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Discrete Linear Canonical Transform Based on Hyperdifferential Operators (Aykut Koc, Burak Bartan, and Haldun M. Ozaktas)

Achievements:

- A new approach to define the discrete linear canonical transform (DLCT) by employing operator theory

Basics :

- L= $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{\beta} & \frac{1}{\beta} \\ -\beta + \frac{\alpha\gamma}{\beta} & \frac{\alpha}{\beta} \end{bmatrix} = \begin{bmatrix} \frac{\alpha}{\beta} & -\frac{1}{\beta} \\ \beta - \frac{\alpha\gamma}{\beta} & \frac{\gamma}{\beta} \end{bmatrix}$,
  $AD - BC = 1$ (Unitary transform)
- LCT as a linear integral transform :

$$C_L f(u) = \sqrt{\beta} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} exp i\pi(u^2 - 2\beta u u' + \gamma u'^2) f(u') du'$$

- Every triplet (α, β, γ) corresponds to a different LCT.

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- $C_{L_1L_2}f(u) = C_{L_1}C_{L_2}f(u)$ and $C_{L_2L_1}f(u) = f(u)$, if $L_2 = L_1^{-1}$
- Scaling : $L_M = \begin{bmatrix} M & 0 \\ 0 & \frac{1}{M} \end{bmatrix}$ $C_{L_M} = M_M f(u) = \sqrt{\frac{1}{M}} f(\frac{u}{M})$
- Fractional Fourier Transform (FRT) : $L_{F_{1c}^a} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $\theta = \pi a/2$ and a is the fractional order, $a = 1 \implies$ FRT $=$ FT. $F_{1c}^a f(u) = \int_{-\infty}^{\infty} K(u,u')f(u')du'$
- Chirp Multiplication : $L_{Q_p} = \begin{bmatrix} 1 & 0 \\ -q & 1 \end{bmatrix}$, $C_{Q_q}f(u) = Q_q f(u) = \exp(-i\pi q u^2)f(u)$
- Iwasawa decomposition : $C_L = Q_q M_M F_{1c}^a \implies L = L_{Q_p} L_M L_{F_{1c}^a}$, a, q, M be chosen appropriately.

Hyperdifferential forms

- $Q_q = \exp(-i2\pi q \frac{U^2}{2})$, $M_M = \exp(-i2\pi \ln(M) \frac{UD+DU}{2})$ and $F_{1c}^{a} = \exp(-ia\pi^2 \frac{U^2+D^2}{2})$
- $Uf(u) = uf(u)$ and $Df(u) = \frac{1}{i2\pi}\frac{df(u)}{du}$ and $U = FDF^{-1}$
- abstract operators being replaced by matrix operators, $\mathbf{C}_L = \mathbf{Q}_q \mathbf{M}_M \mathbf{F}_{1c}^{a}$
- $\mathbf{Q}_q = \exp(-i2\pi q \frac{\mathbf{U}^2}{2})$, $\mathbf{M}_M = \exp(-i2\pi \ln(M) \frac{\mathbf{UD}+\mathbf{DU}}{2})$ and $\mathbf{F}_{1c}^{a} = \exp(-ia\pi^2 \frac{\mathbf{U}^2+\mathbf{D}^2}{2})$
- $u = nh,\ U_{m,n} = \begin{cases} \frac{\sqrt{N}}{\pi}\sin(\frac{\pi n}{N}) & \text{for } m = n \\ 0, & \text{for } m \neq m \end{cases}$
- $\mathbf{D} = \mathbf{FUF}^{-1}$, $\mathbf{F}$: DFT
- $\mathbf{C}_L$: Unitary/ $\mathbf{D}, \mathbf{U}$: Hermitian/ $\mathbf{Q}_q, \mathbf{M}_M, \mathbf{F}_{1c}^{a}$: Unitary

Sinusoidal Parameter Estimation from Signed Measurements via Majorization-Minimization Based RELAX† (Jiaying Ren, Tianyi Zhang, Jian Li, and Petre Stoica)

Achievements:

- present a majorization-minimization (MM) based 1bRELAX algorithm, referred to as 1bMMRELAX, for sinusoidal parameter estimation using signed measurments obtained via one-bit sampling
- introduce a proper majorizing function and develop the MM procedure for minimizing the negative log-likelihood function for the signed measurements.

Problem formulations:

- $1 - D$ sinusoidal signal

$$s_t(\theta) = \sum_{k=1}^{K} A_k sin(\omega_k t + \phi_k) = \sum_{k=1}^{K} a_k \sin(\omega_k t) + b_k cos(\omega_k t),$$

  K : the number of sinusoids, $A_k \in \mathbb{R}^+$: amplitude, $\omega_k \in [0, \pi]$: frequency, $\phi_k \in [0, 2\pi]$: phase (of the $k-$th sinusoid component)

- $y_n = sign(s_n(\theta) + e_n - h_n)$, additive noise $e \in \mathbb{R}^N$: iid Gaussian with mean zero and unknown variance σ^2, $h_n \in \mathbb{R}^N$: known threshold vector.
- likelihood function of the signed measurements : $L_\beta = \prod_{n=0}^{N-1} \Phi(y_n \frac{s_n(\theta) - h_n}{\sigma}) = \prod_{n=0}^{N-1} \Phi[y_n \frac{(\sum_{k=1}^{K} a_k \sin(\omega_k n) + b_k \cos(\omega_k n)) - h_n}{\sigma}]$,
- $\Phi(x)$: cdf of standard normal distribution, unknown parameter : $\beta = [\theta^T, \sigma]^T$

continue ..

- Goal : estimating the parameter vector β and K based on the signed measurement vector $y \in [-1,1]^N$
- Maximum Likelihood Estimation: ML estimate of the parameter vector by minimizing the negative log-likelihood function: $\overset{\wedge}{\tilde{\beta}} = \arg\min_{\tilde{\beta}} l(\tilde{\beta}) = \arg\min_{\tilde{\beta}} \prod_{n=0}^{N-1} \Phi[y_n((\sum_{k=1}^{K} \tilde{a}_k \sin(\omega_k n) + \tilde{b}_k \cos(\omega_k n)) - \lambda h_n)]$
- $\lambda = \frac{1}{\sigma}, \tilde{a}_k = \frac{1}{\sigma} a_k, \tilde{b}_k = \frac{1}{\sigma} b_k, \tilde{\beta} = [\tilde{\theta}^T, \lambda]^T$, and $\tilde{\theta} = [\tilde{a}_1, \tilde{b}_1, \tilde{\omega}_1, \ldots, \tilde{a}_K, \tilde{b}_K, \tilde{\omega}_K]$

Majorization function

- With an auxiliary vector $x(\tilde{\beta}) = (x_n(\tilde{\beta}))_{n=0}^{N-1} = y_n(s_n(\tilde{\theta}) - \lambda h)_{n=0}^{N-1}$ and $x^i = x(\tilde{\beta}^i)$, the estimate obtained at the $i-$th MM iteration,

continue

- $G(\tilde{\beta}|\tilde{\beta}^i) = \tilde{G}(x(\tilde{\beta})|x^i) = \sum_{n=0}^{N-1} f(x_n^i) + f'(x_n^i)(x_n(\tilde{\beta}) - x_n^i) + \frac{1}{2}(x_n(\tilde{\beta}) - x_n^i)^2$, majorizes the objective function
- computational complexity: 1bRELAX($O(CKN^2 + CK^2N^2)$) where C is the number of iterations required to achieve practical convergence
- 1bMMRELAX : $O(KN^2)$

Group Greedy Method for Sensor Placement (Chaoyang Jiang, Zhenghua Chen, Rong Su, and Yeng Chai Soh)

Achievements:

Providing necessary and sufficient conditions for convergence of existing greedy algorithms and newly proposed group greedy algorithm for sensor placement

Problem Statemant

- physical field $\zeta = \tilde{\Phi}\alpha \in \mathbb{R}^N$, where $\tilde{\Phi} \in \mathbb{R}^{N \times n}$ with $n < N$
- aim is to estimate α from

$$y = H(\zeta + \nu) = H\tilde{\Phi}\alpha + H\nu,$$

$y \in \mathbb{R}^M$, M is the number of sensor observations, $H \in \mathbb{R}^{M \times N}$ whose $i-$th row is $e_{s_i}^T$, $\Phi = H\tilde{\Phi} = [\phi_{s_1}, \ldots, \phi_{s_M}]^T \in \mathbb{R}^{M \times n}$, $s_i \in \mathbf{N} = \{1, 2, \ldots, N\}$ corresponds to $i-$th sensing location, $\phi_{s_i}^T$ is the observation vector, noise $\nu \sim \mathcal{N}(0, \sigma^2 I)$

Measure of recovery performance

- mean squared error (MSE):

$$MSE(\tilde{\alpha}) = \sigma^2 tr((\Phi^T\Phi)^{-1}) = \sum_{i=1}^{n} \frac{\sigma^2}{\lambda_i}$$

- log volume of the confidence ellipsoid (VCE):

$$VCE(\tilde{\alpha}) = \beta - \frac{1}{2}\log\det(\Phi^T\Phi)$$

- worst case error variance (WCEV)

$$WCEV(\tilde{\alpha}) = \sigma^2\lambda_{\max}((\Phi^T\Phi)^{-1})$$

- Goal : error of the estimated $\tilde{\alpha}$ is less than predefined threshold and M is minimized

Combinatorial optimization problem

- min $\quad M = |S|$
  subject to $\quad S \subset \mathbf{N}, f(S) \geq \gamma$
- $f_{MSE}(S) = \frac{n}{\epsilon} - tr(\Psi^{-1}) \geq \frac{n}{\epsilon} - \gamma_A$, $f_{VCE}(S) = \log\det\Psi \geq \gamma_D$, $f_{WVEV} = \lambda_n \geq \frac{1}{\gamma_E}$
- Greedy algorithm : Update stage:
  $s^* = \arg\max_{j \in \mathbf{N} \setminus S} f(S \cup \{j\})$, $S = S \cup \{s^*\}$
  run till constraint is not satisfied
- Dual Problem:
  max $\quad f(S)$
  subject to $\quad S \subset \mathbf{N}, |S| = M$
- A and B be both optimal solutions of dual problem with $M_A = |A| < M_B = |B|$
  $A = \arg\max_{|S|=M_A, S \subset \mathbf{N}} f(S)$ and $b = \arg\max_{|S|=M_b, S \subset \mathbf{N}} f(S)$
- Necessary and sufficient condition: The greedy algorithm can obtain the optimal solution of dual problem iif $\forall\, M_A M_B$, $A \subset B$.

Group greedy Algorithm

- when determining each sensing location select a group of L suboptimal sensor configurations instead of the current optimal one
- Update stage :
  $\forall\, i \in [1:L],\ S_{k+1i} = S_{kl_i^*} \cup \{j_i^*\}$ where
  $\{l_i^*, j_i^*\} = \underset{l_i \in [1:N], j_i \in \mathbf{N} \setminus S_{kl_i}}{\arg\,\mathrm{rankde}_i}\ f(S_{kl_i} \cup \{j_i\})$
  $k = k = 1$, $S = S_{k1}$ and $M = k$
- 'rankdei' represents an operator which ranks all elements of a set in a descending order and returns the $i-$th element
- Necessary and sufficient condition: Denote the optimal solution of dual problem by S_M^*. Let $\mathbb{S}_k = \{S_{k1}, \ldots, S_{kL}\}$. Then group greedy algorithm can provide the optimal solution of dual problem iif for every $k \in [1, M-1]$ there exists $i \in [1, L]$ such that $S_{ki} \subset S_{k+1}^*$

Other papers:

- Information-Theoretic Pilot Design for Downlink Channel Estimation in FDD Massive MIMO Systems (Yujie Gu ; Yimin D. Zhang)
- Unimodality-Constrained Matrix Factorization for Non-Parametric Source Localization (Junting Chen ; Urbashi Mitra)
- Delta-Ramp Encoder for Amplitude Sampling and its Interpretation as Time Encoding (Pablo Martinez-Nuevo ; Hsin-Yu Lai ; Alan V. Oppenheim)
- Localization from Incomplete Euclidean Distance Matrix: Performance Analysis for the SVD-MDS Approach (Huan Zhang ; Yulong Liu ; Hong Lei)
- Advanced Low-Complexity Multicarrier Schemes Using Fast-Convolution Processing and Circular Convolution Decomposition (AlaaEddin Y. M. Loulou ; Juha Yli-Kaakinen ; Markku K. Renfors)

Other papers:

- Correction of Corrupted Columns through Fast Robust Hankel Matrix Completion (Shuai Zhang ; Meng Wang)
- On the Use of the Z Transform of LTI Systems for the Synthesis of Steered Beams and Nulls in the Radiation Pattern of Leaky-Wave Antenna Arrays (Rafael Verdu-Monedero ; Jose Luis Gomez-Tornero)
- Semi-Blind Inference of Topologies and Dynamical Processes over Dynamic Graphs (Vassilis N. Ioannidis ; Yanning Shen ; Georgios B. Giannakis)
- Covariance Matrix Estimation from Linearly-Correlated Gaussian Samples (Wei Cui ; Xu Zhang ; Yulong Liu)
- New Saddle-Point Technique for Non-Coherent Radar Detection with Application to Correlated Targets in Uncorrelated Clutter Speckle (Josef Alexander Zuk ; Stephen Bocquet ; Luke Rosenberg)

Other papers:

- Antithetic Dithered 1-bit Massive MIMO Architecture: Efficient Channel Estimation via Parameter Expansion and Pseudo Maximum Likelihood (David K. W. Ho ; Bhaskar D. Rao)
- Frequency Synchronization for Uplink Massive MIMO with Adaptive MUI Suppression in Angle-domain (Yinghao Ge ; Weile Zhang ; Feifei Gao ; Geoffrey Ye Li)
- Parameter Estimation of Heavy-Tailed AR Model with Missing Data via Stochastic EM (Junyan Liu ; Sandeep Kumar ; Daniel P. Palomar)
- Learning To Detect (Neev Samuel ; Ami Wiesel ; Tzvi Diskin)
- Underdetermined DOA Estimation for Wide-Band Stationary Sources in Unknown Noise Environment (L. Huang ; Q. Zhang ; S. Wu ; So)