# Algorithms to find the Optimum Operating Points for D2D Communication

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# Outline

# System Model

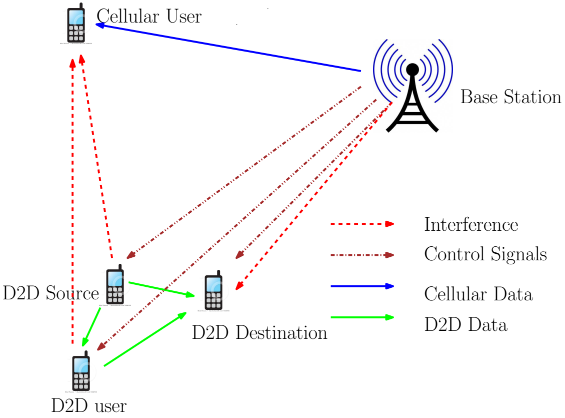


Figure: Downlink In-band D2D

# Recap

- Objective:
  - Optimal D2D routing to maximize end-to-end throughput
    - Fixed Rate Scheme: Find optimal rate
    - Fixed Power Scheme: Find optimal transmit powers
  - Satisfy interference constraint at the cellular users
- Parameters:
  - $\gamma$ - Target D2D SINR
  - $\gamma_b$ - min. cellular SNR
  - $\gamma_d$ - max. interference constraint to the cellular user
- Algorithms
  - Downlink in-band D2D
  - Uplink in-band D2D

# Fading

## Downlink in-band D2D

- Objective: Probabilistic guarantees for protection of the cellular users, while maximizing the throughput for the D2D users.
- Approach:
  - A cellular user assumed at a location where the SNR is greater than $\gamma_b$ with probability $p_b$
  - At this user, interference caused by D2D communication should be less than $\gamma_d$ with probability greater than $p_d$
  - This fixes the maximum power D2D transmitters can use
- Note that in the path loss model, $p_b$ and $p_d$ were both 1

## Location of cellular user

- $$\Pr\left(P + 20 \ \log\left(|h|\right) - 10\alpha \ \log(D_{\max}) \geq \gamma_b\right) = p_b \tag{1}$$
- Given $p_b$, $D_{\max}$ can be calculated:

$$D_{\max} = \left(-10^{\left(\frac{P-\gamma_b}{10}\right)} \ \ln(p_b)\right)^{\frac{1}{\alpha}}$$

**Proof.**

Equation 1 can be rewritten as

$$\Pr\left(10^{\frac{P}{10}} \ |h|^2 D_{\max}^{-\alpha} \geq 10^{\frac{\gamma_b}{10}}\right) = p_b$$

$$\Pr\left(|h|^2 \geq D_{\max}^{\alpha} 10^{\frac{\gamma_b - P}{10}}\right) = p_b$$

Since $|h|^2 \sim exp(1)$, this is equivalent to

$$e^{-\left(D^{\alpha}{}_{\max} \ 10^{\frac{\gamma_b - P}{10}}\right)} = p_b$$

The result follows. □

## Interference at the cellular user I

$$\Pr\left(P_{d_T,BS_i} - 10\alpha\ \log\left(D_{d_T,BS_i} - D\text{max}\right) + 20\ \log\left(|h|\right) \le \gamma_d\right) \ge p_d \quad (2)$$

- Given $D\text{max}$,

$$P^{\text{max}}_{d_T,BS_i} = 10\log\left(\frac{10^{\frac{\gamma_d}{10}}\ (D_{d_T,BS_i} - D\text{max})^{\alpha}}{-\ ln(1-p_d)}\right)$$

$$P^{\text{max}}_{d_T} = \min_{1\le i\le N} P^{\text{max}}_{d_T,BS_i}.$$

## Proof.

Equation 2 can be rewritten as:

$$\Pr\left(10^{\frac{P_{d_T,BS_i}}{10}}\left(D_{d_T,BS_i}-D\text{max}\right)^{-\alpha}|h|^2 \leq 10^{\frac{\gamma_d}{10}}\right) \geq p_d$$

$$\Pr\left(|h|^2 \geq 10^{\frac{\left(\gamma_d - P_{d_T,BS_i}\right)}{10}}\left(D_{d_T,BS_i}-D\text{max}\right)^{\alpha}\right) \leq 1-p_d$$

Since $|h|^2 \sim exp(1)$, this is equivalent to

$$e^{-\left(10^{\frac{\left(\gamma_d - P^{\text{max}}_{d_T,BS_i}\right)}{10}}\left(D_{d_T,BS_i}-D_{\text{max}}\right)^{\alpha}\right)} = 1-p_d$$

The result follows. □

- Transmit power required:

$$P_{d_T}^{d_R} = P_{\text{int}}^{d_R} + \gamma + 10\alpha \ \log(d_{d_T}^{d_R}) - 20\log(|h|)$$

- Feasibility
  Link $d_t \longrightarrow d_R$ declared infeasible if $P_{d_T}^{d_R} > P_{d_T}^{\text{max}}$.
- We have the feasible D2D links.

- Previous algorithms with the following modifications will find the optimum operating points:
  - Definition of $P_{d_T}^{\text{max}}$
  - D2D SINR should also take into account the channel gains.

# Uplink In-band D2D

- Interference at D2D receiver and the BS
- With CSI: Algorithm similar to the one presented earlier
- Without CSI:
  - Cellular link in outage if SINR below a threshold
  - Since cellular users employ power control, link in outage if interference grater than a threshold
  - Interference at the BS less than $\gamma_d$ with probability greater than $p_u$

$$\Pr\left(P_{d_T,BS} - 10\alpha \ \log\left(D_{d_T,BS}\right) + 20 \ \log\left(|h|\right) \leq \gamma_d\right) \geq p_u \qquad (3)$$

  - $P^{\max}_{d_T,BS}$ can be then calculated as :

$$P^{\max}_{d_T} = 10 \log \left( \frac{10^{\frac{\gamma_d}{10}} \ (D_{d_T,BS})^{\alpha}}{- \ ln(1 - p_u)} \right)$$

## Proof.

Equation 3 can be rewritten as:

$$\Pr\left(10^{\frac{P_{d_T}}{10}} \left(D_{d_T,BS}\right)^{-\alpha} |h|^2 \leq 10^{\frac{\gamma_d}{10}}\right) \geq p_u$$

$$\Pr\left(|h|^2 \geq 10^{\frac{\left(\gamma_d - P_{d_T}\right)}{10}} \left(D_{d_T,BS_i}\right)^{\alpha}\right) \leq 1 - p_u$$

Since $|h|^2 \sim exp(1)$, this is equivalent to

$$e^{-\left(10^{\frac{\left(\gamma_d - P_{d_T}^{\max}\right)}{10}} \left(D_{d_T,BS_i}\right)^{\alpha}\right)} = 1 - p_u$$

The result follows. □

# Setup

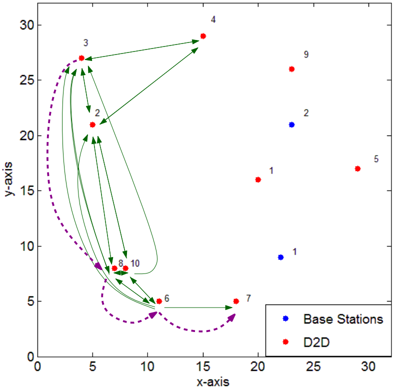


Figure: Locations of the BSs and D2D users.

# Downlink In-band D2D

## Fixed Rate Scheme

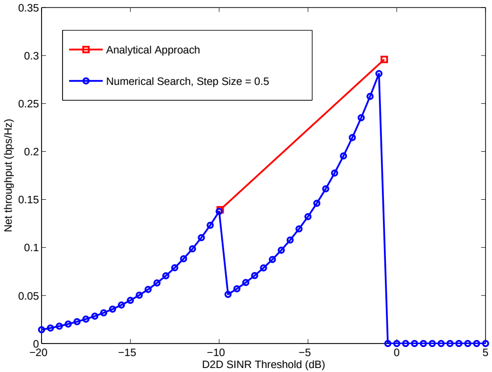


Figure: Fixed rate scheme: Illustration of the numerical search approach and the analytical approach in the Rayleigh fading model.

# Downlink In-band D2D

## Fixed Power Scheme

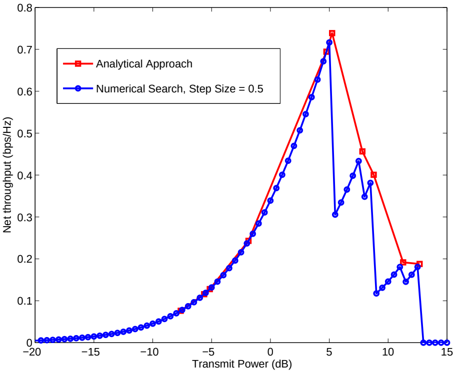


Figure: Fixed power scheme: Illustration of the numerical search approach and the analytical approach in the Rayleigh fading model.

# Uplink In-band D2D

## Fixed Rate Scheme

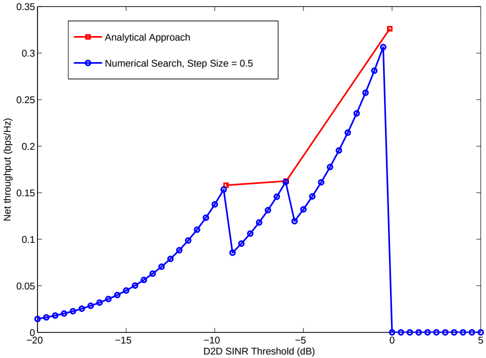


Figure: Fixed rate scheme: Illustration of the numerical search approach and the analytical approach in the *uplink inband* D2D model with Rayleigh fading.

# Modified Dijkstra's Algorithm for the Fixed Rate Scheme: An Alternative

# Previous Algorithm I

Step 1 Find the power ($P_{d_S}^{\max}$) at which the source $d_S$ is allowed to transmit. Find the corresponding SINR ($\gamma_{d_S}^{d_D}$) at $d_D$. Call this SINR $\gamma_1$.

Step 2 Consider the path $d_S \longrightarrow d_i \longrightarrow d_D$.
The maximum SINR at which some two-hop path will exist:

$$\gamma_2 = \max_{i \neq D,S} \left( \min(\gamma_{d_S}^{d_i}, \gamma_{d_i}^{d_D}) \right)$$

A peak exists here only if $\gamma_2$ is greater than $\gamma_1$.

Step 3 Repeat Step 2 for all possible three hop paths, and determine

$$\gamma_3 = \max_{i \neq D,S,j \neq D,S,i} \left( \min(\gamma_{d_S}^{d_i}, \gamma_{d_i}^{d_j}, \gamma_{d_j}^{d_D}) \right)$$

A peak exists here if $\gamma_3$ is greater than the previous $\gamma$ (i.e., $\gamma_2$, or $\gamma_1$ if $\gamma_2$ does not exist).

# Previous Algorithm II

Step 4 Repeat with increasing number of hops, a point $\gamma_f$ is found such that when $\gamma > \gamma_f$, $d_S$ and $d_D$ are no longer connected in the graph $G_\gamma(V, E)$.

Note: The existence of $\gamma_f$ is guaranteed by the fact that the $G_{\gamma_j}(V, E)$ always has fewer links than $G_{\gamma_k}(V, E)$ for all $j$ and $k$ s.t. $j > k$.

Step 5 Set $\gamma_{\text{opt}} = \arg\max_{1 \le i \le M, \gamma_i \text{ exists}} R_{\text{eff}}(\gamma_i)$

# Modified Dijkstra's Algorithm

- Modifications:
  - Link weights: Rates corresponding to the max. transmit power
  - Metric at each step:$\{\min(\text{Rates}), \text{Number of hops}\}$
  - If more than one paths merge at a node, select the one that maximizes $\frac{\min(\text{Rates})}{\text{Number of hops}}$
- Output: The optimum SINR threshold that achieves the maximum end-to-end throughput

# THANKS!!!