Whitespace Identification Using Randomly Deployed Sensors

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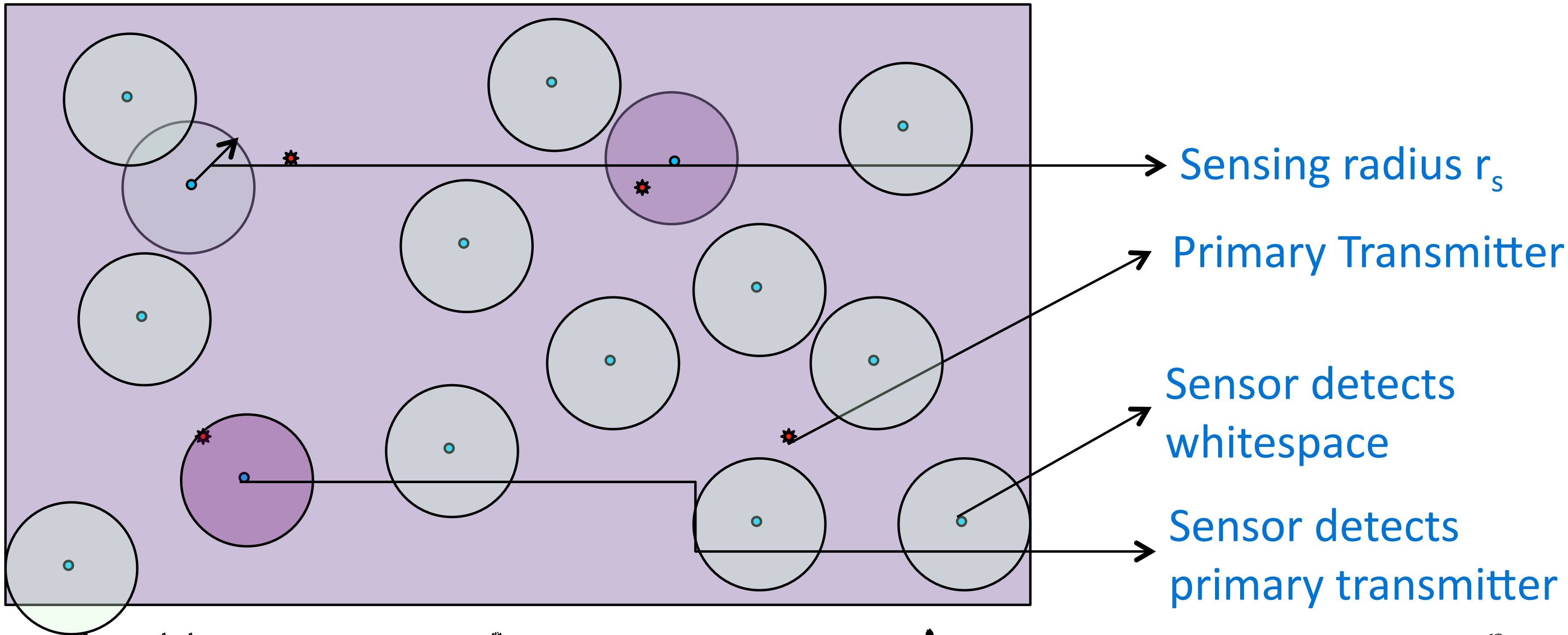
Whitespace Identification

- Regions not covered by any transmitter
- Applications:
  - Identify "dead zones"
  - Areas safe for CR transmission

System Model

- Consider n randomly deployed sensors
- Sensing radius r_s



- Challenge: opt. sensing radius + min. error?

Approach

- WLOG, consider $L = [0,1]$ as the region of interest
- Delpoy n sensors uniformly at random on L
- Each sensor returns 1 if there is a pri. tx. within $2r_s$, and 0 o/w
- Whitespace recovered
  = union of $2r_s$ regions around sensors that return 0

Main Results

1. Both the whitespace recovery error (to be defined) & r_s optimally scale as $\log(n)/n$
2. This holds even if sensors make erroneous measurements!
3. Derive optimal r_s in terms of sum tx. localization error
   1. Needs txs to have a min. separation
   2. Involves estimating num. txs also
4. Derive the sensor distbn. that min. the Pr.(miss a tx)

Comments on the Results

- The results hold "with high probability" as n -> ∞
- For ex., we show that if $r_s = \Theta\left(\frac{\log n}{n}\right)$ we have a localization err. $\Theta\left(\frac{\log n}{n}\right)$ with prob. -> 1 as n -> ∞
- The results extend to 2d
  - Just have square roots of 1d results
  - For simplicity, will focus on 1d here

System Model

- Unit length segment L
- M pri. tx., arbitrarily located
  - Will consider random locs later
- n sensors thrown uniformly At random locations (UARL) on L
  - Sensors retn. $b=1$ if there is a pri. tx. within $r_s(n)$, $b=0$ otherwise
- Whitespace = union of regions around sensors that retn. 0

Problem Statement

- If x_i and b_i are sensor locs and their readings, $i = 1, 2, \ldots, n$

$$\mathcal{A}_{\text{void}} = \bigcup_{i=1}^{n} (1 - b_i)[\min(x_i - r_s, 0), \max(x_i + r_s, 1)].$$

- Let $\ell(\mathcal{A}) = \int_{x \in \mathcal{A}} \mathrm{d}x$
- We are interested in

$$\min_{r_s(n), \epsilon(n)} \lim_{n \to \infty} P\left((1 - A_{\text{void}}) \leq \epsilon(n)\right) = 1$$

Result on 1-Coverage [Kumar et. al. MobiCom04]

- Consider n sensors UARL on L
  - Radio range r(n)
- A pt. x is "covered" if there is at least 1 sensor in [x-r(n), x+r(n)]
- **Lemma 1:** If $\lim_{n\to\infty} \frac{r(n)}{\frac{\log n}{n}} \to 0$
- Then

$$\lim_{n\to\infty} P(\text{all points in } \mathcal{L} \text{ are covered}) < c_2$$

- Where $c_2 < 1$

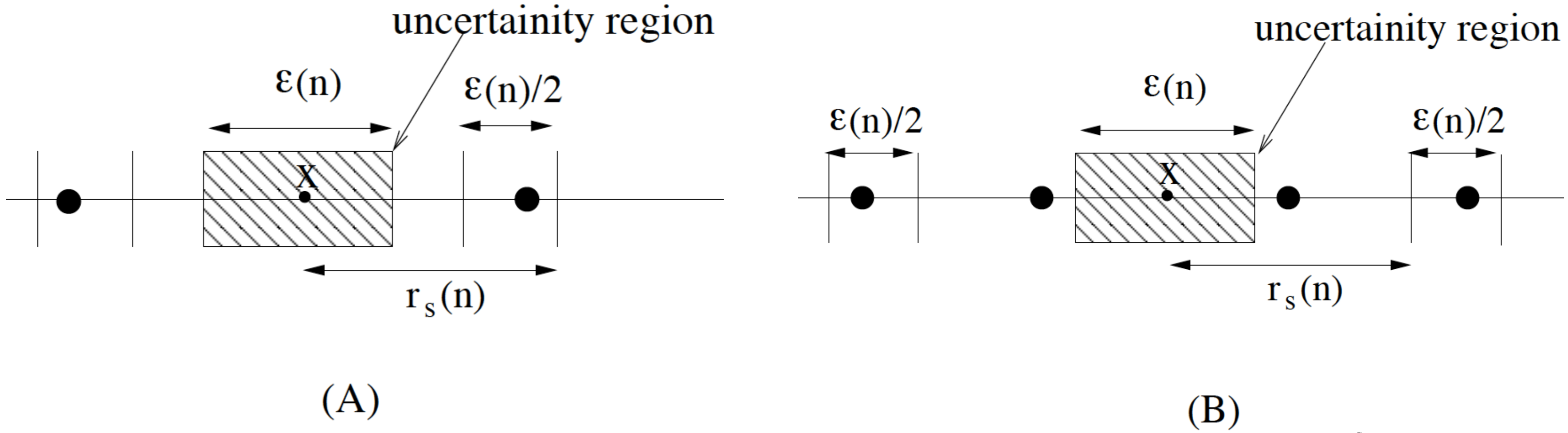
Lower Bound

- Theorem 1: For the whitespace recovery problem, if $\lim_{n\to\infty} \frac{\epsilon(n)}{\frac{\log n}{n}} \to 0$ or $\lim_{n\to\infty} \frac{r_s(n)}{\frac{\log n}{n}} \to 0$

$$P\left(\left(1 - A_{\text{void}}\right) \leq \epsilon(n)\right) < 1$$

Proof

- Suffices to let M=1
- To have (1-A_{void}) ≤ $\epsilon(n)$, need



- That is, need all intervals of length $r_s(n)$ and $\epsilon(n)$ to have at least 1 sensor

- We now use Lemma 1
- If either $2r_s(n)$ or $\epsilon(n)$ is less than O(log n/n), then there exists an interval of width $2r_s(n)$ or $\epsilon(n)$ with prob. at least $1-c_2 > 0$
- Then, either (A) or (B) will not happen
- Thus, if $\lim_{n\to\infty} \frac{\epsilon(n)}{\frac{\log n}{n}} \to 0$ or $\lim_{n\to\infty} \frac{r_s(n)}{\frac{\log n}{n}} \to 0$ cannot have

$$P\left((1 - A_{\text{void}}) \leq \epsilon(n)\right) < 1$$

A Chernoff Bound

- **Lemma 2**: Let $X_1, X_2, \ldots, X_n$ be i.i.d. Bernoulli with $m = E(X_i)$
- Let $X = \sum_{i=1}^{n} X_i$
- Then, for $0 < d < 1$,
  $P(X < (1-d)m) \leq \exp(-nd^2m/2)$

The Lower Bound is Tight!

- **Theorem 2:** If $r_s(n) = \Theta\left(\frac{\log n}{n}\right)$, then with $\epsilon(n) = \Theta\left(\frac{\log n}{n}\right)$,

$$P\left(1 - A_{\text{void}} \leq \epsilon(n)\right) \to 1$$

- **Proof:** Let rs(n) = c log n/n
- Divide L into non-overlapping segments z_1, ..., $z_{(n/c \log n)}$, each of length c log n/n
- Let z_k contain N_k sensors

- From Lemma 2, $P(N_k < \frac{c \log n}{2}) \leq n^{-c/8}$
- Union bound:

$$P(N_k < \frac{c \log n}{2} \text{ for any } k) \; < c_4 n^{1-c/8}$$

- c_4 scales logarithmically with n
- Choose c big enough: all intervals z_k contain ≥ c log n/2 sensors w.h.p.
- Since there are M tx., there are at least $\left(\frac{n}{c \log n}\right) - 3M$ intervals with all sensor readings = 0
- Thus, $A_{\text{void}} \; > \; \left(\left(\frac{n}{c \log n}\right) - 3M\right) \left(\frac{c \log n}{n}\right)$, or

$$(1 - A_{\text{void}}) \leq 3M \left(\frac{c \log n}{n}\right)$$

Unreliable Sensors

- Sensors make errs w/ prob. p<½
- **Theorem 3:** If $r_s(n) = \Theta\left(\frac{\log n}{n}\right)$, then with $\epsilon(n) = \Theta\left(\frac{\log n}{n}\right)$, $P\left(1 - A_{\text{void}} \leq \epsilon(n)\right) \to 1$ asymptotically in n
- **Proof:** Omitted.
  - Uses a majority rule for making 0/1 decision about each z_k and bounds its error prob.

Transmitter Localization

- Want to estimate the number of transmitters M & their locations
- Estimating M:
  - Num. tx = num. disjoint region with sensors that returned 1
  - Tx. locations = geom. centroids of regions w/ sensors that returned 1
- Issue: A region with sensors that returned 1 could have > 1 txs

Optimal Scaling Result

- Assume any 2 primary txs are separated by at least $\delta(n) > 0$
- Want to solve:

$$\min_{r_s(n),\delta(n),\epsilon(n)} \lim_{n\to\infty} P\left(\sum_{i=1}^{\max\{M,\hat{M}\}} |x_i - \hat{x}_i| \leq \epsilon(n)\right) = 1$$

- Theorem 4:
If $r_s(n) = \delta(n) = \epsilon(n) = \Theta\left(\frac{\log n}{n}\right)$, $P\left(\sum_{i=1}^{\max\{M,\hat{M}\}} |x_i - \hat{x}_i| \leq \epsilon(n)\right) \to 1$
- Main message: if the min. separation > log n/n, the localization error is invariant to the knowledge of num. tx.

Optimum Distribution of Sensor Locations

- Suppose we know the distribution of pri. txs.
- Want optimum sensor distbn $f_\lambda(x)$
- Specifically, we wish to solve

$$P_f = \min_{f_\lambda(x):\int_0^1 f_\lambda(x)\mathrm{d}x=1} \int_0^1 \left(1 - 2r_s f_\lambda(x)\right)^n f_X(x)\mathrm{d}x$$

- Solution

$$f_\lambda(x) = \left(1 - \left(\frac{\mu}{2nr_s f_X(x)}\right)^{\frac{1}{n-1}}\right)\frac{1}{2r_s}$$

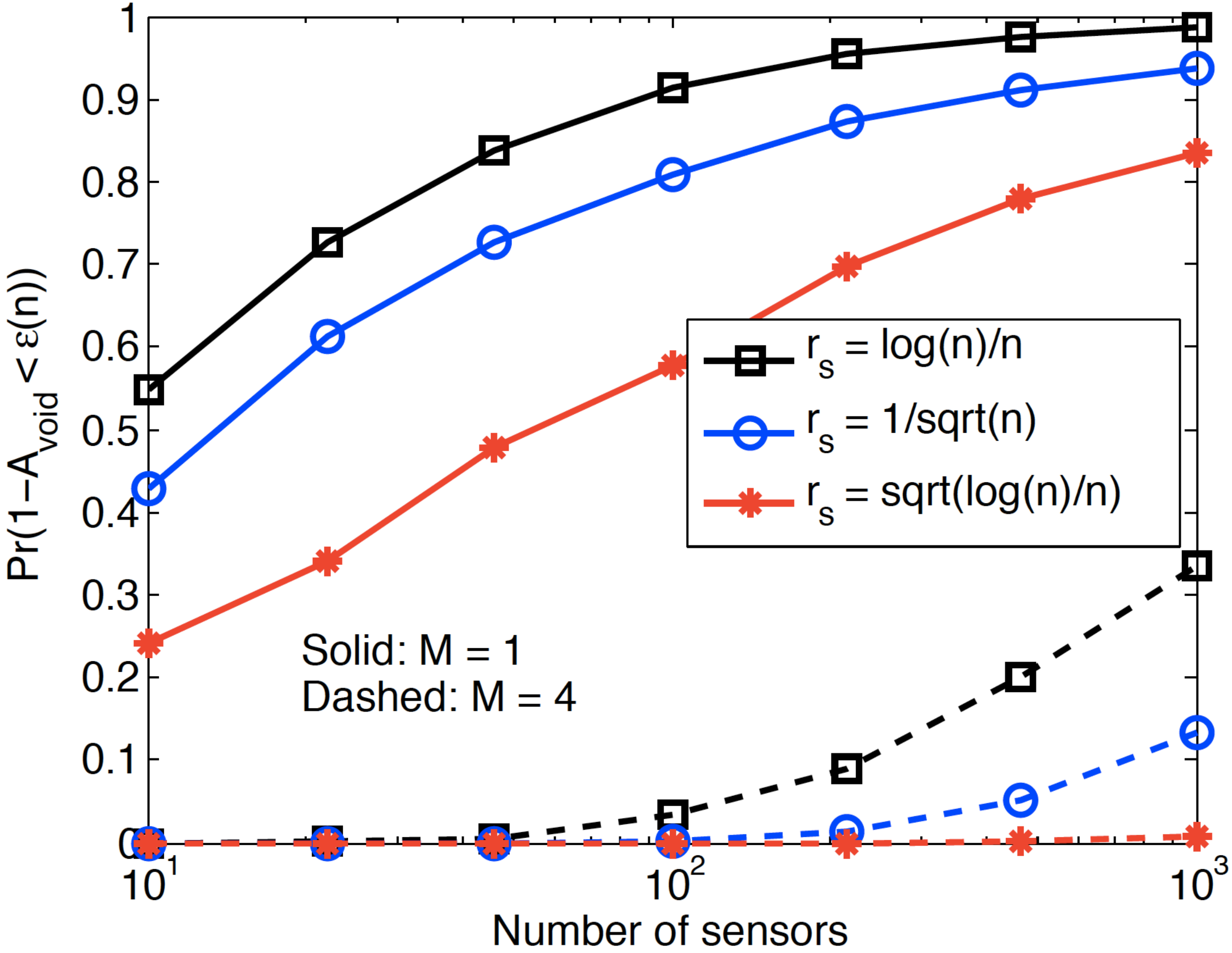
Minimum P_f

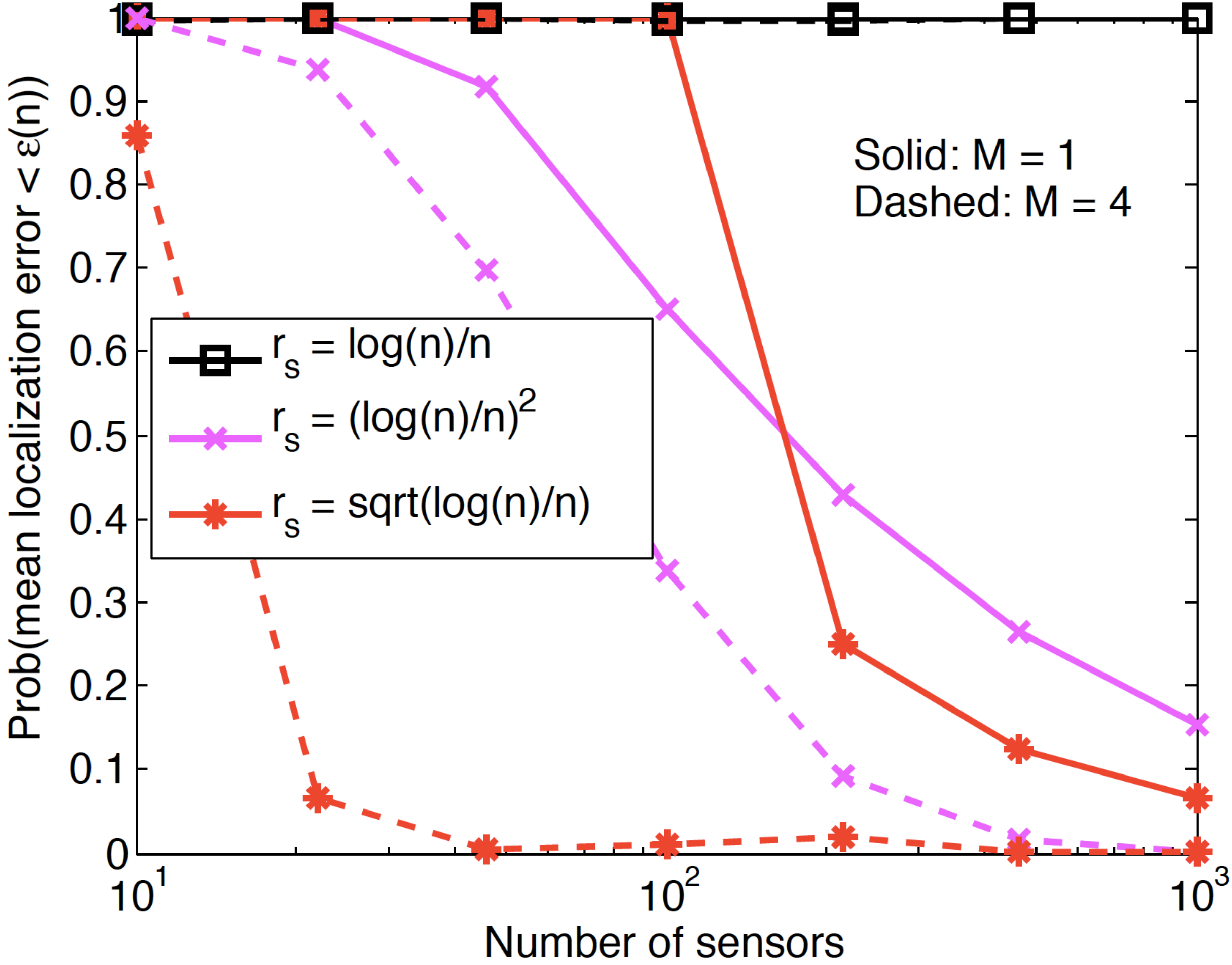
- Can show that

$$P_f^{(\mathrm{opt})} = \frac{(1-2r_s)^n}{\left[\int_0^1 (f_X(x))^{-\frac{1}{n-1}} \mathrm{d}x\right]^{n-1}}$$

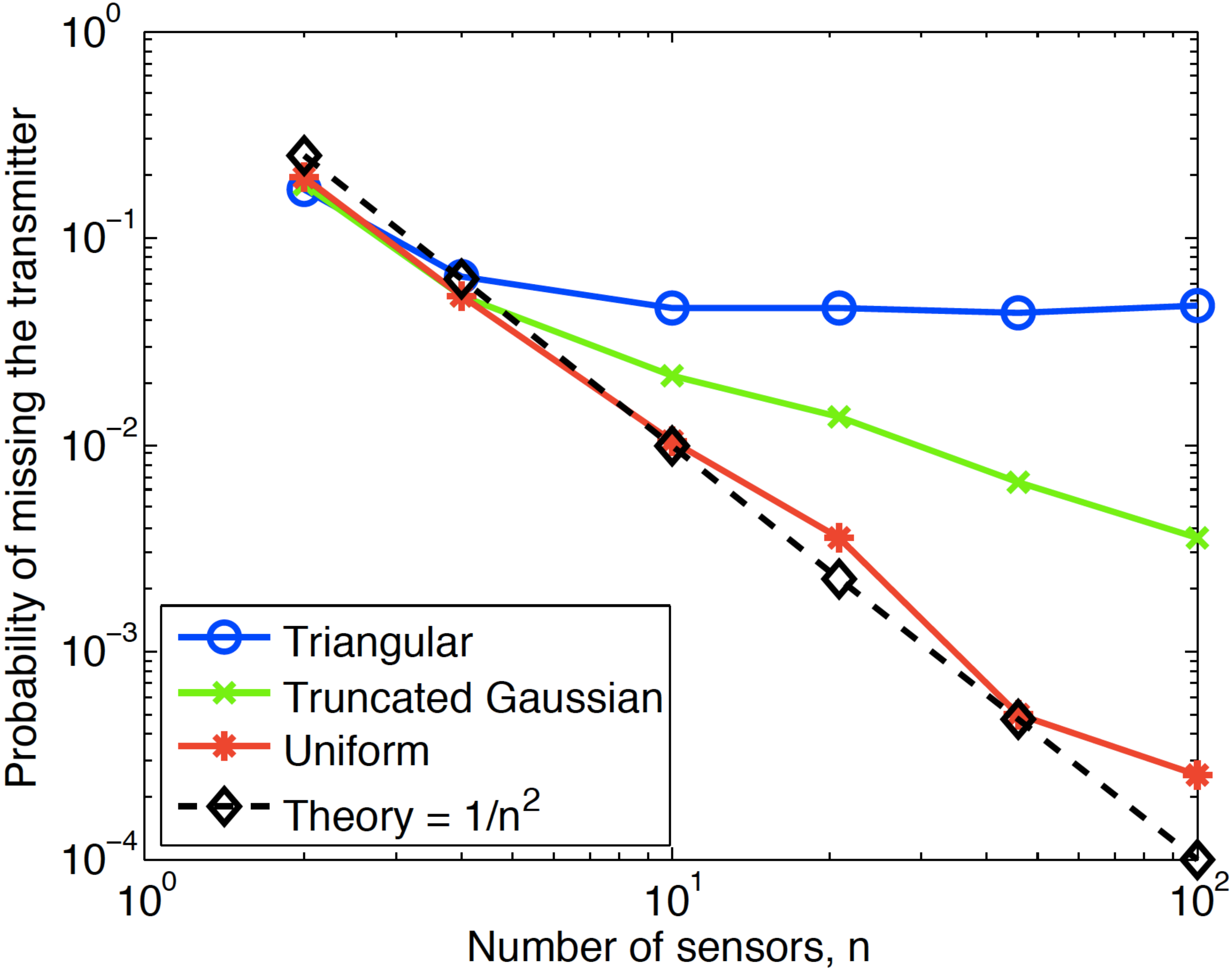
- When $r_s = (\log n)/n$, P_f is inversely proportional to n^2

Simulation Results





Prob(mean localization error < ε(n))
Number of sensors
Solid: M = 1
Dashed: M = 4
r_s = log(n)/n
r_s = (log(n)/n)^2
r_s = sqrt(log(n)/n)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10^1
10^2
10^3



Probability of missing the transmitter
Number of sensors, n
Triangular
Truncated Gaussian
Uniform
Theory = 1/n^2

Conclusions

- Derived limiting behavior of whitespace recovery error as $n \to \infty$
- Whitespace recovery error and sensor radio range optimally scale as $\log n/n$
- Extensions
  - Unreliable sensors
  - Transmitter localization error
  - Optimal distribution of sensors
- Future work
  - Handling fading/shadowing
  - Temporal variations
  - Sensor/primary transmitter mobility