Journal Watch
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- Sparse phase retrieval: Recover $x \in \mathbb{R}^n$, s-sparse, from magnitude only measurements

$$y_i = |a_i^\top x|, \quad i \in [m]$$

- Used in modeling imaging systems where only light intensity is measurable, not phase
- Contributions
  - Recovery algorithm CoPRAM with sample complexity $s^2 \log n$ when a_i are Gaussian
  - Under power law decay assumption on coefficients of x, sample complexity shown to be $s \log n$
  - Results for block sparse case

- The algorithm: good initialization + alternating minimization
  - Compute

$$M_{jj} = \frac{1}{m}\sum_{i=1}^{m} y_i^2 a_{ij}^2$$

    and declare indices of top M_{jj} as support
  - Initialize x as top singular vector of

$$M_{\hat{S}} = \frac{1}{m}\sum_{i=1}^{m} y_i^2 a_{i\hat{S}} a_{i\hat{S}}^\top$$

  - An alternating minimzation step

- Sparse representation over redundant dictionaries
  - Represent any/most $x \in \mathbb{R}^n$ as a linear combination of $k < d$ columns of dictionary $\Phi \in \mathbb{R}^{d \times n}$
  - Minimum n that allows this representation
- Contributions
  - For certain regimes of error and sparsity level, can have universal representation with moderate redundancy
  - Results for both random and deterministic x
  - Minimum overcompleteness scales roughly as $\left(\frac{1}{\varepsilon}\right)^{\frac{d}{k}-1}$

- Normalized k-sparse representation error: For a given Φ, x

$$\varepsilon(\Phi, x) = \min_{\alpha \in \mathbb{R}^n} \frac{\|x - \Phi\alpha\|}{\|x\|} \quad \text{s.t.} \quad \|\alpha\|_0 \leq k$$

  Require $\varepsilon(\Phi, x) \leq \varepsilon$

- Parameters of interest: Sparsity ratio, Overcompleteness ratio

$$s = \frac{k}{d}, \quad o = \frac{n}{d}$$

- Characterize minimal overcompleteness s.t. all/most $x \in \mathbb{R}^n$ have a sparse representation

- Mixture of linear regressions (MoLR)

$$Y_i = R_i(X_i^\top \theta^*) + \varepsilon_i, \quad i \in [n],$$

  where $X_i \overset{iid}{\sim} \mathcal{N}(0, I)$
  $R_i \overset{iid}{\sim} \text{Rademacher}(1/2)$
  $\varepsilon_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$
  Estimate θ^*

- Contributions
  - Convergence guarantees for EM applied to MoLR, provided good initialization
  - Guarantees based on cosine similarity between target θ^* and initialization θ^0

- Connections to phase retrieval: Squaring observations in the symmetric MoLR gives phase retrieval model

$$Y_i^2 = \tilde{Y}_i = |X_i^\top \theta^*|^2$$

- Can use ideas from phase retrieval for the initialization step

- For iteration t, authors provide upper bound on the error $\|\theta^* - \theta^{(t)}\|$

Other interesting papers

- Optimization-Based AMP for Phase Retrieval: The Impact of Initialization and ℓ_2 Regularization. *J. Ma, J. Xu, and A. Maleki*
- Noisy Adaptive Group Testing: Bounds and Algorithms. *J. Scarlett*
- Determining the Number of Samples Required to Estimate Entropy in Natural Sequences. *A. D. Back, D. Angus, and J. Wiles*
- Symmetry, Saddle Points, and Global Optimization Landscape of Nonconvex Matrix Factorization. *X. Li, J. Lu, R. Arora, J. Haupt, H. Liu, Z. Wang, and T. Zhao*