E9 203: Homework - 2

Assigned on: 31 Jan. 2020

Topic

- Experiments in sparse signal recovery[1]

Problems

Consider the tone signal example we discussed in class:

$$y(t) = \sum_{i=1}^{k} a_k \sin(2\pi f_k t + \theta_k), \quad t \in [0, 1].$$

Sample the signal at 512 samples per second to get the samples $y[k] = y(kT_s), k = 0, 1, \ldots, 511$, where $T_s = 1/512$s. Let $\mathbf{y}$ denote a column vector containing the $N = 512$ samples.

Write a Matlab script that lets you pick m, the number of measurements, k, the number of sinusoidal components, $f_i, i = 1, 2, \ldots, k$, and $\theta_i, i = 1, 2, \ldots, k$. Sample the sum of the sinusoids as mentioned above. Apply a Hanning window on the samples, i.e., multiply each of the 512 samples by a length-512 Hanning window. Multiply the windowed samples with a random measurement matrix $B \in \mathbb{R}^{m \times N}$, to get the $m \times 1$ observation vector. Use the ℓ_1 magic (or any other) program to "recover" the frequency-domain sparse vector $\mathbf{Y}$, and compute the corresponding time-domain signal $\mathbf{y}$ from the recovered frequency domain signal.

1. For $k = 1$, generate plots showing the original signal and the recovered signal, in both time and frequency domain, for different cases:
   (a) $m = 10$, $f = 100$, $a = 1$, $\theta = 0$.
   (b) $m = 40$, $f = 100$, $a = 1$, $\theta = 0$.
   (c) $m = 40$, $f = 300$, $a = 1$, $\theta = 0$.

   Important: Comment on your results.

2. What is the role of the Hanning window in the problem?

3. Vary $k = 1, 2, \ldots$. Choose different frequencies, phases and all amplitudes = 1. Explicitly report the values of frequencies and phases you choose. For each k, consider different values of m, run the recovery algorithm 100 times. Call a recovery *successful* if the mean square error between the original signal and the recovered signal is below a threshold, say 10^{-3}. Find the smallest m at which you get a successful recovery at least 90 times. Plot $100 \times k/N$ (the percentage sparsity) on the y-axis and the $100 \times m/N$ (the percentage number of measurements) on the x-axis. Repeat for successful recovery at least 40 times, 60 times, 80 times, and all 100 times, and plot the result on the same graph. This is called a *phase transition diagram*. Can you guess why?

[1] This homework is based on: Shlomo Engelberg, "Compressive Sensing", IEEE Instrumentation and Measurement Magazine, Feb. 2012.

4. What happens to the phase transition diagram as the different frequencies that constitute the tones move closer to one-another?

5. Repeat the above part with highly scaled amplitudes (highly unequal to each other). Explicitly report how you chose the amplitudes. What do you observe?

6. Plot the run time (sec) as a function of N for different values of k, with m chosen such that the recovery is successful all 100 times.