Transition probabilities for DTMC modelling of Dual and Mono Energy harvesting links

Mohit K. Sharma and Chandra R. Murthy

Abstract

This report contains the transition probabilities for the DTMC modeling of the dual and mono energy harvesting links in [1]. The notations used here assumes the same meaning as defined in [1].

I. PDP OF BATTERYLESS EHNS

For fast fading channels, the PDP is given as

$$P_{\mathrm{D}}(K) = \left(1 - \rho_t \rho_r e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{E_s \sigma_c^2}}\right)^K, \tag{1}$$

where (1), again, is written using (4).

In the case of HARQ-CC, the PDP of dual EH links without energy buffer is given as

$$P_{\mathrm{D}}(K) = \sum_{m=0}^{K} \binom{K}{m} (\rho_t \rho_r)^m (1 - \rho_t \rho_r)^{K-m} p_{\mathrm{D}_m} \tag{2}$$

where p_{D_m} is the probability that the transmitted packet is dropped, when both nodes simultaneously harvest energy in exactly m out of the K slots in the current frame. For the slow fading case, p_{D_m} is given by (6), while for fast fading channels it is given by (**??**), with $i = j = 0, m_t = m_r = \Psi_1 = m,$ and $L_n = 1 \quad \forall \quad n.$

A. *Transition Probability Matrix, **G**, for dual EH links*

The probability of transition from state (i_1, j_1, r) to (i_2, j_2, s) is $G_{i_1,j_1,r}^{i_2,j_2,s} = \Pr\left(B_{n+1}^t = i_2, B_{n+1}^r = j_2, U_{n+1} = s \middle| B_n^t = i_1, B_n^r = j_1, U_n = r\right)$, where $i_1, i_2 \in \{0, 1, \ldots, B_{\max}^t\}$, $j_1, j_2 \in \{0, 1, \ldots, B_{\max}^r\}$ and $r, s \in \{-1, 0, \ldots, K\}$. For $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $j_1 \geq R$, the $G_{i_1,j_1,r}^{i_2,j_2,s}$ is written as follows

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r+1, \\ \rho_t\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = r+1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = r+1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = -1, \\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R, s = r+1, \\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{3a}$$

For $r \in \{0, \ldots, K-1\}$, $L_r - 1 \leq i_1 < L_r$ and $j_1 \geq R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r+1, \\ \rho_t\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = r+1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3b}$$

For $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $R-1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, , s = r + 1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3c}$$

For $r \in \{0, \ldots, K-1\}$, $L_r - 1 \leq i_1 < L_r$ and $R-1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - L_r + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = 0, j_2 = 0, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, , s = r, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3d}$$

For $r \in \{0, \ldots, K-1\}$, $0 \leq i_1 \leq L_r - 2$ or $0 \leq j_1 \leq R - 2$

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r, & i_2 = i_1 + 1, j_2 = j_1 + 1, s = r, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = r, \\ (1-\rho_r)\rho_t, & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{3e}$$

For $r=-1$, $i_1 \geq 0$ and $j_1 \geq 0$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t\rho_r, & i_2 = i_1+1, j_2 = j_1+1, s=-1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1+1, s=-1, \\ (1-\rho_r)\rho_t, & i_2 = i_1+1, j_2 = j_1, s=-1, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s=-1, \\ 0, & \text{otherwise.} \end{cases} \tag{3f}$$

In (3a) – (3d), Pr$[\gamma_n < \gamma_0]$ for the slow and the fast Rayleigh fading channels and ARQ is given as

$$p_{\text{out},\ell} \triangleq \text{Pr}[\gamma_\ell < \gamma_0] = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{L_\ell E_s \sigma_c^2}}, \tag{4}$$

where $\mathcal{N}_0$ denotes the power spectral density of the AWGN at the receiver. While for HARQ-CC with slow fast fading channels it is obtained using $\Psi_1 = n$ in the following equation For *slow fading* channels

$$\begin{aligned} p_{\text{D}}(i,j,m_t,m_r) &= \text{Pr}\left[|h|^2 < \frac{\gamma_0 \mathcal{N}_0}{\sum_{m=1}^{\Psi_1} P_m}\right], &(5)\\ &= 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_c^2 \sum_{m=1}^{\Psi_1} P_m}}, &(6) \end{aligned}$$

and for fast fading channel it is obtained using $\Psi_1 = n$ in (16) of [2].

The terms in the above transition probability expressions are obtained by considering the events that need to occur for the particular transition to happen. For e.g., in (3a) transition mentioned in first case happens if both transmitter and receiver harvest the energy in current slot, and the decoding failure occurs in current attempt. Note that, in the above, for simplicity, the transition probabilities are written for infinite buffer size at both transmitter and receiver. However, the above expressions can be trivially modified for finite battery case.

B. Transition Probability Matrix $\boldsymbol{G}_m$

The probability of transition from state (i,r) to (j,s) is $G_{(m)ij}^{rs} = Pr(B_{(M+1)K} = j, U_{n+1} = s | B_{MK} = i, U_n = r)$, where $i,j \in \{0,1,\ldots,\infty\}$ and $r,s \in \{-1,0,\ldots,K\}$.

For $r \in \{0, \ldots, K-1\}$ and $i \geq L_r$

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r+1, \\ \rho_t Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho_t) Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r, s = r+1, \\ (1-\rho_t) Pr\left[\gamma_n > \gamma_0\right], & j = i - L_r, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{7a}$$

For $r \in \{0, \ldots, K-1\}$ and $L_r - 1 \leq i < L_r$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r+1, \\ \rho_t Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho_t), & j = i, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{7b}$$

For $r \in \{0, \ldots, K-1\}$ and $0 \leq i \leq L_r - 2$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t, & j = i+1, s = r, \\ (1-\rho_t), & j = i, s = r, \\ 0, & \text{otherwise.} \end{cases} \tag{7c}$$

For $r = -1$ and $i \geq 0$,

$$G^{rs}_{(m)ij} = \begin{cases} \rho_t, & j = i+1, s = -1, \\ (1-\rho_t), & j = i, s = -1, \\ 0, & \text{otherwise.} \end{cases} \tag{7d}$$

In (7a) and (7b), Pr$[\gamma_n < \gamma_0]$, for ARQ is written using (4), while for HARQ-CC with slow fading channels it can be computed using (6) with $\Psi_1 = n$. For fast fading channels one need to use (16) in [2] with $\Psi_1 = n$.

REFERENCES

[1] M. K. Sharma and C. R. Murthy, "Packet drop probability analysis of dual energy harvesting links with retransmissions," submitted for publication.

[2] M. Sharma and C. Murthy, “Packet drop probability analysis of ARQ and HARQ-CC with energy harvesting transmitters and receivers,” in *Proc. IEEE GlobalSIP*, Dec. 2014, pp. 148–152.