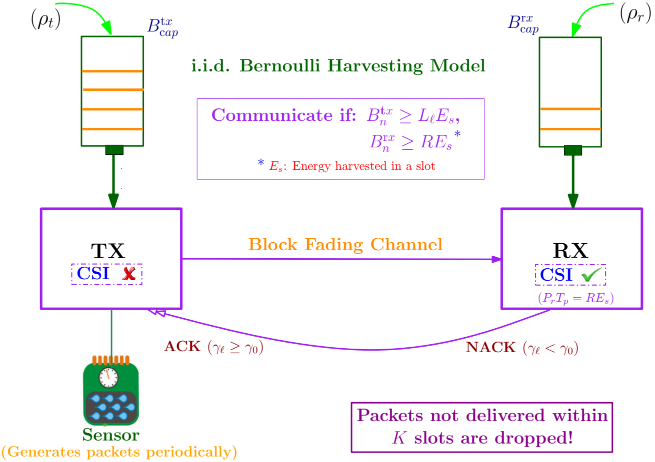
On Optimal Transmit Power Policies for Dual EH Links with Retransmissions

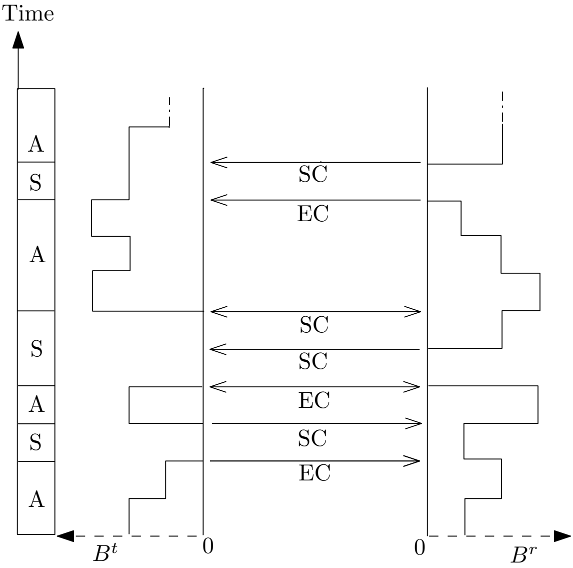
October 12, 2015

Outline

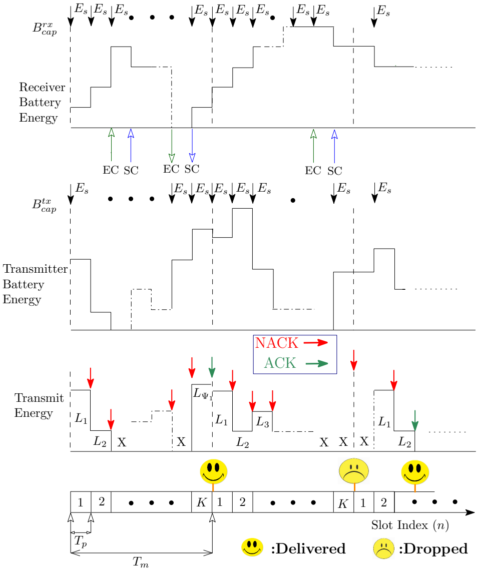
System Model



CSWP Protocol



System Dynamics



Goal

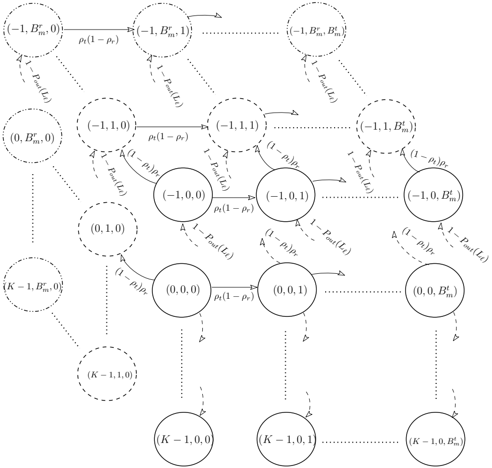
- Battery evolution follows *energy neutrality constraint*

$$B_{n+1}^t = \begin{cases} \min(B_n^t + 1 - L_\ell \mathbb{1}_{\{L_\ell \geq B_n^t, R \geq B_n^r\}}, B_{\max}^t), & \text{if energy harvested in } n^{\text{th}} \text{ slot,} \\ B_n^t - L_\ell . \mathbb{1}_{\{L_\ell \geq B_n^t, R \geq B_n^r\}}, & \text{if no energy harvested in } n^{\text{th}} \text{ slot} \end{cases}$$

- Given
  - harvesting profiles (ρ_t, E_s) and (ρ_r, E_s)
  - Battery size $(B_{\max}^t, B_{\max}^r)$
  - Channel statistics
  - Channel coherence time (Slow/Fast)
  - Retransmission protocol (ARQ/HARQ-CC)

- Goal is to find optimal policy $\mathcal{L} = \{L_1, L_2, \ldots, L_K\}$ s.t. PDP is minimized

PDP Analysis



PDP Analysis

- DTMC state in n^{th} slot (B_n^t, B_n^r, u_n)
- $u_n \in \{-1, 0 \dots, K-1\}$
  - $u_n = 0$: New packet
  - $u_n = -1$: ACK received
  - $u_n = r$: r failed attempts
- PDP

$$
\begin{aligned}
P_{\mathrm{D}}(K) &= \sum_{(i,j)} \pi(i,j) P_{\mathrm{D}}(K|i,j,u_n=0), \qquad \text{where} \\
P_{\mathrm{D}}(K|i,j,u_n=0) &= \sum_{m_t=0}^{K} \sum_{m_r=0}^{K} \binom{K}{m_t} \binom{K}{m_r} \rho_t^{m_t} \rho_r^{m_r} (1-\rho_t)^{K-m_t} \\
&\qquad (1-\rho_r)^{K-m_r} p_{\mathrm{D}}(i,j,m_t,m_r)
\end{aligned}
$$

$\pi(i,j)$

- Obtained by solving the Chapman-Kolomogrov equation using the transition probabilities
- Transition probabilities : $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $R-1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, , s = r + 1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{else.} \end{cases}$$

Problem formulation

- Goal:

$$\begin{aligned}\text{minimize}_{\mathcal{L}=\{L_1,L_2,\ldots,L_K\}} \quad & P_{\text{D}}(K) = \sum_{(i,j)} \pi(i,j) P_{\text{D}}(K|i,j,u_n = 0) \\ \text{subject to} \quad & 0 \le L_i \le L_{\max} \\ \text{and} \quad & \text{energy neutrality constraint}\end{aligned}$$

- Challenges
  - Objective is nonconvex
  - Feasibility set is collection of all policies resulting in sample paths over which DTMC evolution satisfies the energy neutrality constraint

Reformulation

- Let $\mathcal{I} = \{(i,j)|0 \leq i \leq B^t_{max}, 0 \leq i \leq B^r_{max}\}$
- Consider $\mathcal{I} = \mathcal{I}_1 \bigcup \mathcal{I}_2$, where

$$\mathcal{I}_2 : \text{ set of states where all } K \text{ attempts possible}$$

- Bounds on objective function (Is this tight?)

$$P_{\text{D}}\left(K|(i_2,j_2)\in\mathcal{I}_2, r=0\right)\Big|_{\bar{\mathbf{P}}^*} \leq P_D(K) < P_{\text{D}}\left(K|(i_2,j_2)\in\mathcal{I}_2, r=0\right)\Bigg|_{\bar{\mathbf{P}}^*} + \sum_{(i_1,j_1)\in\mathcal{I}_1} \pi(i_1,j_1)\Big|_{\bar{\mathbf{P}}^*}$$

where

$$\bar{\mathbf{P}}^* = \arg\min_{\bar{\mathbf{P}}} P_{\text{D}}\left(K|(i_2,j_2) \in \mathcal{I}_2, r=0\right)$$

- Can we obtain a nice feasibility set?

Harvesting Unconstrained Regime

For a given EH profile and channel,

- ARQ and slow fading

$$\frac{1}{K}\sum_{t=1}^{K} L_t p_{\text{out}}(L_{t-1}) < \rho_t, \text{ and } \frac{R}{K}\sum_{t=1}^{K} p_{\text{out}}(L_{t-1}) < \rho_r,$$

- ARQ and fast fading

$$\frac{1}{K}\sum_{t=1}^{K} L_t \prod_{p=1}^{t-1} p_{\text{out}}(L_p) < \rho_t, \text{ and } \frac{R}{K}\sum_{t=1}^{K}\prod_{p=1}^{t-1} p_{\text{out}}(L_p) < \rho_r,$$

- HARQ-CC

$$\frac{1}{K}\sum_{k=1}^{K} L_k p_{\text{out}}(k-1) \leq \rho_t, \text{ and } \frac{R}{K}\sum_{k=1}^{K} p_{\text{out}}(k-1) \leq \rho_r,$$

respectively, where $p_{\text{out}}(k-1)$ denotes the probability of outage in $(k-1)^{th}$ attempt

Percentage difference between lower and upper bound

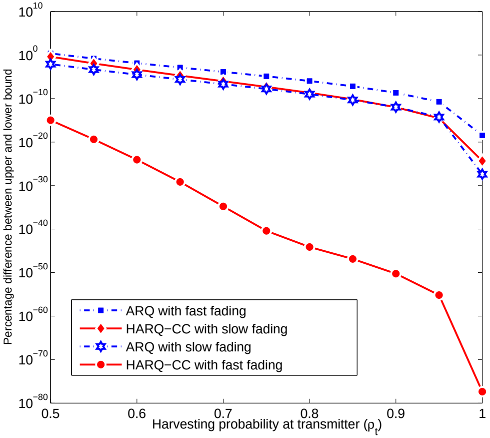


Figure: The parameters are $E_s = 5$ dB, $\gamma_0 = 10$ dB, $\rho_r = 0.9$, $R = 1$, $B^t_{\max} = B^r_{\max} = 25$ and $K = 4$. The policy used is [1 1 2 2].

Theoretical Guarantees

- For mono EH links, in HUR $\sum_{(i_1,j_1)\in\mathcal{I}_1} \pi(i_1,j_1) = \Theta(e^{-r^*|\mathcal{I}_2|})$ proof: will discuss in end.
- For dual EH links : ? (work in progress)

Reformulated Problem

$$\min_{\bar{\mathbf{L}}=\{L_1,\ldots,L_K\}} p_{\mathrm{D}}(i,j,m_t,m_r),$$

$$\text{subject to} \quad \sum_{\ell=1}^{K} L_\ell . p_{\mathrm{out}}(\ell-1) \leq K\rho_t,$$

$$\sum_{\ell=1}^{K} \chi^\ell . p_{\mathrm{out}}(\ell-1) \leq \frac{K\rho_r}{R},$$

$$0 \leq L_i \leq L_{\max} \qquad \forall \quad 1 \leq i \leq K,$$

where,

$$\chi^\ell = \begin{cases} 1, & \text{if } L_\ell \neq 0, \\ 0, & \text{otherwise}, \end{cases}$$

and $p_{\mathrm{out}}(\ell - 1)$ represents the probability of the event that the first $\ell - 1$ attempts are failed.

It is easy to observe, that solution only depends on $\chi = \sum_{\ell=1}^{K} \chi_\ell$ for e.g., for $K = 3$, and $\chi = 2$, the constraints written as

$$\chi^\ell = \begin{cases} \begin{cases} L_1 + L_2 p_{\text{out}}(L_1) \leq K\rho_t \\ 1 + p_{\text{out}}(L_1) \leq \frac{K\rho_r}{R} \end{cases} & \text{if } L_1 \neq 0, L_2 \neq 0, L_3 = 0 \\ \begin{cases} L_2 + L_3 p_{\text{out}}(L_2) \leq K\rho_t \\ 1 + p_{\text{out}}(L_2) \leq \frac{K\rho_r}{R} \end{cases} & \text{if } L_1 = 0, L_2 \neq 0, L_3 \neq 0 \\ \begin{cases} L_1 + L_3 p_{\text{out}}(L_1) \leq K\rho_t \\ 1 + p_{\text{out}}(L_1) \leq \frac{K\rho_r}{R} \end{cases} & \text{if } L_1 \neq 0, L_2 = 0, L_3 \neq 0 \end{cases}$$

Instead of 2^K nonconvex subproblems only K nonconvex subproblems need to be solved

ARQ and slow fading

$$\textbf{(P3)} \qquad \min_{\bar{\mathbf{L}}=\{L_1,\ldots,L_K\}} 1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{L_K E_s \sigma_c^2}\right)},$$

$$\text{subject to} \sum_{\ell=1}^{K} L_\ell \cdot \left(1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{L_{\ell-1} E_s \sigma_c^2}\right)}\right) \leq K\rho_t,$$

$$\sum_{\ell=1}^{K} \chi^\ell \cdot \left(1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{L_{\ell-1} E_s \sigma_c^2}\right)}\right) \leq \frac{K\rho_r}{R},$$

$$0 \leq L_1 \leq L_2 \ldots \leq L_K \leq L_{\max},$$

$$\chi^\ell \in \{0, 1\} \qquad \forall \quad 1 \leq \ell \leq K,$$

for $\chi = K'$, using the Taylor series expansion of e^{-x}, the problem is written as,

$$\min_{\bar{\mathbf{Z}}=\{Z_1,\ldots,Z_{K'}\}} Z_{K'},$$

$$\text{subject to} \quad \frac{\sum_{\ell=1}^{K'} Z_\ell^{-1} \cdot \left(\sum_{i=0}^{\infty} \frac{Z_{\ell-1}^{2i+1}}{(2i+1)!}\right)}{\frac{K\rho_t}{s} + \sum_{\ell=2}^{K'} Z_\ell^{-1} \cdot \left(\sum_{i=1}^{\infty} \frac{Z_{\ell-1}^{2i}}{(2i)!}\right)} \leq 1,$$

$$\frac{\sum_{\ell=1}^{K'} \left(\sum_{i=0}^{\infty} \frac{Z_{\ell-1}^{2i+1}}{(2i+1)!}\right)}{\frac{K\rho_r}{R} + \sum_{\ell=2}^{K'} \left(\sum_{i=1}^{\infty} \frac{Z_{\ell-1}^{2i}}{(2i)!}\right)} \leq 1,$$

$$0 \leq Z_1^{-1} \leq Z_2^{-1} \ldots \leq Z_{K'}^{-1} \leq L_{\max},$$

where, $s = \frac{\gamma_0 \mathcal{N}_0 T_P}{E_s \sigma_c^2}$, and $Z_\ell = \frac{s}{L_\ell}$.

Geometric Programming: Terminology

- Monomial: $f : \mathbf{R}^{\mathbf{n}}_{++} \to \mathbf{R}$:

$$f(\mathbf{x}) = d x_1^{a^{(1)}} x_2^{a^{(2)}} \dots x_n^{a^{(n)}}$$

where, $d \geq 0$, and $a^{(j)} \in \mathbf{R}, j = 1, \dots, n$

- Posynomial: Sum of monomials

$$f(\mathbf{x}) = \sum_{k=1}^{K} d_k x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}}$$

where, $d_k \geq 0$, $a_k^{(j)} \in \mathbf{R}$, $k = 1, 2, \dots, K$, $j = 1, 2, \dots, n$

- Examples:

$$\text{Posynomial : } \quad 2x_1^{-\pi} x_2^{0.5} + 3x_1 x_3^{100}, \frac{x_1}{x_2}$$

$$\text{Not a Posynomial : } \quad x_1 - x_2, \frac{x_1 + x_2}{x_3 + x_1}$$

Geometric Programming: Standard form

Standard form: (non convex problem)

$$\begin{aligned} \min \quad & f_0(\mathbf{x}) & & \\ \text{subject to} \quad & f_i(\mathbf{x}) & \leq 1 \quad \forall \quad & i = 1, 2, \dots, m \\ & h_\ell(\mathbf{x}) & = 1 \quad \forall \quad & i = 1, 2, \dots, M \end{aligned}$$

where,

$$f(\mathbf{x}) = \sum_{k=1}^{K_i} d_{ik} x_1^{a_{ik}^{(1)}} x_2^{a_{ik}^{(2)}} \dots x_n^{a_{ik}^{(n)}}$$

$$\text{and,} \quad h_\ell(\mathbf{x}) = d_\ell x_1^{a_\ell^{(1)}} x_2^{a_\ell^{(2)}} \dots x_n^{a_\ell^{(n)}}$$

Geometric Programming: Convex form

- Let $y_i = \log x_i$, $b_{ik} = \log d_{ik}$, and $b_\ell = \log d_\ell$

$$\begin{aligned}
\min \quad P_0(\mathbf{y}) &= \log \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \mathbf{y} + b_{0k}) \\
\text{s. t.} \quad P_i(\mathbf{y}) &= \log \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \mathbf{y} + b_{ik}) \leq 0 \quad \forall \quad i = 1, 2, \ldots, m \\
q_\ell(\mathbf{y}) &= \mathbf{a}_\ell^T \mathbf{y} + b_\ell = 0 \quad \forall \quad \ell = 1, 2, \ldots, M
\end{aligned}$$

- References:

  - M. Chiang, "Geometric Programming for Communication Systems," *Foundations and Trends of Communication and Information Theory*, vol. 2, no 1-2, pp 1-156, Aug. 2005
  - S. Boyd, S. J. Kim, L. Vandenberghe, and A. Hassibi, "A Tutorial on Geometric Programming,"*Optimization and Engineering*, pp 67-127, April 2007.

Approximation for posynomial ratio

- Let $g(\mathbf{x}) = \sum_i u_i(\mathbf{x})$. Approximate a ratio of polynomials $\frac{f(\mathbf{x})}{g(\mathbf{x})}$ with $\frac{f(\mathbf{x})}{\tilde{g}(\mathbf{x})}$ where

$$\tilde{g}(\mathbf{x}) = \prod_i \left(\frac{u_i(\mathbf{x})}{\alpha_i} \right)^{\alpha_i} \leq g(\mathbf{x})$$

- Directly follows from AM-GM inequality $\sum_i \alpha_i v_i \geq \prod_i v_i^{\alpha_i}$
- If, $\alpha_i = \frac{u_i(\mathbf{x_0})}{g(\mathbf{x_0})} \quad \forall \quad i$, for any fixed $\mathbf{x}_0 > 0$, then

$$\tilde{g}(\mathbf{x}_0) = g(\mathbf{x}_0)$$

- $\tilde{g}(\mathbf{x}_0)$, it is the best local monomial approximation $g(\mathbf{x}_0)$ near x_0, in the sense of first order Taylor approximation.

Algorithm to Solve Complementary GP

1. **Initialize**: $\mathbf{Z}^0 = \{Z_1, Z_2, \ldots, Z_{K'}, 0, \ldots, 0\}$, where $\mathbf{Z}^0$ is any feasible vector.
2. $p \leftarrow 0$
3. Evaluate the denominator posynomial, $G_a(\mathbf{Z})$, and $G_b(\mathbf{Z})$ in constraints with the given $\mathbf{Z}^p$.
4. For each term $\mathbf{V}_\ell^q$ in the denominator posynomials, compute $\beta_\ell^q = \frac{V_\ell^q(\mathbf{Z}^p)}{G_q(\mathbf{Z}^p)}$, where $q = a, b$
5. Approximate the denominator posynomial of constraints into a monomial with weights β_ℓ^q
6. Solve the resulting GP, and $p \leftarrow p + 1$.
7. Go to step 3, and use $\mathbf{Z}^p$ obtained in step 6

Algorithm to find optimal policies

1. For $K' = 1$ to K
2. Use previous Algorithm to solve for corresponding subproblem
3. If$Z_{K'} \leq Z_{K'-1}$ then $\mathbf{Z}_{\text{opt}} \leftarrow \mathbf{Z}^p = \{Z_1, Z_2, \ldots, Z_{K'}, 0, \ldots, 0\}$

ARQ and fast fading channel

(P4)

$$\min_{\bar{\mathbf{L}}=\{L_1,\ldots,L_K\}} \prod_{i=1}^{K} \left(1 - e^{-\left(\frac{s}{L_i}\right)}\right),$$

subject to

$$\sum_{\ell=1}^{K} L_\ell . \prod_{i=1}^{\ell-1} \left(1 - e^{-\left(\frac{s}{L_i}\right)}\right) \leq K\rho_t,$$

$$\sum_{\ell=1}^{K} \chi^\ell . \prod_{i=1}^{\ell-1} \left(1 - e^{-\left(\frac{s}{L_i}\right)}\right) \leq \frac{K\rho_r}{R},$$

$$0 \leq L_i \leq L_{\max}, \qquad \forall \quad 1 \leq i \leq K$$

$$\chi^\ell \in \{0, 1\}, \qquad \forall \quad 1 \leq \ell \leq K.$$

Transformation to CGP

$$\min_{\bar{\mathbf{z}}=\{t,Z_1,\dots,Z_{K'}\}} \quad t,$$

subject to

$$\prod_{\ell=1}^{K'}(A_\ell - B_\ell) \le t,$$

$$\frac{Z_1^{-1} + Z_2^{-1}A_1 + Z_3^{-1}(A_1A_2 + B_1B_2) + \cdots}{\frac{K\rho_t}{s} + Z_2^{-1}B_1 + Z_3^{-1}(A_1B_2 + A_2B_1) + \cdots} \le 1,$$

$$\frac{1 + A_1 + A_1A_2 + B_1B_2 + \cdots}{\frac{K\rho_r}{R} + B_1 + (A_1B_2 + A_2B_1) + \cdots} \le 1,$$

$$0 \le s.Z_i^{-1} \le L_{\max}, \qquad \forall \quad 1 \le i \le K,$$

where $A_\ell = \sum_{i=0}^{\infty} \frac{Z_\ell^{2i+1}}{(2i+1)!}$, $B_\ell = \sum_{i=1}^{\infty} \frac{Z_\ell^{2i}}{(2i)!}$, and $Z_\ell = \frac{s}{L_\ell}$

HARQ-CC slow fading channel

$$\textbf{(P5)} \qquad \min_{\bar{\mathbf{L}}=\{L_1,\ldots,L_K\}} 1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{E_s \sigma_c^2 \sum_{\ell=1}^{K} L_\ell}\right)},$$

$$\text{subject to} \sum_{\ell=1}^{K} L_\ell . \left(1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{E_s \sigma_c^2 . \sum_{i=1}^{\ell-1} L_i}\right)}\right) \leq K\rho_t,$$

$$\sum_{\ell=1}^{K} \chi^\ell . \left(1 - e^{-\left(\frac{\gamma_0 \mathcal{N}_0 T_P}{E_s \sigma_c^2 . \sum_{i=1}^{\ell-1} L_i}\right)}\right) \leq \frac{K\rho_r}{R},$$

$$0 \leq L_i \leq L_{\max}, \qquad \forall \quad 1 \leq i \leq K,$$

$$\chi^\ell \in \{0, 1\}, \qquad \forall \quad 1 \leq \ell \leq K.$$

Transformation to CGP

$$\min_{\bar{\mathbf{z}}=\{Z_1,\ldots,Z_K'\}} Z_{K'},$$

subject to

$$\frac{\sum_{\ell=1}^{K'} Z_\ell^{-1} . \left(\sum_{i=0}^{\infty} \frac{Z_{\ell-1}^{2i+1}}{(2i+1)!} + \sum_{i=1}^{\infty} \frac{Z_\ell^{2i}}{(2i)!}\right)}{\frac{K\rho_t}{s} + \sum_{\ell=1}^{K'} Z_\ell^{-1} . \left(\sum_{i=0}^{\infty} \frac{Z_\ell^{2i+1}}{(2i+1)!} + \sum_{i=1}^{\infty} \frac{Z_{\ell-1}^{2i}}{(2i)!}\right)} \leq 1,$$

$$\frac{\sum_{\ell=1}^{K'} \left(\sum_{i=0}^{\infty} \frac{Z_{\ell-1}^{2i+1}}{(2i+1)!}\right)}{\frac{K\rho_r}{R} + \sum_{\ell=2}^{K'} \left(\sum_{i=1}^{\infty} \frac{Z_{\ell-1}^{2i}}{(2i)!}\right)} \leq 1,$$

$$Z_i . Z_{i-1}^{-1} \leq 1, \qquad \forall \quad 1 \leq i \leq K',$$

$$\frac{Z_i^{-1}}{Z_{i-1}^{-1} + \frac{L_{\max}}{s}} \leq 1, \qquad \forall \quad 1 \leq \ell \leq K',$$

where, $Z_{K'} = \frac{s}{\sum_{i=1}^{K'} L_i}$

HARQ-CC and fast fading channel

$$p_{\mathrm{D}}(i,j,m_t,m_r) = 1 - \sum_{i=1}^{M_{\Psi_1}} \sum_{j=1}^{\tau_i} \sum_{k=0}^{j-1} \frac{\chi_{i,j}(\mathbf{L}_{\Psi_1})}{k!} \left(\frac{X}{L_{\{i\}}}\right)^k e^{-\left(\frac{x}{L_{\{i\}}}\right)},$$

where, $X = \frac{\gamma_0 \mathcal{N}_0 T_p}{E_s}$, $\mathbf{L}_{\Psi_1} = \mathrm{diag}(\frac{L_1}{\sigma_c^2}, \frac{L_2}{\sigma_c^2}, \ldots, \frac{L_{\Psi_1}}{\sigma_c^2})$, and $L_{\{1\}}, L_{\{2\}}, \cdots, L_{\{M_{\Psi_1}\}}$ denote the distinct nonzero elements of $\mathbf{L}_{\Psi_1}$. τ_i denotes the multiplicity of $L_{\{i\}}$, and $\chi_{i,j}(\mathbf{L}_{\Psi_1})$ denotes the (i,j)th characteristics coefficients of $\mathbf{L}_{\Psi_1}$ such that

$$\begin{aligned} \det(\mathbf{I} + u\mathbf{L}_{\Psi_1})^{-1} &= \frac{1}{(1+uL_{\{1\}})^{\tau_1} \cdots (1+uL_{\{M_{\Psi_1}\}})^{\tau_{M_{\Psi_1}}}}, \\ &= \sum_{i=1}^{M_{\Psi_1}} \sum_{j=1}^{\tau_1} \frac{\chi_{i,j}(\mathbf{L}_{\Psi_1})}{(1+uL_{\{i\}})^j}, \end{aligned}$$

where u is a scalar constant such that $\det(\mathbf{I} + u\mathbf{L}_{\Psi_1}) \neq 0$.

Approximation for GP

- In general, some of the powers L_i, $1 \leq i \leq K$ may be equal or distinct.
- Depending on the number of distinct, and similar elements in $\bar{\mathbf{L}} = \{L_1, \ldots, L_K\}$, the objective and constraints in above problem would have different expressions, for e.g. for $\ell = 2$

$$p_{out}(\ell) = \begin{cases} 1 - e^{-\left(\frac{X}{L_1\sigma_c^2}\right)}\left[1 + \frac{X}{L_1\sigma_c^2}\right] & \text{if} \quad L_1 = L_2 \\ 1 - \left[\frac{L_1 e^{-\left(\frac{X}{L_1\sigma_c^2}\right)} - L_2 e^{-\left(\frac{X}{L_2\sigma_c^2}\right)}}{L_1 - L_2}\right] & \text{if} \quad L_1 \neq L_2 \end{cases}$$

- Using a result from [Chaitanya et.al. Theorem 1], we can approximate the $p_{out}(\ell)$, as following,

$$p_{out}(\ell) \approx \frac{X^\ell}{\ell! L_1 L_2 \cdots L_\ell}. \tag{8}$$

Simulation Result

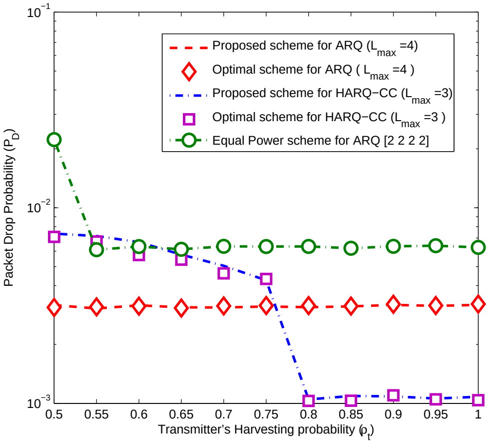


Figure: Performance comparison of proposed scheme with equal power allocation, and optimal power allocation for slow fading channels with ARQ, and HARQ-CC. Parameters chosen are $E_s = 12$ dB, $\gamma_0 = 10$ dB, $K = 4$, $B^t_{\max} = B^r_{\max} = 40E_s$.

Simulation Result

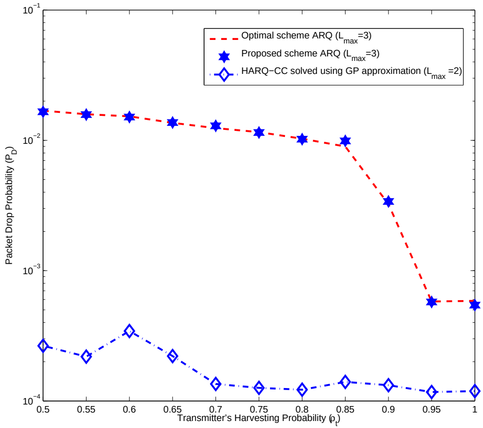


Figure: Parameters chosen are $E_s = 5$ dB, $\gamma_0 = 12$ dB, $K = 4$ while the energy buffer size, are $B^t_{\max} = B^r_{\max} = 40E_s$, and $B^t_{\max} = B^r_{\max} = 60E_s$, for ARQ, and HARQ-CC, respectively.

Comparison with State-of-Art

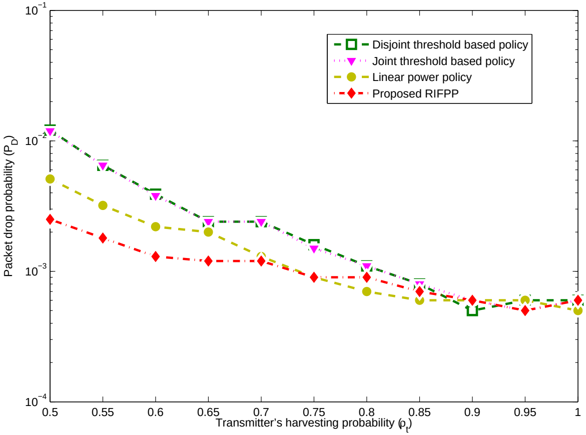


Figure: Parameters chosen are $E_s = 5$ dB, $\gamma_0 = 12$ dB, $K = 4$ while the energy buffer size, are $B^t_{\max} = 2000E_s$ $B^r_{\max} = 500E_s$.

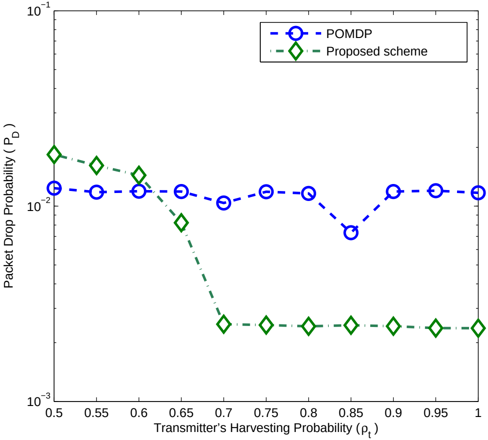
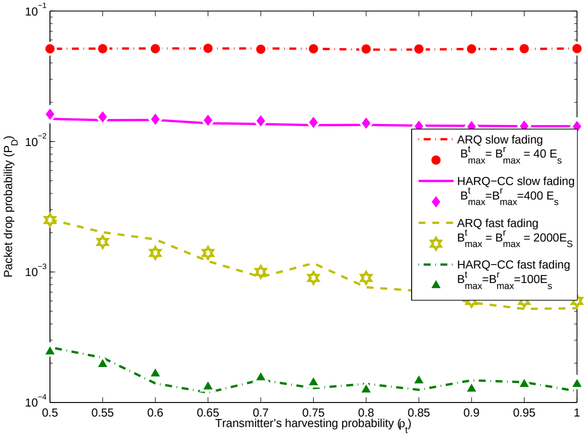


Figure: Performance comparison with the policies designed using POMDP assuming the access to perfect SoC information, provided with inaccurate SoC information. The root-mean-square, and the maximum error, in the SoC estimation, are assumed to be 5%, and 30%, respectively. In HUR, the policies obtained using proposed shceme performs better in comparison of optimal policy. Parameters chosen are $E_s = 5$ dB, $\gamma_0 = 12$ dB while number of transmissions and energy buffer size are $K = 4$ and $B^t_{\max} = 20E_s$, respectively, and $L_{\max} = 2$.

Convergence of utility to infinite battery



"Well, Ladies and Gentlemen, I'm sure my little talk has made you all think."