Zero-Crossings Based Spectrum Sensing Under Noise Uncertainties

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- Motivation
- System model
- Weighted Zero-Crossings based Detector (WZCD)
- Robustness to noise uncertainties : parameter, model
- Simulation results

Spectrum Sensing

- In most of the CR-SS literature,

$$\begin{aligned} \mathcal{H}_0 &: \text{(signal absent)}\, Y_i = n_i \\ \mathcal{H}_1 &: \text{(signal present)}\, Y_i = h_i\, s_i + n_i,\ i = 1, 2, \cdots, M \end{aligned}$$

- Classical Goodness-of-Fit Test formulation

$$\mathcal{H}_0 \quad : \quad Y_i \sim f_{\mathbb{N}},\ i \in \mathcal{M}$$

- Threshold : chosen s.t. for $\alpha_f \in [0, 1]$,

$$p_f \triangleq \mathcal{P}\{\text{reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is true}\} \leq \alpha_f.$$

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- Receiver noise in communication systems $\sim \mathcal{N}(0, \sigma_{\mathsf{G}}^2)$: bad assumption!
- In signal processing for telecommunication systems, under $\mathcal{H}_0$: [Middleton1999]

$$Y_i = Y_i^{(\mathcal{G})} + Y_i^{(\mathcal{A})} + Y_i^{(\mathcal{B})}$$

Gaussian Class A Class B

$$(1-\epsilon) f_{\mathcal{G}} + \epsilon f_{\mathcal{I}} \qquad \mathcal{S}\alpha\mathcal{S}(\alpha)$$

- Approximations due to [Vatsola1984] and [Middleton1999] (and earlier works of Middleton)

- As seen earlier,
  - $Y_i^{(\mathcal{G})} + Y_i^{(\mathcal{A})} \sim (1-\epsilon)f_{\mathcal{G}} + \epsilon f_{\mathcal{I}}$, with $0 < \epsilon \ll 1$.
    - $f_{\mathcal{G}} \stackrel{d.}{=} \mathcal{N}(0, \sigma_G^2)$
    - $f_{\mathcal{I}} \stackrel{d.}{=} \mathcal{N}(0, \sigma_I^2)$ [Vatsola1984], [AazhangPoor1987], or
      $f_{\mathcal{I}} \stackrel{d.}{=} \mathcal{L}(0, \sigma_I^2)$ [MillerThomas1976], with $\sigma_I^2/\sigma_G^2 \in (10, 100)$.
  - $Y_i^{(\mathcal{B})} \sim \mathcal{S}\alpha\mathcal{S}(\alpha)$
    - Characteristic function of the $\mathcal{S}\alpha\mathcal{S}(\gamma_0, \alpha)$ distribution

$$\Phi_B(w, \gamma_0, \alpha) = \exp\left(-\gamma_0|w|^{\alpha}\right), \quad \gamma_0 > 0,\ 0 < \alpha \leq 2. \qquad (1)$$

    - No closed form for PDF; except for the cases $\alpha = 2$ (Gaussian), $\alpha = 1$ (Cauchy) and $\alpha = 0.5$ (Lévy)
- Should not ignore the Gaussian component! (opposed to [Chavali2012], and references therein)

Noise Uncertainties

- Noise Model Uncertainty (NMU)
  - Uncertainty in knowledge of $f_{\mathbb{N}} \Rightarrow$ either class A, B or both, and PDF of $f_{\mathcal{I}}$
- Noise Parameter Uncertainty (NPU)
  - Uncertainty in the parameter set $(\sigma_{\mathrm{G}}^2, \sigma_{\mathrm{I}}^2, \epsilon, \alpha)$

An Existing Technique

- Proposed by [Shen et al. 2011], called the Blind Detector (BD).
- Robust to the classical noise variance uncertainty, when $f_{\mathbb{N}} \stackrel{d.}{=} \mathcal{N}(0, \sigma_{\mathrm{G}}^2)$
- M observations are divided into n windows of m samples each. Choose n to be small.
- Calculate sample mean and sample variances from each window. Their ratio is student-t distributed with parameter $m - 1$.
- Over the obtained n samples, run an Anderson-Darling GoFT.

Weighted Zero-Crossings based Sensing

- In an earlier work, [KedemSlud1982] have studied for the Gaussian case (both i.i.d. and correlated)
- Consider the first k order difference operators:

$$
\begin{aligned}
\nabla Y_i &\triangleq Y_i - Y_{i-1} \\
\nabla^2 Y_i &= \nabla(\nabla Y_i) = Y_i - 2Y_{i-1} + Y_{i-2} \\
&\vdots \\
\nabla^k Y_i &= \sum_{j=0}^{k} \binom{k}{j} (-1)^j Y_{i-j}, \quad i \geq k+1. \qquad (2)
\end{aligned}
$$

Weighted Zero-Crossings based Sensing

- The k^{th} order ZC is said to occur, if the sign of $\nabla^{k-1} Y_i$ is different from that of $\nabla^{k-1} Y_{i+1}$

$$\Delta_{j,M} \triangleq \begin{cases} D_{1,M}, & j = 1, \\ D_{j,M} - D_{j-1,M}, & j = 2, \cdots, k-1 \\ (M-1) - D_{k-1,M}, & j = k, \end{cases} \quad (3)$$

$$\mu_{j,M} \triangleq \mathbb{E}\Delta_{j,M}, j = 1, \cdots, k, \quad (4)$$

- For a given set of weights w_j, a Ψ_w^2 Statistic-based Detector (Ψ_wSD) is given by

$$\Psi_w^2 \underset{\sim\mathcal{H}_0}{\overset{\nsim\mathcal{H}_0}{\gtrless}} \tau_{\text{w}}^{\Psi}, \quad (5)$$

- We consider the following cases for comparison with BD
- Equal and unit weights: [KedemSlud1982] $\Psi_1^2 \sim \chi_3^2(11)$. Choose the threshold τ_1^Ψ such that

$$Q_{\frac{3}{2}}\left(\sqrt{11}, \sqrt{\tau_1^\Psi}\right) = \alpha_f, \tag{6}$$

- Exponential weights: $\Psi_w^2 \sim \mathcal{F}(17.5, 7)$. Choose the threshold $\tau_{\mathrm{m\Psi SD}}$ such that

$$1 - \mathcal{I}\left(\frac{17.5\tau_{\mathrm{m\Psi SD}}}{17.5\tau_{\mathrm{m\Psi SD}} + 7}, 8.75, 3.5\right) = \alpha_f, \tag{7}$$

- Note that the statistic and the threshold are independent of variance of $f_{\mathbb{N}}$!

Robustness to NPU and NMU: Intuition

- When $f_{\mathcal{I}} \sim \mathcal{N}(0, \sigma_{\mathrm{G}}^2)$, no problem at all.
- The PDF of any member of the $\mathcal{S}\alpha\mathcal{S}$ family, for $1 \leq \alpha \leq 2$ can be written as [West1987]

$$p(X) = \int_0^\infty \left(\frac{1}{\sigma}\right) g\left(\frac{X}{\sigma}\right) h(\sigma)\, d\sigma, \tag{8}$$

- where $g(\cdot)$ is the standard Gaussian PDF, and $h : \mathbb{R}^+ \to \mathbb{R}^+$ is some function (can as well be a PDF).
- West extends this result to exponential family, which includes the Laplace distribution.

More Advantages

- For large M, the number of zero-crossings for any symmetric distribution is $M/2$.
- Works with distributions with infinite variance.
- Works with distributions with infinite mean! $\Rightarrow$ works as long as the median exists.
- Given that it is a GoFT, can be used with any signal and fading models.
- Computational complexity: same as the energy detector, and less intense that the blind detector.
- One disadvantage

Primary, and Fading models for simulations

- Primary signal models
  - Model 1 : Constant primary
  - Model 2 : Sinusoid primary
- Fading models
  - Rayleigh fading (i.i.d., and first order ARMA correlated)

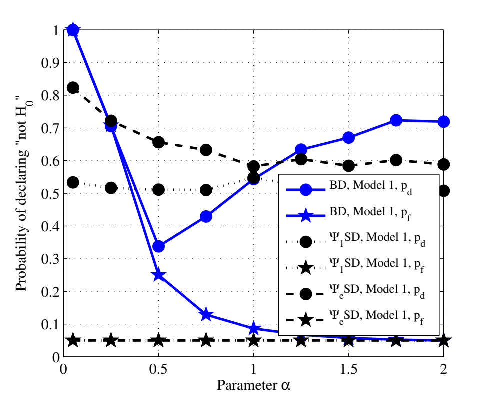

Figure: Detection of primary model 1 under Rayleigh fading, with Gaussian + $\mathcal{S}\alpha\mathcal{S}$ model.

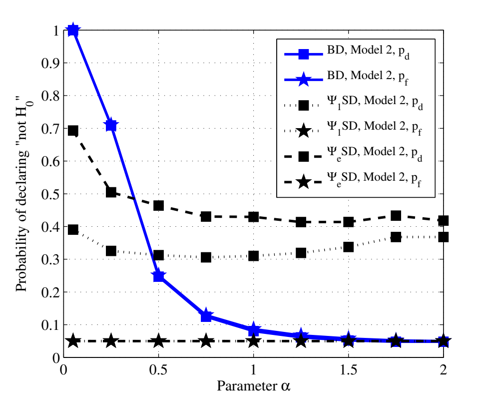

Figure: Detection of primary model 2 under Rayleigh fading, with Gaussian + $\mathcal{S}\alpha\mathcal{S}$ model.

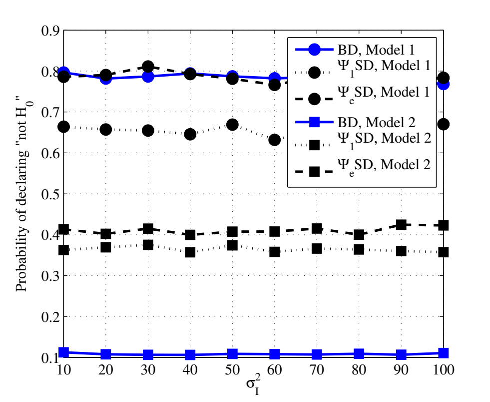

Figure: Detection of primary models 1 and 2 under Rayleigh fading, with ϵ-mixture model, $\epsilon = 0.05$, and $f_{\mathcal{I}} \sim \mathcal{N}(0, \sigma_{\mathrm{I}}^2)$.

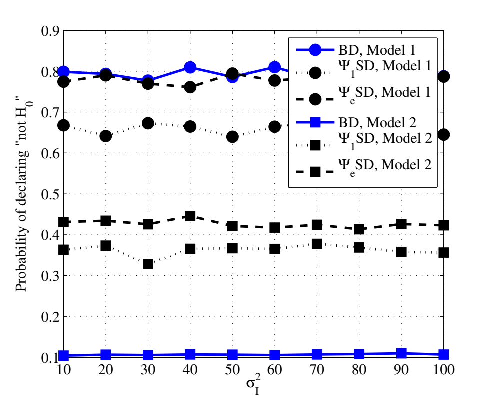

Figure: Detection of primary models 1 and 2 under Rayleigh fading, with ϵ-mixture model, $\epsilon = 0.05$, and $f_{\mathcal{I}} \sim \mathcal{L}(\sigma_{\mathrm{I}}^2)$.

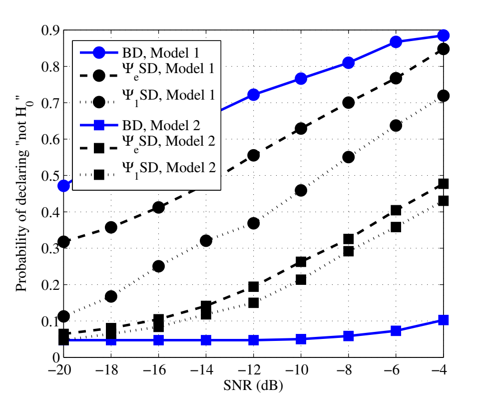

Figure: Detection of primary models 1 and 2 under pure Gaussian noise, with noise variance uncertainty$=3dB$, $M=300$, $\alpha_f=0.05$. Average p_f obtained through simulations for BD, ΨSD and mΨSD are 0.0498, 0.05, and 0.0501, respectively.

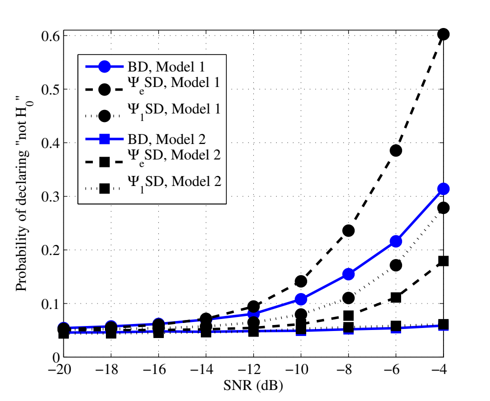

Figure: Detection of primary models 1 and 2 under first order AR correlated fading (with $\rho = 0.5$) and pure Gaussian Noise, with noise variance uncertainty$= 3dB$, $M = 300$, $\alpha_f = 0.05$.

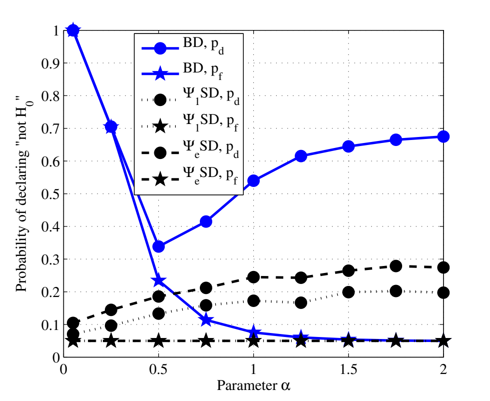

Figure: Detection of primary model 1 under Gaussian + class A + class B noises, with noise variance uncertainty$= 3dB$, $M = 300, \alpha_f = 0.05$, $\epsilon = 0.05$, $f_{\mathcal{I}} \sim \mathcal{N}(0, 100\sigma_G^2)$.

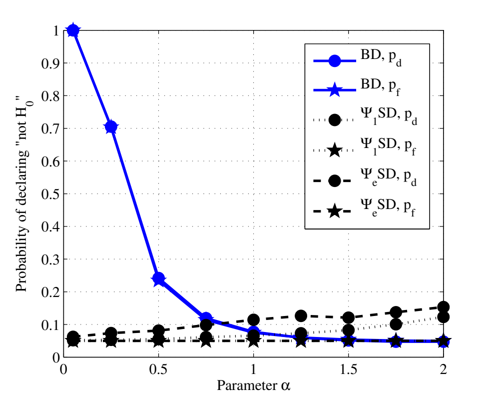

Figure: Detection of primary model 2 under Gaussian + class A + class B noises, with noise variance uncertainty$= 3dB$, $M = 300$, $\alpha_f = 0.05$, $\epsilon = 0.05$, $f_{\mathcal{I}} \sim \mathcal{N}(0, 100\sigma_G^2)$.

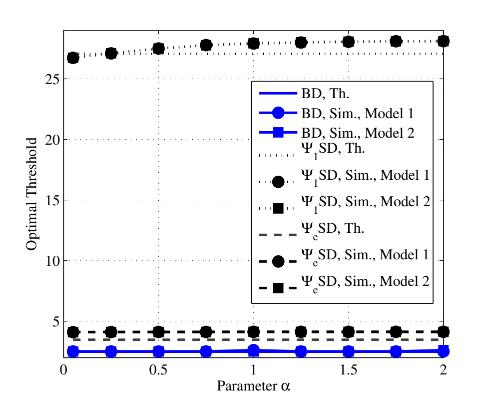

Figure: Optimal threshold calculation under Gaussian + class A + class B noises, with noise variance uncertainty$= 3dB$, $M = 300$, $\alpha_f = 0.05$, $\epsilon = 0.05$, $f_{\mathcal{I}} \sim \mathcal{N}(0, 100\sigma_{\mathrm{G}}^2)$.