

On the Throughput of Large MIMO Beamforming Systems with Channel Aging

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Abstract—We study the problem of throughput optimization in large MIMO beamforming systems with channel aging, as the number of transmit and receive antennas is increased. We quantify the effective channel coherence time in terms of the inter-training interval, which is the time after which the channel state information needs to be re-estimated to avoid loss in performance due to outdated channel estimates. Conventional wisdom suggests that the coherence time of large MIMO systems should decrease with increasing number of antennas, because the system has a large number of independently varying channel coefficients. On the contrary, we show that the effective channel coherence time *increases* with the number of antennas. We optimize the throughput with respect to the symbol and pilot energies, the number of antennas, frame duration, and target SNR. This analysis brings out the important fact that the effective channel coherence time critically depends on the underlying signaling scheme, and not only on the temporal correlation coefficient of the wireless channel.

I. INTRODUCTION

Recently, much effort has been put into the study and performance analysis of large multiple input multiple output (MIMO) systems [1]. This is because the use of large MIMO systems can potentially lead to increased spectral efficiency and data throughput [2]. Further, the channel hardening effect that occurs in the large number of antennas regime can lead to low complexity decoding [3]. However, the benefits offered by *beamforming based* large MIMO systems are predicated on the availability of accurate channel state information (CSI) at both the transmitter and the receiver. This may not always hold true due to various physical impairments such as pilot contamination, channel estimation errors and channel aging [4], [5]. Single user large MIMO systems have been analyzed under different non-idealities in [6]–[8]. However, while a substantial body of literature exists on the analysis and mitigation of the effects caused due to pilot contamination and channel estimation errors [4], [9], the effect of increasing the number of antennas on the performance of a large MIMO system with channel aging is relatively unexplored, and is the focus of this work.

Channel aging refers to the mismatch between the CSI at the time instant when the channel is estimated and when it is used for data transmission. It occurs due to the time-varying nature of the channel [5]. This discrepancy between the channel

coefficients being used for beamforming/data detection and the true channel values leads to degraded system performance. The effect of channel aging on the achievable rates of large MIMO systems has been analyzed in [5], [10]–[12]. Closed form bounds for the achievable sum rate and deterministic equivalents of the SINR for maximal ratio combining and regularized linear receivers in the presence of channel aging are derived in [13], [14]. Mitigation of channel aging effects using channel prediction is also considered in [13], [14]. The combined effects of phase noise and channel aging are considered in [15] by means of an autoregressive model. In [16], the uplink performance of massive MIMO systems in a multi-cell environment under the joint effects of pilot contamination and channel aging is analyzed. Different from the aforementioned studies, in this paper, we analyze the performance of a beamforming based large MIMO system with the outage capacity as the performance metric.

In this letter, we study the performance of a point to point large MIMO beamforming system in terms of the effective throughput. For this purpose, we first characterize the maximum usable frame length of the large MIMO system in terms of the temporal correlation coefficient of the channel, number of antennas and the target SINR. The maximum usable frame length, or the effective channel coherence time, is defined as the time after which the channel needs to be re-estimated in order to maintain a certain performance level. We show that the effective coherence time *increases* logarithmically with the number of antennas. However, the training overhead scales linearly with the number of antennas, due to which, the data transmission duration decreases as the number of antennas is increased. Hence, there exists an optimal number of antennas beyond which the throughput drops even though the usable frame length increases. Our key contributions are as follows:

- We characterize the effective coherence time of the channel in terms of the number of antennas;
- We maximize the throughput in terms of the number of transmit antennas, signal and pilot energies, and frame length, to find the optimum system configuration.

The key finding of this work is that, in large MIMO systems, more (antennas) is not always better. First, a larger number of antennas improves the system throughput because of the diversity gain offered by the channel. Second, tracking a larger number of independently time-varying parameters requires a correspondingly large number of training symbols. Third, the channel coherence time itself increases logarithmically with the number of antennas. The combined effect of these three

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effects is that there is an optimal number of antennas in a large MIMO system, that leads to the best throughput performance. Another finding of this work is that the choosing the frame duration based on the effective channel coherence time is important, as it strikes a balance between the rate loss resulting from outdated channel estimates and the overhead in involved in re-estimating the channel.

II. SYSTEM MODEL

We consider a large MIMO system consisting of N_t transmit antennas and N_r receive antennas, resulting in a baseband channel matrix at the n th time instant $\mathbf{H}[n] \in \mathbb{C}^{N_r \times N_t}$. The individual entries $h_{ij}[n]$ of $\mathbf{H}[n]$ are modeled as independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, denoted by $\mathcal{CN}(0, 1)$.

At time $n = 0$, the channel can be expressed in terms of its least squares (maximum likelihood) estimate at the receiver $\hat{\mathbf{H}}$ and an orthogonal error matrix $\tilde{\mathbf{H}}$ with i.i.d. zero mean, unit variance, complex Gaussian entries, as follows [5], [12]:

$$\mathbf{H}[0] = \sqrt{1 - \frac{N_0}{\mathcal{E}_p}} \hat{\mathbf{H}} + \sqrt{\frac{N_0}{\mathcal{E}_p}} \tilde{\mathbf{H}} \quad (1)$$

with \mathcal{E}_p being the energy of each pilot signal and N_0 being the two sided power spectral density of the noise at the receiver. Considering the pilot power to be variable, we can assume that one training symbol is required per transmit antenna [17], resulting in a total training duration of N_t symbols. Thus, in each frame of duration T_c symbols, the first N_t symbols are used for training, and the remaining $T_c - N_t$ symbols are used for data transmission. Clearly, the number of antennas should always be less than the frame duration, i.e., $N_t < T_c$.

In order to model the effect of channel aging, $h_{ij}[n]$ are assumed to evolve as [18]

$$h_{ij}[n] = \rho h_{ij}[n-1] + \sqrt{1 - \rho^2} \eta_{ij}[n] \quad (2)$$

where $\eta_{ij}[n] \sim \mathcal{CN}(0, 1)$ is the innovation component at discrete time n and $\rho \triangleq J_0(2\pi f_d T_s)$ is the autocorrelation coefficient. Here, $J_k(x)$ is the k th order Bessel function of the first kind, f_d is the maximum Doppler frequency, and T_s is the sampling period. Hence, we can express $h_{ij}[n]$ as

$$h_{ij}[n] = \rho^n h_{ij}[0] + \sqrt{1 - \rho^{2n}} z_{ij}[n], \quad (3)$$

with an innovation component $z_{ij}[n] \sim \mathcal{CN}(0, 1)$.

Plugging (1) into (3), we get

$$\mathbf{H}[n] = \rho^n \sqrt{1 - \frac{N_0}{\mathcal{E}_p}} \hat{\mathbf{H}} + \rho^n \sqrt{\frac{N_0}{\mathcal{E}_p}} \tilde{\mathbf{H}} + \sqrt{1 - \rho^{2n}} \mathbf{Z}[n] \quad (4)$$

where $\mathbf{Z}[n]$ is the innovation matrix with (ij) th entry $z_{ij}[n]$.

We perform singular value decomposition on the estimated channel $\hat{\mathbf{H}}$ to obtain the dominant left and right singular vectors \mathbf{u} and \mathbf{v} respectively, such that $\hat{\mu} \triangleq \mathbf{u}^H \hat{\mathbf{H}} \mathbf{v}$ is the largest singular value of $\hat{\mathbf{H}}$. The transmit beamforming vector, \mathbf{v} , is assumed to be communicated to the transmitter via an ideal feedback channel. This model is equally applicable under a frequency division duplex system with forward link training followed by reverse link transmit beamforming vector

feedback, as well as under a time division duplex system exploiting channel reciprocity.

Now, if the symbol $s[n]$ to be sent over the channel is multiplied with \mathbf{v} prior to transmission, and the received symbol vector is premultiplied with \mathbf{u}^H , then the received symbol can be expressed as,

$$y[n] = \mu[n]s[n] + \sqrt{N_0}w[n], \quad (5)$$

with $w[n] \sim \mathcal{CN}(0, 1)$, and $\mu[n] \triangleq \mathbf{u}^H \mathbf{H}[n] \mathbf{v}$. Using (4), $\mu[n] = \rho^n \sqrt{1 - \frac{N_0}{\mathcal{E}_p}} \hat{\mu} + \rho^n \sqrt{\frac{N_0}{\mathcal{E}_p}} \tilde{\mu} + \sqrt{1 - \rho^{2n}} \xi[n]$, where $\hat{\mu} \triangleq \mathbf{u}^H \hat{\mathbf{H}} \mathbf{v}$ and $\xi[n] \triangleq \mathbf{u}^H \mathbf{Z}[n] \mathbf{v}$. It can easily be shown that $\tilde{\mu}$ and ξ are i.i.d. $\mathcal{CN}(0, 1)$. Hence, we can rewrite (5) as

$$y[n] = \rho^n \sqrt{1 - \frac{N_0}{\mathcal{E}_p}} \hat{\mu} s[n] + \sqrt{\frac{N_0}{\mathcal{E}_p}} \rho^n \tilde{\mu} s[n] + \sqrt{1 - \rho^{2n}} \xi[n] s[n] + \sqrt{N_0} w[n]. \quad (6)$$

In the above expression, all terms but the first act as noise and interference, and therefore, the SINR of the received signal at the n th instant can be written as

$$\gamma[n] \triangleq \frac{\rho^{2n} \left(1 - \frac{N_0}{\mathcal{E}_p}\right) |\hat{\mu}|^2 \mathcal{E}_s}{(\rho^{2n} \frac{N_0}{\mathcal{E}_p} + (1 - \rho^{2n})) \mathcal{E}_s + N_0}. \quad (7)$$

where $\mathcal{E}_s \triangleq E[|s[n]|^2]$ is the transmit symbol energy. Now, $|\hat{\mu}|^2 = \lambda_{\max}(\hat{\mathbf{H}}^H \hat{\mathbf{H}})$ where $\lambda_{\max}(\hat{\mathbf{H}}^H \hat{\mathbf{H}})$ is the largest eigenvalue of $\hat{\mathbf{H}}^H \hat{\mathbf{H}} \in \mathbb{C}^{N_t \times N_t}$. It has been shown in [19, Ch. 7] that, in the large antenna regime, $\lambda_{\max}(\hat{\mathbf{H}}^H \hat{\mathbf{H}})$ converges to its deterministic equivalent:

$$\lim_{N_t \rightarrow \infty} \lambda_{\max}(\hat{\mathbf{H}}^H \hat{\mathbf{H}}) \xrightarrow{\text{a.s.}} N_t (1 + \sqrt{c})^2, \quad (8)$$

where $c \triangleq \frac{N_r}{N_t}$, $0 \leq c \leq 1$. Therefore, $|\hat{\mu}|^2$ can be replaced by $N_t (1 + \sqrt{c})^2$, and the SINR at the n th instant becomes

$$\gamma[n] = \frac{\rho^{2n} \left(1 - \frac{N_0}{\mathcal{E}_p}\right) N_t (1 + \sqrt{c})^2 \mathcal{E}_s}{(\rho^{2n} \frac{N_0}{\mathcal{E}_p} + (1 - \rho^{2n})) \mathcal{E}_s + N_0}. \quad (9)$$

III. EFFECTIVE CHANNEL COHERENCE TIME

Note that $\gamma[n]$ decays with n , therefore, for a system with channel aging, outage can be defined as the event of the received SINR $\gamma[n]$ falling below a target γ_{th} . Following this, the usable frame duration, or the effective coherence time, T_c , can be defined as the largest n such that $\gamma[n] \geq \gamma_{\text{th}}$. Letting $\gamma[T_c] = \gamma_{\text{th}}$ and substituting into (9), we obtain

$$\frac{\rho^{2T_c} \left(1 - \frac{N_0}{\mathcal{E}_p}\right) N_t (1 + \sqrt{c})^2 \mathcal{E}_s}{(\rho^{2T_c} \frac{N_0}{\mathcal{E}_p} + (1 - \rho^{2T_c})) \mathcal{E}_s + N_0} = \gamma_{\text{th}}. \quad (10)$$

Rearranging (10), we get

$$T_c = \frac{\log \left(\left(1 - \frac{N_0}{\mathcal{E}_p}\right) N_t (1 + \sqrt{c})^2 - \gamma_{\text{th}} \left(\frac{N_0}{\mathcal{E}_p} - 1\right) \right)}{2 \log(1/\rho)} - \frac{\log \left(\gamma_{\text{th}} \left(1 + \frac{N_0}{\mathcal{E}_s}\right) \right)}{2 \log(1/\rho)}. \quad (11)$$

This is the maximum time for which the system can be used prior to being trained again using N_t pilot symbols. Hence, each frame of duration T_c can be divided into two non-overlapping parts, for training and data transmission, and with durations N_t and T_{data} , respectively, such that $N_t + T_{\text{data}} = T_c$. It can be observed that for a given target SINR, γ_{th} , the effective channel coherence time increases logarithmically with the number of antennas, whereas the training time increases linearly. The usable data duration, T_{data} , will therefore be maximized for some finite value of N_t .

Also, the effective throughput (in nats) for a given target SINR can be expressed as

$$R \triangleq \left(1 - \frac{N_t}{T_c}\right) \log(1 + \gamma_{\text{th}}). \quad (12)$$

Here, T_c and γ_{th} depend on each other, as well as on the number of transmit antennas, and the pilot and data powers.

IV. THROUGHPUT OPTIMIZATION

In this section, we formulate the maximization of the effective throughput as a constrained optimization problem in N_t , \mathcal{E}_p , \mathcal{E}_s , γ_{th} , and T_c .

To characterize the constraints on the data and pilot powers, we limit the system transmit power as \mathcal{E}_t , such that

$$N_t \mathcal{E}_p + (T_c - N_t) \mathcal{E}_s = T_c \mathcal{E}_t. \quad (13)$$

Note that (12) is valid only if $T_c \geq N_t$. Hence, the problem of throughput maximization can be expressed as

$$\max_{N_t, \gamma_{\text{th}}, \mathcal{E}_s, \mathcal{E}_p, T_c} \left(1 - \frac{N_t}{T_c}\right) \log(1 + \gamma_{\text{th}}) \quad (14a)$$

subject to

$$\gamma_{\text{th}} = \frac{\rho^{2T_c} \left(1 - \frac{N_0}{\mathcal{E}_p}\right) N_t (1 + \sqrt{c})^2 \mathcal{E}_s \mathcal{E}_p}{\rho^{2T_c} N_0 \mathcal{E}_s + \mathcal{E}_p (N_0 + (1 - \rho^{2T_c}) \mathcal{E}_s)} \quad (14b)$$

$$T_c \geq N_t \quad (14c)$$

$$\mathcal{E}_p = \frac{T_c}{N_t} \mathcal{E}_t - \frac{T_c - N_t}{N_t} \mathcal{E}_s \quad (14d)$$

$$\mathcal{E}_p, \mathcal{E}_s, N_t, \gamma_{\text{th}} > 0. \quad (14e)$$

Defining the fractional training duration $\alpha \triangleq \frac{N_t}{T_c}$ and substituting (14b), (14d) in (14a), we can rewrite the above as the optimization problem in (15) on the top of the next page, subject to $0 \leq \alpha \leq 1$ and $\mathcal{E}_s, N_t > 0$.

We first consider optimizing the throughput in terms of the symbol energy for given values of N_t and α . In this case, since \mathcal{E}_s occurs only inside the log term, the above problem can be reformulated as

$$\min_{\mathcal{E}_s > 0} \frac{\rho^{\frac{2N_t}{\alpha}} N_0 \mathcal{E}_s + \left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha}\right) \mathcal{E}_s\right) (N_0 + (1 - \rho^{\frac{2N_t}{\alpha}}) \mathcal{E}_s)}{\rho^{\frac{2N_t}{\alpha}} \left(1 - \frac{N_0}{\left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha}\right) \mathcal{E}_s\right)}\right) \mathcal{E}_s \left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha}\right) \mathcal{E}_s\right)}. \quad (16)$$

It is easy to show that the above problem has a closed form solution with the optimal symbol energy \mathcal{E}_s^* given by

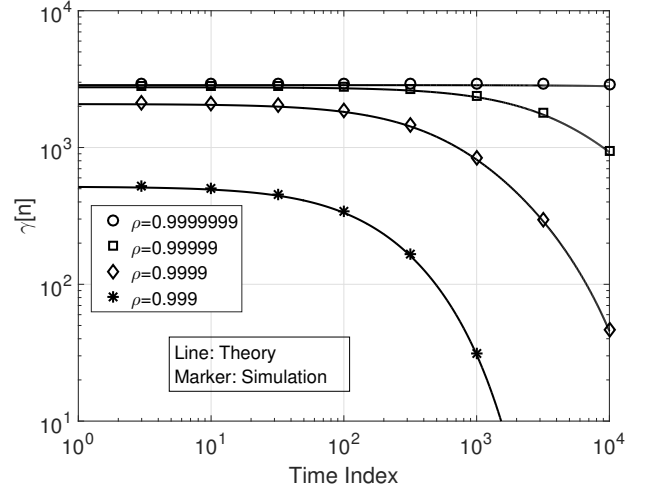


Fig. 1. Received SINRs for different values of ρ and with $N_t = 200$.

$$\mathcal{E}_s^* = \frac{\left(\frac{1-\alpha}{\alpha}\right) \frac{\mathcal{E}_t}{\alpha} - \sqrt{\left(\frac{1-\alpha}{\alpha}\right) \frac{\mathcal{E}_t}{\alpha} \left[\frac{1-\alpha}{\alpha} N_0 + \frac{\mathcal{E}_t}{\alpha} - N_0\right]}}{\left[\left(\frac{1-2\alpha}{\alpha}\right) \left(\frac{1-\alpha}{\alpha}\right)\right]}. \quad (17)$$

Substituting this in (15), we obtain a useful expression for studying the throughput behavior of the system for different values of α and N_t . For example, given a particular value of N_t , it is easy to numerically optimize (15) over $0 \leq \alpha \leq 1$ to obtain the optimal fractional training duration that maximizes the throughput. Also, it can be numerically maximized over both N_t and α . We illustrate these through the simulation results presented in the next section.

V. NUMERICAL RESULTS

In this section, we validate our derived expressions and study the performance of large MIMO beamforming systems using Monte Carlo simulations, for different values of the channel correlation coefficient, ρ . In all these experiments, the system SNR $\frac{\mathcal{E}_t}{N_0}$ is fixed at 10 dB. Also, the ratio of receive to transmit antennas, c , is set as 0.1. All the results are obtained by averaging over 100 independent realizations of sequences of matrices representing the temporally correlated channel.

Figure 1 shows the theoretical and simulated values of the SINR $\gamma[n]$ against the time index for different values of the channel correlation coefficient ρ , for a 200 transmit antenna system. Note that $\rho = 0.9999999$, $\rho = 0.9999$, and $\rho = 0.999$ correspond respectively to low, medium and high user mobilities. At a carrier frequency of 2 GHz and a sampling rate of 500 kHz, these correspond to a mobile speeds of 3, 70 and 270 km/hr, respectively. From (9), it is seen that $\gamma[n]$ decreases roughly exponentially with time n , and its slope depends on $\log \rho$, i.e., the closer ρ is to one, the smaller the slope. This is corroborated by the behavior of the curves for the three values of ρ considered here. Further, we see that the SINR improves with increasing ρ . This is also expected because the more correlated the channel is with the estimate obtained at time $n = 0$, the better the SINR.

$$\max_{N_t, \alpha, \mathcal{E}_s} (1 - \alpha) \log \left(1 + \frac{\rho^{2N_t/\alpha} \left(1 - \frac{N_0}{\left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha} \right) \mathcal{E}_s \right)} \right) N_t (1 + \sqrt{c})^2 \mathcal{E}_s \left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha} \right) \mathcal{E}_s \right)}{\rho^{2N_t/\alpha} N_0 \mathcal{E}_s + \left(\frac{\mathcal{E}_t}{\alpha} - \left(\frac{1-\alpha}{\alpha} \right) \mathcal{E}_s \right) (N_0 + (1 - \rho^{2N_t/\alpha}) \mathcal{E}_s)} \right) \quad (15)$$

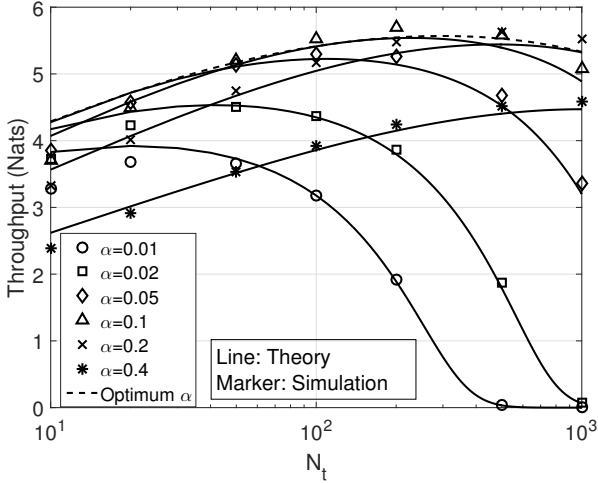


Fig. 2. Effective throughput for different number of transmit antennas and different values of α for $\rho = 0.9999$.

In Fig. 2, the simulated performance of the beamforming system for different values of the fractional training duration α is plotted against the number of transmitting antennas N_t . The channel correlation coefficient in this case is fixed at $\rho = 0.9999$. The dotted line in the figure plots the predicted maximum throughput, optimized over α for the given number of antennas. For a given value of α , the maximum effective throughput first increases with N_t , and then starts decreasing. Also, the envelope of the simulated throughputs closely follow the predicted optimal throughput. Finally, there is a mismatch between the theory and simulations for smaller values of N_t ; this is because the deterministic equivalent (8) is valid only asymptotically as $N_t \rightarrow \infty$.

Figure 3 shows the effective throughput optimized over α for different number of transmit antennas. It is observed that the optimal number of antennas maximizing the system throughput reduces as the effects of channel aging worsen. Hence, a larger number of antennas may adversely affect the system performance for high user mobilities, and it is therefore desirable to use only a subset of the total available number of antennas in this case.

In Fig. 4, we plot the optimized throughput as a function of the system SNR $\left(\frac{\mathcal{E}_t}{N_0} \right)$ for different number of transmit antennas and $\rho = 0.9999$. It is observed that the number of antennas maximizing the throughput decreases with \mathcal{E}_t/N_0 . This is because, as \mathcal{E}_t/N_0 increases, the effect of aging acts as the dominant component of the interference, which gets exacerbated for larger N_t , in turn, limiting the performance.

VI. CONCLUSIONS

In this letter, we considered a large MIMO beamforming system under channel aging and derived the dependence of the

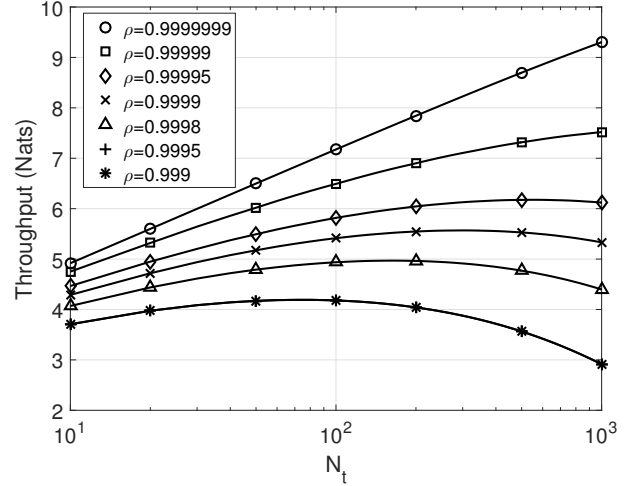


Fig. 3. Optimized throughput versus the number of transmit antennas for different values of ρ .

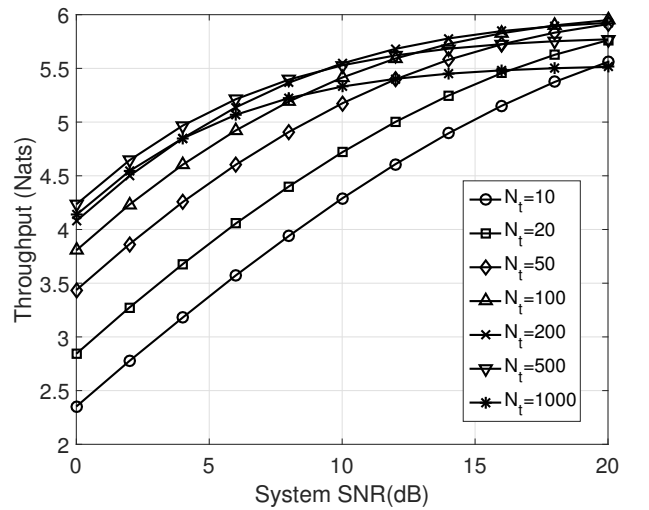


Fig. 4. Optimized throughput versus the system SNR $\left(\frac{\mathcal{E}_t}{N_0} \right)$ for different numbers of transmit antennas and with $\rho = 0.9999$.

effective coherence time of such a system on the number of antennas and the temporal correlation coefficient of the channel. Further, we used the effective coherence time to derive an expression for the net system throughput. We maximized the throughput in terms of the pilot to data power ratio, frame duration, and the number of transmit antennas. The analysis revealed that the number of antennas that maximizes the throughput decreases as the effects of channel aging worsen, or as the system SNR increases. The extension of the single-user/single-beam analysis to the multi-user/multi-beam case is an interesting direction for future work.

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