncated channel inversion

Assumptions

- Battery at each node: storage efficiency η
- Supercapacitor assumed perfectly efficient
- The load seen by battery within a CP slot is constant (due to the presence of supercapacitor)

Notation

Symbol	Meaning
$P_{h,i}$	Harvested power, i-th node, in some CP slot, $P_h = [P_{h,1}, \ldots, P_{h,K}]$
$P_{a,i}$	*Allotted power*, i-th node, in some CP slot, $P_a = [P_{a,1}, \ldots, P_{a,K}]$
$R_i(P_a)$	Average rate of i-th node in a CP slot with allotted power vector P_a
$\gamma_{m,i}(P_a)$	Channel gain threshold for xmission i-th node, allotted power vector P_a
$D_i(P_a)$	Duty cycle of i-th node in a CP slot with allotted power vector P_a
$F_\gamma(\cdot)$	CDF of the channel power gain
$f_{P_{h,i}}(\cdot)$	PDF of the harvested power at the i-th node

Proposed Approach

- First Stage: choose duty cycle D_i and gain threshold $\gamma_{m,i}$ at each node as functions of P_a
  - Requires knowledge of sum power allotted at the nodes
  - Assumes identical channel statistics
  - Maximizes channel statistics-averaged sum throughput for the given P_a
- Second Stage: choose power allocation vector P_a as a function of the harvested power vector P_h
  - Requires knowledge of harvested power at each node
  - Tries to maximize the sum throughput over the harvested power statistics

First Stage Problem Setup

- To choose: duty cycle D_i and gain threshold $\gamma_{m,i}$ at each node as functions of P_a
- Average data rate of the i-th node

$$R_i(P_a) = R_{on} D_i \left[1 - F_\gamma(\gamma_{m,i})\right]$$

- Optimization problem

$$\max_{D_i \geq 0, \gamma_{m,i} \geq 0, \sum_i D_i = 1} \sum_{i=1}^{K} R_i(P_a)$$

- Subject to the power constraint

$$D_i \int_{\gamma_{m,i}}^{\infty} \frac{P_0}{\gamma} f_\gamma(\gamma) d\gamma = P_{a,i}, \quad i = 1, \ldots, K$$

Assumes identical channel statistics $f_\gamma(\gamma)$ from each node to destination

First Stage Solution

- Using Lagrange mult. λ with $\sum_{i=1}^{K} D_i = 1$, can show

$$R_{on} \int_{\gamma_{m,i}}^{\infty} \left[1 - \frac{\gamma_{m,i}}{\gamma}\right] f_\gamma(\gamma) d\gamma = \lambda, \;\; i = 1, 2, \ldots, K$$

- The solution $\gamma_{m,i}$ in the above does not depend on $P_{a,i}$!
- Therefore $\gamma_m \triangleq \gamma_{m,i}$ is the same for all $i \Rightarrow D_i^{opt} = \frac{P_{a,i}}{\sum_{i=1}^{K} P_{a,i}}$
- The optimum channel gain threshold satisfies

$$\int_{\gamma_m}^{\infty} \frac{f_\gamma(\gamma)}{\gamma} d\gamma = \frac{\sum_{i=1}^{K} P_{a,i}}{P_0}$$

- And the optimized sum throughput

$$R_\Sigma(P_a) \triangleq \sum_{i=1}^{L} R_i^{opt}(P_a) = R_{on}\left[1 - F_\gamma(\gamma_m)\right]$$

Second Stage Problem Setup

- Now lets optimize P_a
- Optimization problem

$$\max_{P_a(P_h)\geq 0} E\left\{R_\Sigma(P_a)\right\}$$

The expectation is over the joint distribution of P_h

- Subject to energy neutrality at node i, $1 \leq i \leq K$:

$$\int_0^\infty \left(A_i P_{a,i} + (1 - A_i)\left(\pi_h + \frac{P_{a,i} - \pi_h}{\eta}\right)\right) f_{P_{h,i}}(\pi_h) d\pi_h = E\left\{P_{h,i}\right\}$$

where $A_i = 1$ if $P_{a,i} < \pi_h$ and 0 otherwise.

In the above, the variable of integration π_h is the power harvested at the i-th node

Simplification of Second Stage

- Recall: $R_\Sigma(P_a)$ depends only on $\sum_{i=1}^{K} P_{a,i}$ (through γ_m)
- Now, when $\sum P_{h,i} > \sum P_{a,i}$, some nodes can deposit energy into battery
- When $\sum P_{h,i} < \sum P_{a,i}$, some nodes must draw energy from battery
- Energy efficient design: sum power energy neutrality $**$

$$\int_0^\infty \left(A\,P_{a,\Sigma} + (1-A)\left(\pi_h + \frac{P_{a,\Sigma} - \pi_h}{\eta}\right)\right) f_{P_{h,\Sigma}}(\pi_h) d\pi_h = E\left\{P_{h,\Sigma}\right\}$$

where $A = 1$ if $P_{a,\Sigma} < \pi_h$ and 0 otherwise.

Note: $\pi_h = \sum_{i=1}^{K} P_{h,i}$ is the sum power harvested and $f_{P_{h,\Sigma}}(\cdot)$ is its PDF

- Tries to minimize wasteful cycling of energy through battery

Second Stage Solution

- New optimization problem

$$\max_{P_{a,\Sigma}(P_{h,\Sigma})} R_{\Sigma}(P_a) \triangleq R_{on}\left[1 - F_{\gamma}(\gamma_m)\right]$$

subject to $**$

- Same as the "outer stage optimization" in our past work!
  - Working with a fictitious EHS which harvests the sum power, and under a sum-power energy neutrality constraint
- Optimal solution:

$$P_{a,\Sigma}^{opt} = \begin{cases} P_1 & \text{if } P_{h,\Sigma} \geq P_1; \\ P_2 & \text{if } P_{h,\Sigma} \leq P_2; \\ P_{h,\Sigma} & \text{if } P_2 < P_{h,\Sigma} < P_1. \end{cases}$$

Second Stage Solution (Cont'd)

- For optimality, P_1 and P_2 should satisfy

$$\left.\frac{dR_\Sigma}{dP_{a,\Sigma}}\right|_{P_{a,\Sigma}=P_1} = \eta \left.\frac{dR_\Sigma}{dP_{a,\Sigma}}\right|_{P_{a,\Sigma}=P_2}$$

- For energy neutrality, P_1 and P_2 should satisfy

$$\int_0^{P_2} [P_2 - \pi_h] f_{P_{h,\Sigma}}(\pi_h) d\pi_h = \eta \int_{P_1}^{\infty} [\pi_h - P_1] f_{P_{h,\Sigma}}(\pi_h) d\pi_h$$

Power Allocation for Each Node (1/2)

- Given $P_{a,\Sigma}^{opt}$, to find $P_{a,i}$ that satisfy EN at each node
- Recall sum power allocation rule:

$$P_{a,\Sigma}^{opt} = \begin{cases} P_1 & \text{if } P_{h,\Sigma} \geq P_1; \\ P_2 & \text{if } P_{h,\Sigma} \leq P_2; \\ P_{h,\Sigma} & \text{if } P_2 < P_{h,\Sigma} < P_1. \end{cases}$$

- Many ways of allotting $P_{a,i}$ s.t. $\sum P_{a,i} = P_{a,\Sigma}$
- Individual energy neutrality conditions

$$\int_0^\infty \left(A_i P_{a,i} + (1 - A_i)\left(\pi_h + \frac{P_{a,i} - \pi_h}{\eta}\right)\right) f_{P_{h,i}}(\pi_h) d\pi_h = E\left\{P_{h,i}\right\}$$

where $A_i = 1$ if $P_{a,i} < \pi_h$ and 0 otherwise.

Power Allocation for Each Node (2/2)

- Proposed allotment scheme:

$$P_{a,i} = \begin{cases} \left[P_{h,i} - \frac{P_{h,\Sigma}-P_1}{K} - P_{e,i}\right]^+ & \text{if } P_{h,\Sigma} \geq P_1; \\ P_{h,i} + \frac{P_2 - P_{h,\Sigma}}{\eta K} & \text{if } P_{h,\Sigma} \leq P_2; \\ P_{h,i} & \text{if } P_2 < P_{h,\Sigma} < P_1. \end{cases}$$

- Let $P_{e,i}^{(j)}$ denote the $P_{e,i}$ in CP slot j
- Update equation for $P_{e,i}$ is as follows:

$$P_{e,i}^{(j+1)} = \begin{cases} \left[-P_{h,i} + \frac{P_{h,\Sigma}-P_1}{K} + P_{e,i}^{(j)}\right]^+ & \text{if } P_{h,\Sigma} \geq P_1; \\ P_{e,i}^{(j)} & \text{if } P_{h,\Sigma} \leq P_2; \\ P_{e,i}^{(j)} & \text{if } P_2 < P_{h,\Sigma} < P_1. \end{cases}$$

Note: in the above, $P_{h,i}$, $P_{h,\Sigma}$ are the harvested powers in the j-th CP slot.

Simulation Result

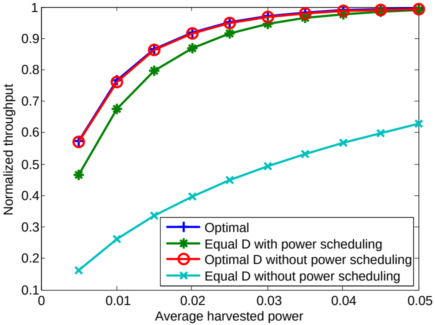

Figure: Normalized throughput *Vs.* $E\{P_h\}$, for $\eta = 0.8$, $K = 10$. Harvested power uniformly distributed between $[0, 2E\{P_h\}]$

Conclusions

- We solved duty cycling and power allocation problems of an EHS network
- The model proposed here is generic and can be used with several performance metrics
- Future work includes solving the optimization problem of minimizing the energy storage requirement

The End

Thank you very much!
Questions?