Single-RF Spatial Modulation Relying on Finite-Rate Phase-Only Feedback: Design and Analysis

Mishfad Shaikh Veedu, Chandra R. Murthy, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—In this paper, we consider a spatial modulation (SM) based multiple input single output (MISO) system relying on a single Radio-Frequency chain equipped with a finite-rate feedback channel to provide quantized channel state information (CSI) to the transmitter. First, under the assumption of Rayleigh flat-fading channels and perfect CSI at the transmitter (CSIT), we analyze the symbol error probability (SEP) of an SM scheme which perfectly compensates the channel phase and employs constellation rotation at the different transmit antennas (TAs). Then, we consider a more practical scenario, where scalar quantization of the channel phase angles is employed, and the quantized CSI is made available to the transmitter via a finite-rate feedback channel. We analyze the SEP-reduction, P_{e_L}, relative to perfect CSIT, imposed by the quantized CSI at the transmitter. We show that at a high feedback rate, P_{e_L} varies as $C'2^{-2B}$, where each channel phase angle is quantized to B bits, and C' is a constant. Furthermore, based on the rotational symmetry of the M-PSK signal constellation, we propose a novel feedback scheme, which requires $(n_t-1)\log_2(M)$ fewer bits of feedback with any performance erosion, where n_t is the number of TAs. We characterize the performance of the SM-MISO system with finite rate feedback and validate our analysis through Monte Carlo simulations.**

***Index Terms*—Spatial modulation, finite rate feedback, scalar quantization, constellation rotation, high-rate analysis.**

I. Introduction

Spatial modulation (SM) is a recent technique in multiple antenna communications, where, along with the classic modulated symbol transmitted from the antenna, additional implicit information is conveyed by the specific index of the transmit antennas (TAs). For a system with n_t TAs, $\log_2 n_t$ bits are conveyed by the index of the TA to be activated and a symbol selected from a constellation $\mathcal{M}$ of size M is transmitted through the selected TA, giving a total rate of $(\log_2 n_t + \log_2 M)$ bits per channel use. Excellent recent tutorial surveys on SM include [1]–[3]. In the early work on SM, e.g., [4]–[11], basic signaling schemes and optimal detectors were proposed and their corresponding symbol error probability (SEP) performance was analyzed. For example, in [12], the authors derive expressions for the average bit error probability (ABEP) of an SM system in terms of the Marcum-Q and hypergeometric functions. However, these expressions cannot be readily extended to the case where partial channel state information (CSI) is available at the transmitter (CSIT) via a finite rate feedback link.

The performance of an SM system can be improved by making CSI available at the transmitter also; see, e.g., [13]–[16]. In practice, the CSI has to be quantized and signaled from the receiver to the transmitter over a finite-rate feedback channel; this is the focus of this paper. In [16], the authors consider SM with finite-rate feedback, and propose adaptive power allocation algorithms for maximizing the minimum distance between constellation points based on the CSIT. A numerical approach is adopted, which, unfortunately, does not lead to tractable performance analysis.

Several variants of SM based on the availability of CSIT have been proposed in the recent literature. Link adaptation techniques were explored in [13]–[15] for minimizing the pairwise error probability (PEP). Selection of the optimal modulation order by searching over different constellations that offer a given spectral efficiency was studied in [13]. This was extended to accommodate antenna selection in [14]. The same authors also proposed algorithms for reducing both the computational complexity and the search space of the above techniques, hence reducing the number of candidates and the feedback load [15]. Ntonin *et al.* proposed an adaptive generalized space shift keying (GSSK) scheme, which exploits the channel's phase information for improving the attainable diversity and coding gains of the modulation scheme [17], [18]. The latter treatise also studied the effects of quantized CSI feedback, but only through Monte Carlo simulations. Explicitly, the performance of transmission over Rayleigh fading channels was not analyzed in closed-form. Hence, this contribution seeks to fill this open problem in the literature by theoretically analyzing the performance of SM with the aid of finite-rate feedback. In [19] and [20], the constellation shaping conceived is both data and channel dependent, unlike the channel-only dependent scheme studied in this work. In [21], a scheme based on phase-shifting the transmit signal based on CSIT is considered, because joint amplitude scaling and phase shifting is energy inefficient and challenging from a power-amplifier design point of view. In classical MIMO systems, the performance under Q-CSIT has been analyzed in detail, especially for beamforming based transmission [22]–

Mishfad Shaikh Veedu is with the Dept. of ECE, P.E.S. Institute of Technology, Bangalore South Campus, Bangalore, India. He was with the Dept. of ECE, Indian Institute of Science, Bangalore, India, during the course of this work. Email: mishfad@gmail.com. Chandra R. Murthy is with the Dept. of ECE, Indian Institute of Science, Bangalore, India. Email: cmurthy@ece.iisc.ernet.in. Lajos Hanzo is with the Faculty of Physical Sciences and Engg., Univ. of Southampton, UK. Email: lh@ecs.soton.uk.

This work was financially supported by the India-UK Advanced Technology Centre and the European Research Council Advanced Fellow Grant.

[24]. These techniques and results are, however, not applicable to SM with finite-rate feedback, since the signaling scheme is fundamentally different. To the best of our knowledge, the performance of SM with quantized CSI feedback has not been studied in the literature to date.

As a first step to this end, we commence by analyzing the performance of a Rayleigh flat-fading multiple input single output (MISO) SM system relying on perfect CSIT. The transmission scheme employs phase compensation at each antenna to cancel the phase shift to be introduced by the channel, followed by deterministic constellation rotation in order to increase the minimum distance of the constellation at the receiver. This philosophy was proposed in the context of generalized space shift keying in [17], [18], and its diversity order was derived more recently in [25].

Against this background, we study the performance of the above scheme under Q-CSIT, obtained via a finite rate feedback channel from the receiver to the transmitter. The receiver sends $(n_t-1)B$ bits of information about the channel to the transmitter once in every channel instantiation, through a delay-free and noise-free feedback link. We consider scalar quantization of the phase angles (note that one angle can be compensated for at the receiver and therefore need not be quantized) for its simplicity and analytical tractability.

The main contributions in this paper are as follows:

1) Assuming perfect CSIT, we derive approximate closed-form analytical expressions for the SEP performance of a phase compensation and constellation rotation based transmission scheme for SM-MISO systems for transmissions over Rayleigh fading channels.
2) We also analyze the performance of the above scheme in a more practical system, when Q-CSIT becomes available via a finite rate feedback channel. Considering the loss in probability of error P_{e_L} as the metric, we derive a theoretical expression for P_{e_L} under Q-CSIT.
3) We show that P_{e_L} decreases exponentially upon increasing the number of feedback bits B per phase angle, as $C'/2^{2B}$, where C' is a constant. This allows us to quantify the feedback rate required for guaranteeing an upper bound on the loss in PEP due to the quantized feedback.
4) We consider the case where M-PSK is employed as the classic signal constellation. We propose a novel phase compensation scheme that exploits the rotational symmetry of the constellation, for achieving the same P_{e_L} performance as the above scheme, while requiring $(n_t-1)\log(M)$ fewer bits of feedback.
5) We extend the phase compensation and constellation rotation scheme to systems equipped with multiple receive antennas (RAs). Using simulations, we compare the average pairwise error probability of the above scheme to that of transmit antenna selection (TAS). We find that TAS outperforms SM relying on CSIT, but the performance gap is reduced as the number of RAs is increased.
6) We empirically study the performance of a more practical case, where the signals of the TAs are correlated, and show that the above scheme is significantly more robust to this correlation than the conventional SM system.

We validate our analysis, and draw conclusions on the design and performance of SM-MISO systems with Q-CSIT, through simulations. For example, we find that a feedback rate of about 3 bits per phase angle is sufficient for achieving a performance comparable to that of the scheme with perfect CSIT, at reasonable SNRs.

In the next section, we elaborate on the system model.

II. System Model

We consider an SM-MISO system equipped with n_t TAs and a single antenna at the receiver. In the complex baseband notation, the wireless channel can be represented by the vector $\mathbf{h} = [h_1, \ldots, h_{n_t}]$, where $h_l \in \mathbb{C}$ denotes the channel signaling from the l^{th} TA to the receiver. For our analysis, we assume that the channel is Rayleigh flat-fading. With SM, the symbol transmitted over the n_t TAs can be represented by $\mathbf{s} \in \mathcal{M}^{n_t}$, where $\mathbf{s} = [0, \ldots, 0, s_l^i, 0, \ldots, 0]^T$, where s_l^i denotes the i^{th} symbol in the signal constellation, being transmitted from the l^{th} TA. Since only one antenna at the transmitter is turned on at any given time, all the remaining entries in $\mathbf{s}$ are zero.

When CSI is available at the transmitter, the data symbol $\mathbf{s}$ is multiplied by a phase compensation matrix $\mathbf{W} = \text{diag}(\mathbf{w})$ that depends on the CSI, to get the channel input $\mathbf{x}$ as $\mathbf{x} = \mathbf{W}\mathbf{s}$. The channel output $\mathbf{y}$ can then be expressed as

$$\mathbf{y} = \sqrt{\rho}\mathbf{h}\mathbf{x} + \mathbf{z}, \tag{1}$$

where $\mathbf{z}$ is the complex additive white Gaussian noise (AWGN) with zero mean and unit variance, and ρ is the SNR.

III. Performance of Phase Compensation And Constellation Rotation with CSIT

In this section, we analyze the attainable performance of a phase compensation and constellation rotation scheme conceived for SM based on perfect CSIT. The phase compensation cancels the phase angle introduced by the channel at each antenna; while the deterministic constellation rotation helps to better separate the constellation points at the receiver. It can also be optimized for reducing the peak-to-mean ratio. Thus, the phase compensation vector $\mathbf{w}$ is given as

$$\mathbf{w} = [1, \exp(-j\phi_2), \ldots, \exp(-j\phi_{n_t})]^T, \tag{2}$$

where $\phi_i = \varphi_i - \varphi_1, 2 \le i \le n_t$, and $\varphi_i = \angle h_i, 1 \le i \le n_t$ is the phase angle of h_i. The above exploits the fact that one phase angle (φ_1) can be compensated for at the receiver, and therefore does not have to be sent to the transmitter over the feedback channel [23]. The constellation rotation scheme is as follows. The signal constellation transmitted on antenna l is rotated by the deterministic phase angle $(l-1)\theta_0$, where θ_0 is dependent on the rotational symmetry of the constellation. For the M-PSK constellation, we set $\theta_0 = 2\pi/(Mn_t)$. Hence, in the sequel, we will use s_l^i to denote the i^{th} symbol from the *rotated* constellation, and transmitted on the l^{th} antenna. The achievable diversity order of this scheme was analyzed in [17], [18], [25]; in this paper, we are interested in analyzing the SEP performance, under quantized CSIT.

The received signal, given by (1), can then be rewritten as

$$\mathbf{y} = \sqrt{\rho}|h_l|s_l^i + \mathbf{z}. \tag{3}$$

At the receiver, we assume perfect knowledge of the CSI $\mathbf{h}$, and consider the joint maximum likelihood (ML) detection of the TA index and the symbol index, given by

$$[l', i'] = \arg\max_{l\in\{1,\ldots,n_t\}, i\in\mathcal{M}} p\left(\mathbf{y}|s_l^i,\mathbf{h}\right) = \arg\max_{l,i} \left|\mathbf{y} - h_l x_l^i\right|^2, \quad (4)$$

where x_l^i denotes the input to the channel when the l^{th} TA is transmitting the i^{th} constellation symbol. The receiver makes a symbol error if

$$\left|y - h_l x_l^i\right|^2 > \min_{m\in\{1,\ldots,n_t\}, k\in\mathcal{M}:(m,k)\neq(l,i)} \left|y - h_m x_m^k\right|^2. \quad (5)$$

The pairwise error probability (PEP) of decoding the transmit symbol (l, i) pair as the pair (l', i'), conditioned on the CSI $\mathbf{h}$, is given by [21]

$$\Omega_{(l,i)}^{(l',i')}|_{\mathbf{h}} = Q\left(\sqrt{\frac{\rho}{2}}\left|h_l x_l^i - h_{l'} x_{l'}^{i'}\right|\right) \quad (6)$$

$$\approx k\exp\left(-\frac{\rho}{4}\left|h_l x_l^i - h_{l'} x_{l'}^{i'}\right|^2\right), \quad (7)$$

where the latter is due to a Chernoff bound based approximation, and k is a constellation-dependent constant. Averaging over the channel statistics, the PEP can then be written as

$$\Omega_{(l,i)}^{(l',i')} = \mathbb{E}_{\mathbf{h}}\left\{\Omega_{(l,i)}^{(l',i')}|_{\mathbf{h}}\right\}. \quad (8)$$

Using the union bound, the probability of error can be written as [26]

$$P_e = \frac{1}{n_t M} \sum_{(l\in\{1,\ldots,n_t, i\in\mathcal{M})} \sum_{(l',i')\neq(l,i)} \Omega_{(l,i)}^{(l',i')}. \quad (9)$$

It thus remains to derive an expression for $\Omega_{(l,i)}^{(l',i')}$. Given the scheme described above, $\Omega_{(l,i)}^{(l',i')}$ in (8) can be written as

$$\Omega_{(l,i)}^{(l',i')} = k\ \mathbb{E}_{\mathbf{h}}\left\{\exp\left(-\frac{\rho}{4}\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2\right)\right\}. \quad (10)$$

When $l = l'$, that is, when decoding the received signal as another symbol from the same TA, it can be shown that we have:

$$\Omega_{(l,i)}^{(l,i')} = \frac{4k}{4 + d_i^{i'2}\rho}, \quad (11)$$

where $d_i^{i'} = \left|s_l^i - s_l^{i'}\right|$. When $l \neq l'$, we arrive at:

$$\Omega_{(l,i)}^{(l',i')} = \frac{k}{c^2 - d^2}\left[1 + \frac{d}{\sqrt{c^2-d^2}}\left(\frac{\pi}{2} + \tan^{-1}\left(\frac{d}{\sqrt{c^2-d^2}}\right)\right)\right] \quad (12)$$

where $c \triangleq 1 + \rho/4$, $d \triangleq \rho\Re\{s_{l'}^{i'}\left(s_l^i\right)^*\}/4$. The proof of (12) is provided in Appendix A. At high SNRs, using the approximation of $c \approx \rho/4$, it may be readily seen that (12) varies as $1/\rho^2$. However, due to (11), we see that the system has a diversity order of one. This is a consequence of the fact that even if CSIT is available, only a single TA is used to transmit each data symbol. In Sec. VI, we illustrate the accuracy of the above error probability expressions through Monte Carlo simulations.

In the next subsection, we discuss the detrimental effect of spatial correlation between the TAs on the performance. We also present a natural extension of the above phase compensation as well as constellation rotation scheme to an SM system relying on multiple antennas at the receiver.

1) Performance Under Spatial Correlation: Above, we considered the MISO channels between the TAs and the RA to be statistically independent. This is valid when the antennas are spaced sufficiently far apart, and it is also a useful assumption because it facilitates tractable performance analysis.

To gain insight into the effect of spatial correlation that may be present in practical systems where the antennas are insufficiently separated to experience independent fading, let us consider the extreme case where the antennas are perfectly correlated. Then, for conventional SM, it is not possible to determine the TA index using the received data symbols, since the channels of the different TAs are identical. The CSIT-based scheme considered in this work, by contrast, relies on rotated versions of the base constellation. For example, when QPSK signaling is employed, the constellation rotation leads to an effective constellation that may be viewed as a higher-order M-PSK constellation. Similarly, non-constant modulus constellations would map to a superposition of rotated versions of the base constellation. This effective constellation remains decodable at the receiver, unlike the conventional SM system.

For intermediate correlation values, there would be a scaling and superposition of the base constellation, where the scaling coefficients would be correlated. In this case, it may be readily seen that the performance impact on the conventional SM system would be much more severe than on the constellation rotation based scheme. We illustrate this through simulations in Sec. VI.

2) Extension to Multiple Antennas at the Receiver: The CSIT-based transmission scheme described in Sec. III is based on compensating the phase rotation introduced by the complex-valued channel between each TA to the RA. When there are $n_r > 1$ RAs, this is not directly applicable, since there are now n_r complex-valued channels between each TA and the receiver. Hence the question is, which of the n_r channels should be used for the phase compensation? Here, we present a natural extension of the phase compensation scheme, that can be applied when there are multiple RAs. We propose to use the above analysis to compute the probability of error when the phase constellation and constellation rotation scheme is applied to each of the RAs in turn, and select the best antenna based on the probability of error.

Note that this involves only n_r probability of error computations, which is not expensive. Moreover, since phase compensation and constellation rotation is applied based on a single selected antenna, this scheme can be implemented with a the aid of a single receive RF chain, preserving the spirit of SM systems, hence requiring minimal hardware cost at both the transmitter and receiver.

In Sec. VI, we demonstrate the performance improvement that can be obtained by selecting the best from n_r RAs. In particular, we show that the proposed scheme significantly reduces the performance gap between the full diversity transmit antenna selection (TAS) scheme and SM relying on CSIT.

IV. SM with Finite Rate Feedback

In this section, we present the main results of this paper, which is the performance analysis of SM-MISO systems with Q-CSIT. Here, the receiver sends a B-bit scalar-quantized (SQ) version of the phase angles that have to be available at the transmitter, once per channel instantiation. Since $(n_t - 1)$ phase angles have to be quantized, the feedback overhead of the system is $(n_t - 1)B$. As in much of the past work on quantized CSIT, we assume that the feedback channel is both delay- and error-free.

Due to the quantization errors, the channel phase compensation vector $\mathbf{w}$ in (2) becomes

$$\mathbf{w} = [1, \exp(-j\hat{\phi}_2), \ldots, \exp(-j\hat{\phi}_{n_t})]^T, \tag{13}$$

where $\hat{\phi}_i,\ 2 \leq i \leq n_t$ is the quantized version of ϕ_i.

Again, we employ SQ at the receiver, where each of the $(n_t - 1)$ phase angle differences are quantized independent of each other to obtain $\hat{\phi}_i$, $2 \leq i \leq n_t$. Now, with Rayleigh flat-fading, the entries of $\mathbf{h}$ are i.i.d. complex Gaussian. Hence ϕ_i is uniformly distributed over $[0, 2\pi)$. With SQ, the quantization regions are $\mathcal{R}_p \triangleq [(2p-1)\pi/N, (2p+1)\pi/N]$, $0 \leq p \leq N-1$, where $N \triangleq 2^B$. If $\phi_i \in \mathcal{R}_p$, the receiver sends the index p back to the transmitter and the transmitter uses the centroid of $\mathcal{R}_p$ as the quantized phase angle, i.e., $\hat{\phi}_i = 2\pi p/N$, whenever $\phi_i \in \mathcal{R}_p$.

In this work, we use the excess probability of error [22],[1] i.e., the difference between the probability of error with Q-CSIT and with perfect CSIT, as our metric for quantifying the performance degradation due to the use of Q-CSIT instead of perfect CSIT.

Along similar lines as (9) , the probability of error with Q-CSIT, $\hat{P}_e$, can be written as

$$\hat{P}_e = \frac{1}{n_t M} \sum_{(l \in \{1,\ldots,n_t\}, i \in \mathcal{M})} \sum_{(l',i') \neq (l,i)} \hat{\Omega}_{(l,i)}^{(l',i')}, \tag{14}$$

where $\hat{\Omega}_{(l,i)}^{(l',i')}$ is the PEP under Q-CSIT, defined as

$$\hat{\Omega}_{(l,i)}^{(l',i')} \triangleq k\ \mathbb{E}_{\mathbf{h}} \left\{ \exp\left(-\frac{\rho}{4} \left| h_l w_l s_l^i - h_{l'} w_{l'} s_{l'}^{i'} \right|^2 \right) \right\}. \tag{15}$$

Hence, the probability of error degradation $P_{e_L} \triangleq \hat{P}_e - P_e$ is given by

$$P_{e_L} = \frac{1}{n_t M} \sum_{(l \in \{1,\ldots,n_t\}, i \in \mathcal{M})} \sum_{(l',i') \neq (l,i)} \Omega_{L_{(l,i)}}^{(l',i')}, \tag{16}$$

where $\Omega_{L_{(l,i)}}^{(l',i')} \triangleq \hat{\Omega}_{(l,i)}^{(l',i')} - \Omega_{(l,i)}^{(l',i')}$ is the PEP$_L$ and $\Omega_{(l,i)}^{(l',i')}$ is the PEP under perfect CSIT given by (10). Note that once we find P_{e_L}, $\hat{P}_e$ can be obtained from $\hat{P}_e = P_e + P_{e_L}$, with P_e calculated by substituting (11) and (12) into (9).

[1] In [22], the authors use the *relative* loss in probability of error, which the excess probability of error normalized by the probability of error with perfect CSIT. The two metrics are similar in terms of capturing the effect of Q-CSIT.

In order to calculate P_{e_L}, we have to analyze $\Omega_{L_{(l,i)}}^{(l',i')}$, which can be written as

$$\Omega_{L_{(l,i)}}^{(l',i')} = k\ \mathbb{E}_{\mathbf{h}} \left\{ \exp\left(-\frac{\rho}{4} \left| h_l w_l s_l^i - h_{l'} w_{l'} s_{l'}^{i'} \right|^2 \right) - \exp\left(-\frac{\rho}{4} \left| |h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'} \right|^2 \right) \right\}. \tag{17}$$

When $l = l'$, the above expression reduces to 0, since $h_l w_l$ being common to the two terms in the first exponential can be brought out, and note that $|w_l| = 1$. As shown in Appendix B, for $l \neq l'$, equation (17) can be written as

$$\Omega_{L_{(l,i)}}^{(l',i')} = \frac{k\rho}{8} \Re\left\{ s_{l'}^{i'} s_l^{i*} \right\} \left(1 - \tilde{N}^2 \left(2 - 2\cos\left(\frac{1}{\tilde{N}} \right) \right) \right) \times \left[-\frac{(c^2 + 2d^2)\left(\frac{\pi}{2} + \tan^{-1}\left(\frac{d}{\sqrt{c^2 - d^2}} \right) \right)}{(c^2 - d^2)^{\frac{5}{2}}} + \frac{(c^2 - 4d^2)}{d(c^2 - d^2)^2} - \left(\frac{1}{d(c^2 - d^2)} \right) \right], \tag{18}$$

where $\tilde{N} \triangleq N/(2\pi)$, $c \triangleq 1 + \frac{\rho}{4}$, and $d \triangleq \frac{\rho}{4} \Re\{ s_{l'}^{i'} \left(s_l^i \right)^* \}$.

Now, the PEP degradation in (18) can be written as

$$\Omega_{L_{(l,i)}}^{(l',i')} = C_{l_i, l'_{i'}} \left(1 - \left(\frac{N}{2\pi} \right)^2 \left(2 - 2\cos\left(\frac{2\pi}{N} \right) \right) \right), \tag{19}$$

where $C_{l_i, l'_{i'}}$ is a constant that depends only on and the pair of symbols s_l^i and $s_{l'}^{i'}$.

From (16), we thus get

$$P_{e_L} = C \left[1 - \left(\frac{N}{2\pi} \right)^2 \left(2 - 2\cos\left(\frac{2\pi}{N} \right) \right) \right], \tag{20}$$

where

$$C = \frac{1}{n_t M} \sum_{(l,i)} \sum_{(l',i') \neq (l,i)} C_{l_i, l'_{i'}}. \tag{21}$$

In the above, C is now a constant that depends only on the signal constellation and on the SNR. The equation (20) is useful for understanding the effect of the number of quantization regions N on P_{e_L}. For example, using the approximation $\cos(x) \approx 1 - x^2/2! + x^4/4!$ which is valid for small x, i.e., for large N, we arrive at:

$$P_{e_L} = \frac{\pi^2 C}{3} \frac{1}{N^2}. \tag{22}$$

Furthermore, it may be readily shown that C decreases with the SNR as $1/\rho^2$ at high SNRs. Hence, we have the following result:

Proposition 1. *(Variation of P_{e_L} with N and ρ): For high-rate quantization and high SNRs, the probability of error degradation P_{e_L} is inversely proportional to the square of both the number N of quantization levels per phase angle, and the SNR, ρ.*

Recall that the probability of error P_e decreases as $1/\rho$. Since, the probability of error degradation P_{e_L} decreases much faster with the SNR than P_e itself, at high SNRs, there will be a constant gap between the perfect and quantized CSIT cases, in terms of the SNR required for achieving a given P_e.

V. Rotational Symmetry based Phase Compensation (RSPC)

In this section, we propose a new phase compensation scheme for reducing the required number of feedback bits, based on the rotational symmetry of the signal constellation.

Again, we consider the M-PSK signal constellation, which has a rotational symmetry of $\frac{2\pi}{M}$. For this constellation, complete phase compensation is not necessary. It is sufficient to de-rotate the channel to the nearest modulo-$\frac{2\pi}{M}$ phase angle. The effective constellation at the receiver would be a rotated version of the constellation, but the rotation angle is a multiple of $2\pi/M$, which is known at the receiver. Hence, data decoding is possible by simply accounting for the constellation rotation at the receiver. Thus, instead of quantizing the channel-induced phase angle which is uniformly distributed in the range $[0, 2\pi)$, it suffices to quantize a phase angle that is uniformly distributed in the range $[0, 2\pi/M)$. This reduces the feedback rate requirement for a given accuracy of quantization by a factor of $\log_2(M)$ bits per phase angle.

Using the scheme described above, the channel phase range of $[0, 2\pi/M)$ is quantized to B_{rspc} bits using a uniform scalar quantizer. Then, by following the steps in the derivation of the probability of error degradation in Sec. IV, P_{e_L} is still given by (22), but with N replaced by MN_{rspc}, where $N_{\text{rspc}} = 2^{B_{\text{rspc}}}$, and the factor M arises because of the fact that the quantization range is now reduced to $[0, 2\pi/M)$ instead of $[0, 2\pi)$. Hence, to achieve the same P_{e_L}, we need $N = MN_{\text{rspc}}$, which translates to $B_{\text{rspc}} = B - \log_2(M)$, *i.e.,* $\log_2(M)$ fewer bits of feedback would suffice per phase angle. Thus, the RSPC scheme proposed in this section achieves the same P_{e_L} performance as in Sec. IV, while requiring $(n_t - 1)\log_2(M)$ fewer bits of feedback.

VI. Simulation Results

In this section, we present our Monte Carlo simulation results for the SEP performance of SM-MISO with both perfect and quantized CSIT, compare them to previous results, and validate the theoretical expressions. The simulation set-up consists of an $n_t = 4$ TA MISO system using uncoded QPSK transmission ($M = 4$). The wireless channel undergoes Rayleigh fading, where the baseband channel coefficients are generated constant for a channel-use and then they are independently faded based on standard complex normal distribution. The SEP is computed by averaging the performance over 10^4 symbol, channel and noise instantiations.

Figure 1 plots the SEP as a function of the SNR in dB for conventional SM transmission [9], SM with phase compensation and constellation rotation based on perfect CSIT that was proposed in [17], [18] and analyzed in this paper, as well as for SQ-based finite-rate feedback using $B = 2, 3, 4$ and 5 bits per phase angle. We see that the perfect CSIT offers about 6 dB SNR advantage over the conventional SM transmission at an SEP of 10^{-2}. Also shown in figure 1 is the theoretical performance in the presence of perfect CSIT derived in (9), and the close agreement of the simulation and union bound based theoretical result is evident from the graph. Finally, we note that 5 bits of feedback per phase angle

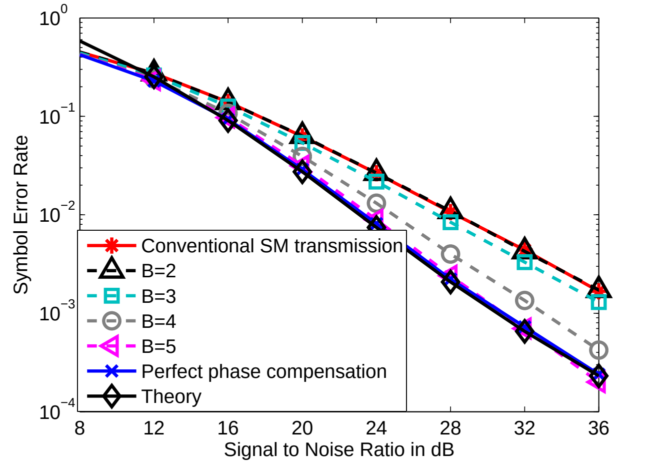

Fig. 1. Comparison of SEP at different feedback rates, for $n_t = 4$ and $M = 4$. Also plotted are the performance of conventional SM and the phase compensation and constellation rotation scheme based on perfect CSIT.

practically closes most of the gap between the conventional SM-MISO and the perfect CSIT based transmission scheme.

Figure 2 shows the SEP as a function of the SNR in dB for the scheme described in Sec. V, which exploits the 4-fold rotational symmetry of the QPSK constellation. It can be seen that the RSPC scheme achieves the same performance as the complete phase compensation scheme in Fig. 1, with $\log_2(M) = 2$ fewer bits of feedback per antenna, as expected.

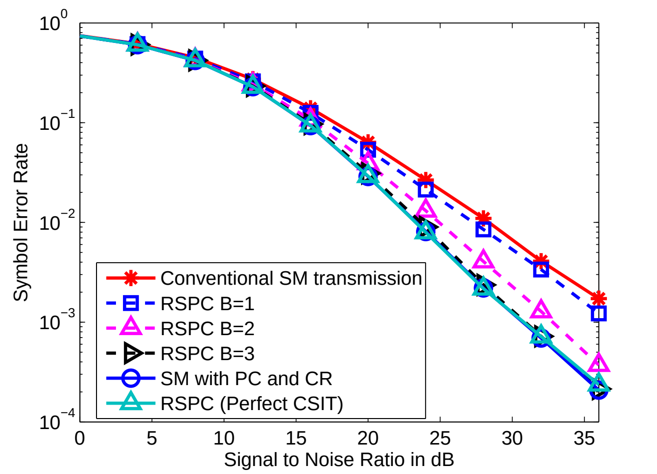

Fig. 2. Comparison of SEP at different feedback rates for the RSPC scheme described in Sec. V, for $n_t = 4$ and $M = 4$.

Next, the variation of the probability of error degradation P_{eL} versus the number of feedback bits B at the SNR values of $\rho = 4, 12$ and 16 dB is shown in Fig. 3. Here, we compare the simulation results to the theoretical calculation of P_{e_L} obtained from (20). The close agreement of the simulation and theoretical result is clear from the graph. It can also be seen that P_{e_L} decreases exponentially with B, as expected from (22).

In Fig. 4, we study the effect of spatial correlation at the transmitter. We model the correlation coefficient between the

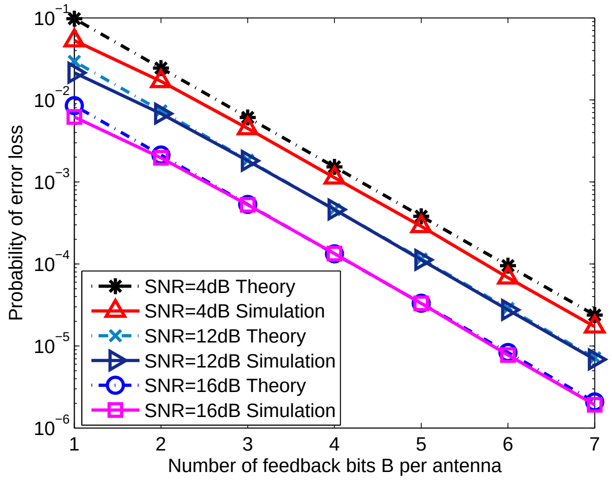

Fig. 3. Variation of P_{e_L} with number of feedback bits per antenna B, for $n_t = 4$ and $M = 4$, and at different SNRs.

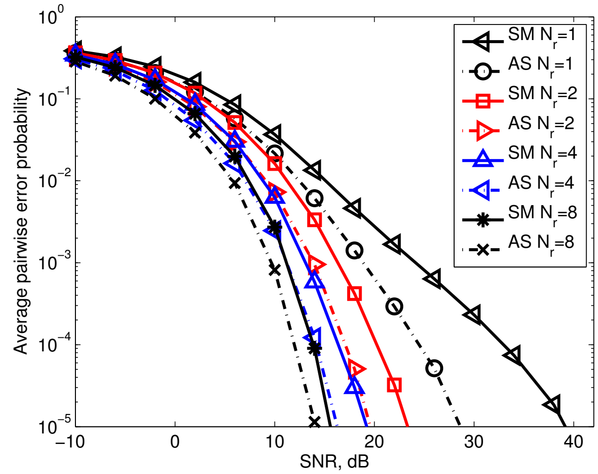

Fig. 5. Comparison of transmit antenna selection and SM with CSIT with multiple antennas at the receiver.

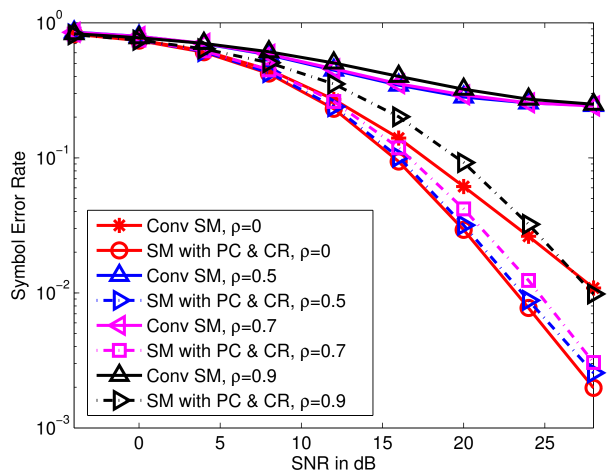

Fig. 4. SEP of SM with phase compensation (PC) and constellation rotation (CR) in the presence of correlated TAs for various values of the correlation coefficient ρ.

antenna indexed by i and j as $\rho^{|i-j|}$, where $\rho \in [0, 1]$, i.e., the $n_t \times n_t$ spatial correlation matrix $\mathbf{R}$ has entries given by $[\mathbf{R}]_{i,j} = \rho^{|i-j|}$ [27]. As expected from the discussion in Sec. III-1, the performance of the conventional SM system significantly deteriorates when the signals of the antennas are spatially correlated (i.e., when $\rho > 0$), while the phase compensation and constellation rotation scheme is significantly more robust, only suffering a modest performance drop even at ρ as high as 0.9.

Fig. 5 compares the constellation rotation and phase compensation scheme against transmit antenna selection (TAS), for various number of RAs. At the receiver, for fairness of comparison and in-line with the spirit of SM systems, we consider a system with multiple antenna elements and a single receive RF chain. With TAS, we select the transmit-receive antenna pair associated with the largest channel magnitude for communication, and employ an uncoded 16-PSK constellation. This corresponds to four bits per channel use, the same as that of the SM system. For the SM system with CSIT, we use the scheme proposed in Sec. III-2 for receive antenna selection, and perform phase compensation and constellation rotation corresponding to the selected receive antenna. We observe that for $n_r = 1$, TAS outperforms SM relying on CSIT, as expected, which is due to the higher diversity gain offered by TAS. However, as n_r increases, the gap reduces significantly, since the SM system is also capable of exploiting the receive diversity.

VII. Conclusions

In this paper, we investigated the impact of both full and quantized CSIT in an SM-MISO system, where the CSI at the transmitter was used for phase compensation and deterministic constellation rotation. In the case of perfect CSIT, we obtained a closed-form analytical expression for the PEP. Furthermore, using the union bound, we presented an expression for the SEP performance also. Next, we analyzed the performance of the scheme when the phase angles are quantized to a finite number of bits using scalar quantization and sent to the transmitter over a delay- and error-free finite-rate feedback link. We derived the SEP degradation relative to the perfect CSIT case, which led to a closed-form expression for the SEP performance under quantized CSIT. We showed that the SEP degradation decreases inversely with the square of the number of quantization levels as well as the SNR. Through simulations, we illustrated the close agreement between the theoretical as well as analytical expressions, and showed that about $B = 5$ bits of feedback per phase angle is sufficient to attain nearly the same performance as with perfect CSIT. Future work could involve extension of the results to the MIMO scenario, the use of vector quantization techniques, and the associated performance analysis.

Acknowledgements

The first author would like to thank Geethu Joseph for several fruitful discussions. The authors would also like to thank the reviewers for their insightful feedback, which have greatly improved this paper.

Appendix A
Calculation of PEP Under Perfect CSIT

In this section, we derive the pairwise error probability (PEP) of decoding the i^{th} symbol transmitted from the l^{th} antenna as the i'^{th} symbol from the l'^{th} antenna, when perfect CSI is available at the transmitter. The PEP, denoted by $\Omega_{(l,i)}^{(l',i')}$, is given by

$$\Omega_{(l,i)}^{(l',i')} = \mathbb{E}_{\mathbf{h}}\left\{\exp\left(-\frac{\rho}{4}\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2\right)\right\}. \quad (23)$$

Let $X = |h_l|$, $Y = |h_{l'}|$, $a = s_l^i$ and $b = s_{l'}^{i'}$. Since X and Y are Rayleigh distributed, (23) can be written as

$$\Omega_{(l,i)}^{(l',i')} = \int\limits_{x=0}^{\infty}\int\limits_{y=0}^{\infty} 2xe^{-x^2}2ye^{-y^2}e^{-\frac{\rho}{4}|ax-by|^2}\mathrm{d}x\mathrm{d}y. \quad (24)$$

Let $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Then, the right hand side of the above can be written as

$$\Omega_{(l,i)}^{(l',i')} = 2\int\limits_{\theta=0}^{\frac{\pi}{2}}\int\limits_{r=0}^{\infty} e^{-r^2\left(1+\frac{\rho}{4}(1-\Re(ab^*)\sin(2\theta))\right)} \times r^3\sin(2\theta)\mathrm{d}r\mathrm{d}\theta.$$

Let $K(\theta) = \left(\underbrace{\left(1+\frac{\rho}{4}\right)}_{c} - \underbrace{\frac{\rho}{4}\Re(ab^*)}_{d}\sin(2\theta)\right)$. Then,

$$\begin{aligned}\Omega_{(l,i)}^{(l',i')} &= 2\int_{\theta=0}^{\pi/2}\sin(2\theta)\int_{r=0}^{\infty} r^3 e^{-K(\theta)r^2}\mathrm{d}r\mathrm{d}\theta \\ &= \int_{\theta=0}^{\pi/2}\frac{\sin(2\theta)}{(c-d\sin(2\theta))^2}\mathrm{d}\theta \\ &= \left[\frac{c\cos(2\theta)}{2(c^2-d^2)(d\sin(2\theta)-c)} - \frac{d\tan^{-1}\left(\frac{d-c\tan(\theta)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}\right]_0^{\pi/2} \\ &= \frac{1}{c^2-d^2}\left[1+\frac{d}{\sqrt{c^2-d^2}}\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{d}{\sqrt{c^2-d^2}}\right)\right)\right].\end{aligned} \quad (25)$$

Appendix B
Calculation of Loss in PEP

The loss in PEP due to the quantization of the CSI available at the transmitter is given by

$$\Omega_{L_{(l,i)}}^{(l',i')} = k\ \mathbb{E}_{\mathbf{h}}\left\{\exp\left(-\frac{\rho}{4}\left|h_l w_l s_l^i - h_{l'}w_{l'}s_{l'}^{i'}\right|^2\right) - \exp\left(-\frac{\rho}{4}\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2\right)\right\}. \quad (26)$$

Using $e^{-A} - e^{-B} = e^{-B}\left[e^{-(A-B)} - 1\right] \approx e^{-B}[B-A]$, which is valid when $(B-A)$ is small, i.e., for high-rate quantization, the right hand side of (26) can be written as $k\rho\mathbb{E}_{\mathbf{h}}\{(\alpha_1 - \alpha_2)\}/4$, where $\alpha_1 \triangleq \exp\left(-\rho\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2/4\right)\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2$ and $\alpha_2 \triangleq \exp\left(-\rho\left||h_l|\, s_l^i - |h_{l'}|\, s_{l'}^{i'}\right|^2/4\right)\left|h_l w_l s_l^i - h_{l'}w_{l'}s_{l'}^{i'}\right|^2$. We will now discuss the simplification of $\mathbb{E}_{\mathbf{h}}\{\alpha_1\}$ and $\mathbb{E}_{\mathbf{h}}\{\alpha_2\}$ separately.

1) Solving for $\mathbb{E}_{\mathbf{h}}\{\alpha_1\}$*:* Let $X = |h_l|$, $Y = |h_{l'}|$, $a = s_l^i$ and $b = s_{l'}^{i'}$. Note that, under M-PSK signaling, $|a| = |b| = 1$. Then, $\mathbb{E}_{\mathbf{h}}\{\alpha_1\}$ can be written as

$$\begin{aligned}&\mathbb{E}_{X,Y}\left\{\exp\left(-\frac{\rho}{4}|aX-bY|^2\right)|aX-bY|^2\right\} \\ &= \int\limits_{x=0}^{\infty}\int\limits_{y=0}^{\infty} 2xe^{-x^2}2ye^{-y^2}|ax-by|^2 e^{-\frac{\rho}{4}|ax-by|^2}\mathrm{d}x\mathrm{d}y\end{aligned} \quad (27)$$

Substituting $x = r\cos(\theta)$, and $y = r\sin(\theta)$, we arrive at

$$\mathbb{E}_{\mathbf{h}}\{\alpha_1\} = 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} r^5\sin(2\theta)\left(1-\Re(ab^*)\sin(2\theta)\right) \times e^{\left(-r^2\left(1+\frac{\rho}{4}-\frac{\Re(ab^*)}{2\sigma^2}\sin(2\theta)\right)\right)}\mathrm{d}r\mathrm{d}\theta. \quad (28)$$

Let $K = c - d\sin(2\theta)$ where $c \triangleq 1+\rho/4$ and $d \triangleq \rho\Re(ab^*)/4$, as before, and $\gamma_1 \triangleq \Re(ab^*)$. Then, we get

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\{\alpha_1\} &= 2\int\limits_{\theta=0}^{\pi/2}\sin(2\theta)\left(1-\gamma_1\sin(2\theta)\right)\int\limits_{r=0}^{\infty} r^5 e^{-Kr^2}\mathrm{d}r\mathrm{d}\theta \\ &= 2\int\limits_{\theta=0}^{\pi/2}\sin(2\theta)\left(1-\gamma_1\sin(2\theta)\right)\frac{1}{K^3}\mathrm{d}\theta.\end{aligned} \quad (29)$$

Substituting for K and simplifying leads to:

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\{\alpha_1\} &= 2\int_{\theta=0}^{\pi/2}\left[\frac{\sin(2\theta)\left(1-\gamma_1\sin(2\theta)\right)}{\left(c-d\sin(2\theta)\right)^3}\right]\mathrm{d}\theta \\ &= \frac{1}{4}\left[\frac{2\left(c^2\gamma_1 - 3cd + 2d^2\gamma_1\right)\tan^{-1}\left(\frac{d-c\tan(\theta)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{\frac{5}{2}}} + \frac{\cos(2\theta)(c^3\gamma_1 + c^2 d - 4cd^2\gamma_1 + 2d^3)}{d(c^2-d^2)^2(d\sin(2\theta)-c)} + \frac{c\cos(2\theta)(c\gamma_1 - d)}{d(c^2-d^2)(d\sin(2\theta)-c)^2}\right]_0^{\frac{\pi}{2}} \\ &= -\frac{\left(c^2\gamma_1 - 3cd + 2d^2\gamma_1\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{d}{\sqrt{c^2-d^2}}\right)\right)}{2\left(c^2-d^2\right)^{\frac{5}{2}}} + \frac{(c^3\gamma_1 + c^2 d - 4cd^2\gamma_1 + 2d^3)}{2cd(c^2-d^2)^2} - \frac{(c\gamma_1 - d)}{2cd(c^2-d^2)}.\end{aligned} \quad (30)$$

2) Solving for $\mathbb{E}_{\mathbf{h}}\{\alpha_2\}$*:* By splitting the expression for $\mathbb{E}_{\mathbf{h}}\{\alpha_2\}$ into different Voronoi regions formed by SQ of the phase angles, we can write

$$\mathbb{E}_{\mathbf{h}}\left\{\alpha_2\right\} = \sum_{p,q=0}^{N-1} P\left(h_l \in \mathcal{R}_p\right) P\left(h_{l'} \in \mathcal{R}_q\right)$$
$$\mathbb{E}_{h_l\in\mathcal{R}_p,h_{l'}\in\mathcal{R}_q}\left\{e^{-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2}\left|h_l w_l s_l^i - h_{l'} w_{l'} s_{l'}^{i'}\right|^2\right\}$$
$$= \frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\mathbb{E}_{h_l\in\mathcal{R}_p,h_{l'}\in\mathcal{R}_q}\left\{e^{-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2}\right.$$
$$\left.\times\underbrace{\left(\left||h_l|\,e^{j\phi_l}w_p s_l^i - |h_{l'}|\,e^{j\phi_{l'}}w_q s_{l'}^{i'}\right|^2\right)}_{\gamma}\right\}. \quad (31)$$

Simplifying, we get

$$\mathbb{E}_{\mathbf{h}}\left\{\alpha_2\right\} = \frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}$$
$$\mathbb{E}_{|h_l|,|h_{l'}|,\phi_l\in\mathcal{R}_p,\phi_{l'}\in\mathcal{R}_q}\left\{e^{\left(-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2\right)}\gamma\right\}$$
$$= \frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\mathbb{E}_{|h_l|,|h_{l'}|}\left\{e^{\left(-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2\right)}\right.$$
$$\left.\mathbb{E}_{\phi_l\in\mathcal{R}_p,\phi_{l'}\in\mathcal{R}_q}\left\{\gamma\right\}\right\}. \quad (32)$$

In arriving at (32), we exploited the fact that scalar quantization of the phase angles is performed, i.e., the channel gain h_i lies inside the Voronoi region $\mathcal{R}_p$, if the channel phase angle ϕ_i lies in $\mathcal{R}_p$. The amplitude $|h_i|$ can, of course, take any nonnegative value. By bringing the summation inside, it can be written as

$$\mathbb{E}_{\mathbf{h}}\left\{\alpha_2\right\} = \mathbb{E}_{|h_l|,|h_{l'}|}\left\{e^{\left(-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2\right)}\right.$$
$$\left.\frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\mathbb{E}_{\phi_l\in\mathcal{R}_p,\phi_{l'}\in\mathcal{R}_q}\left\{\gamma\right\}\right\}. \quad (33)$$

The simplification of $\mathbb{E}_{\phi_l\in\mathcal{R}_p,\phi_{l'}\in\mathcal{R}_q}\left\{\gamma\right\}$ is given by (34) at the top the next page. We thus have

$$\mathbb{E}_{\phi_l\in\mathcal{R}_p,\phi_{l'}\in\mathcal{R}_q}\left\{\gamma\right\} = |h_l|^2 + |h_{l'}|^2 - 2\,|h_l|\,|h_{l'}|\,\gamma_2, \quad (35)$$

where

$$\gamma_2 = \left(\frac{N}{2\pi}\right)^2\int_{\phi_l=\frac{(2p-1)\pi}{N}}^{\frac{(2p+1)\pi}{N}}\int_{\phi_{l'}=\frac{(2q-1)\pi}{N}}^{\frac{(2q+1)\pi}{N}}$$
$$\left[\Re\left\{s_{l'}^{i'}s_l^{i*}\right\}\cos\left((\phi_l-\phi_{l'})-(\hat{\phi}_p-\hat{\phi}_q)\right)\right.$$
$$\left.-\Im\left\{s_{l'}^{i'}s_l^{i*}\right\}\sin\left((\phi_l-\phi_{l'})-(\hat{\phi}_p-\hat{\phi}_q)\right)\right]\mathrm{d}\phi_l\mathrm{d}\phi_{l'}. \quad (36)$$

Let $A = \hat{\phi}_p - \hat{\phi}_q = \frac{2\pi}{N}(p-q)$. Then,

$$\int_{\phi_{l'}=\frac{(2q-1)\pi}{N}}^{\frac{(2q+1)\pi}{N}}\int_{\phi_l=\frac{(2p-1)\pi}{N}}^{\frac{(2p+1)\pi}{N}}\cos\left((\phi_l-\phi_{l'})-A\right)\mathrm{d}\phi_l\mathrm{d}\phi_{l'}$$
$$= 2 - 2\cos\left(\frac{2\pi}{N}\right). \quad (37)$$

Similarly,

$$\int_{\phi_{l'}=\frac{(2q-1)\pi}{N}}^{\frac{(2q+1)\pi}{N}}\int_{\phi_l=\frac{(2p-1)\pi}{N}}^{\frac{(2p+1)\pi}{N}}\sin\left((\phi_l-\phi_{l'})-A\right)\mathrm{d}\phi_l\mathrm{d}\phi_{l'} = 0. \quad (38)$$

Hence, from (36),

$$\gamma_2 = \Re\left\{s_{l'}^{i'}s_l^{i*}\right\}\left(2-2\cos\left(\frac{2\pi}{N}\right)\right). \quad (39)$$

Substituting (39) and (35) in (33), we get

$$\mathbb{E}_{\mathbf{h}}\{\alpha_2\} = \mathbb{E}_{|h_l|,|h_{l'}|}\left\{e^{\left(-\frac{\rho}{4}\left||h_l|s_l^i-|h_{l'}|s_{l'}^{i'}\right|^2\right)}\right.$$
$$\left.\left(|h_l|^2 + |h_{l'}|^2 - 2\,|h_l|\,|h_{l'}|\,\gamma_2\right)\right\}, \quad (40)$$

which is similar to (27) and can be written, following the same procedure, as

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\{\alpha_2\} &= 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} r^5\sin(2\theta)\left(1-\gamma_2\sin(2\theta)\right) \\ &\quad e^{\left(-r^2\left(1+\frac{\rho}{4}-\frac{\rho}{4}\Re(ab^*)\sin(2\theta)\right)\right)}\mathrm{d}r\mathrm{d}\theta \\ &= 2\int_{\theta=0}^{\pi/2}\left[\frac{\sin(2\theta)\left(1-\gamma_2\sin(2\theta)\right)}{\left(c-d\sin(2\theta)\right)^3}\right]\mathrm{d}\theta. \quad (41)\end{aligned}$$

Now, (41) is similar to the first equation in (30). Hence, it can be simplified using the same procedure as in (30), and thus, the right hand side of $\Omega_{L_{(l,i)}}^{(l',i')} = \mathbb{E}_{\mathbf{h}}\left\{\frac{k\rho}{4}\left(\alpha_1-\alpha_2\right)\right\}$ can be written as

$$\frac{k\rho}{8}\left(\gamma_1-\gamma_2\right)\left[-\frac{\left(c^2+2d^2\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{d}{\sqrt{(c^2-d^2)}}\right)\right)}{\left(c^2-d^2\right)^{\frac{5}{2}}}\right.$$
$$\left.+\frac{(c^2-4d^2)}{d(c^2-d^2)^2}-\left(\frac{1}{d(c^2-d^2)}\right)\right] \quad (42)$$

$$= \frac{k\rho}{8}\Re\left\{s_{l'}^{i'}s_l^{i*}\right\}\left(1-\left(\frac{N}{2\pi}\right)^2\left(2-2\cos\left(\frac{2\pi}{N}\right)\right)\right)$$
$$\left[-\frac{\left(c^2+2d^2\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{d}{\sqrt{(c^2-d^2)}}\right)\right)}{\left(c^2-d^2\right)^{\frac{5}{2}}}\right.$$
$$\left.+\frac{(c^2-4d^2)}{d(c^2-d^2)^2}-\left(\frac{1}{d(c^2-d^2)}\right)\right]$$
$$= C_{l_i,l'_{i'}}\left(1-\left(\frac{N}{2\pi}\right)^2\left(2-2\cos\left(\frac{2\pi}{N}\right)\right)\right), \quad (43)$$

where

$$C_{l_i,l'_{i'}} = \frac{k\rho}{8}\left[-\frac{\left(c^2+2d^2\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{d}{\sqrt{(c^2-d^2)}}\right)\right)}{\left(c^2-d^2\right)^{\frac{5}{2}}}\right.$$
$$\left.+\frac{(c^2-4d^2)}{d(c^2-d^2)^2}-\left(\frac{1}{d(c^2-d^2)}\right)\right]\Re\left\{s_{l'}^{i'}s_l^{i*}\right\}. \quad (44)$$

$$\mathbb{E}_{\phi_l \in \mathcal{R}_p, \phi_{l'} \in \mathcal{R}_q} \{\gamma\} = \left(\frac{N}{2\pi}\right)^2 \int\limits_{\phi_l \in \mathcal{R}_p, \phi_{l'} \in \mathcal{R}_q} ||h_l| e^{j\phi_l} w_p s_l^i - |h_{l'}| e^{j\phi_{l'}} w_q s_{l'}^{i'}|^2 \mathrm{d}\phi_l \mathrm{d}\phi_{l'}$$

$$= \left(\frac{N}{2\pi}\right)^2 \int\limits_{\phi_l = \frac{(2p-1)\pi}{N}}^{\frac{(2p+1)\pi}{N}} \int\limits_{\phi_{l'} = \frac{(2q-1)\pi}{N}}^{\frac{(2q+1)\pi}{N}} \left(\left| |h_l| e^{j(\phi_l - \hat{\phi}_p)} s_l^i - |h_{l'}| e^{j(\phi_{l'} - \hat{\phi}_q)} s_{l'}^{i'} \right|^2 \right) \mathrm{d}\phi_l \mathrm{d}\phi_{l'}$$

$$= \left(\frac{N}{2\pi}\right)^2 \int\limits_{\phi_l = \frac{(2p-1)\pi}{N}}^{\frac{(2p+1)\pi}{N}} \int\limits_{\phi_{l'} = \frac{(2q-1)\pi}{N}}^{\frac{(2q+1)\pi}{N}} \left(|h_l|^2 + |h_{l'}|^2 - 2\,|h_l|\,|h_{l'}|\ \Re\left\{ s_{l'}^{i'} s_l^{i*} e^{j((\phi_l - \hat{\phi}_p) - (\phi_{l'} - \hat{\phi}_q))} \right\} \right) \mathrm{d}\phi_l \mathrm{d}\phi_{l'} \quad (34)$$

The above expression for the loss in PEP $\Omega_{L_{(l,i)}}^{(l',i')}$ can be used to study the effects of Q-CSIT instead of perfect CSIT in our schemes, as detailed in Section IV.

References

[1] M. Di Renzo, H. Haas, and P. M. Grant, "Spatial modulation for multiple-antenna wireless systems: a survey," *IEEE Commun. Mag.*, vol. 49, no. 12, pp. 182–191, Dec. 2011.

[2] M. D. Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: challenges, opportunities and implementation," *Proc. IEEE*, vol. 102, no. 1, pp. 56–103, Jan. 2014. [Online]. Available: http://eprints.soton.ac.uk/354175/

[3] P. Yang, M. Di Renzo, Y. Xiao, S. Li, and L. Hanzo, "Design guidelines for spatial modulation," *IEEE Commun. Surveys Tuts.*, vol. PP, no. 99, pp. 1–1, 2014.

[4] Y. A. Chau and S.-H. Yu, "Space modulation on wireless fading channels," in *Proc. IEEE Veh. Technol. Conf. Fall*, Oct. 2001, pp. 1668–1671.

[5] S. Song, Y. Yang, Q. Xiong, K. Xie, B.-J. Jeong, and B. Jiao, "A channel hopping technique I: Theoretical studies on band efficiency and capacity," in *Proc. IEEE Int. Conf. Commun. Circuits Syst.*, Jun. 2004, pp. 229–233.

[6] R. Mesleh, H. Haas, Y. Lee, and S. Yun, "Interchannel interference avoidance in MIMO transmission by exploiting spatial information," in *Proc. IEEE Int. Symp. Pers. Indoor Mobile Radio Commun.*, Sep. 2005, pp. 141–145.

[7] R. Mesleh, H. Haas, C. W. Ahn, and S. Yun, "Spatial modulation - a new low complexity spectral efficiency enhancing technique," in *Proc. CHINACOM*, Oct. 2006, pp. 1–5.

[8] J. Jeganathan, "Space shift keying modulation for MIMO channels," M.A.Sc. thesis, Dept. Elect. Eng., Concordia University, Montreal, QC, Aug. 2008.

[9] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, pp. 2228–2241, Jul. 2008.

[10] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545–547, Aug. 2008.

[11] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, "Space shift keying modulation for MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3692–3703, Jul. 2009.

[12] M. D. Renzo and H. Haas, "Bit error probability of SM-MIMO over generalized fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1124–1144, Mar. 2012.

[13] P. Yang, Y. Xiao, Y. Yu, and S. Q. Li, "Adaptive spatial modulation for wireless MIMO transmission systems," *IEEE Commun. Lett.*, vol. 15, no. 6, pp. 602–605, Jun. 2011.

[14] P. Yang, Y. Xiao, L. Li, Q. Tang, Y. Yu, and S. Li, "Link adaptation for spatial modulation with limited feedback," *IEEE Trans. Veh. Technol.*, vol. 61, no. 8, pp. 3808–3813, Oct. 2012.

[15] P. Yang, Y. Xiao, Y. Yu, L. Li, Q. Tang, and S. Li, "Simplified adaptive spatial modulation for limited-feedback MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, Jul. 2013.

[16] P. Yang, Y. Xiao, B. Zhang, S. Li, M. El-Hajjar, and L. Hanzo, "Power allocation aided spatial modulation for limited-feedback MIMO systems," *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1–1, 2014.

[17] K. Ntontin, M. D. Renzo, A. Perez-Neira, and C. Verikoukis, "Adaptive generalized space shift keying (GSSK) modulation for MISO channels: A new method for high diversity and coding gains," in *Proc. IEEE Veh. Technol. Conf. Fall*, Sep. 2012.

[18] ——, "Adaptive generalized space shift keying," *EURASIP Journal on Wireless Communications and Networking*, no. 1, p. 43, 2013.

[19] X. Guan, Y. Cai, and W. Yang, "On the mutual information and precoding for spatial modulation with finite alphabet," *IEEE Commun. Lett.*, vol. 2, no. 4, pp. 383–386, Aug. 2013.

[20] J. M. Luna-Rivera, D. U. Campos-Delgado, and M. G. Gonzalez-Perez, "Constellation design for spatial modulation," *Procedia Technol.*, vol. 7, pp. 71–78, 2013.

[21] M. D. Renzo and H. Haas, "On transmit diversity for spatial modulation MIMO: impact of spatial constellation diagram and shaping filters at the transmitter," *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 2507–2531, Jul. 2013.

[22] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[23] C. R. Murthy and B. D. Rao, "Quantization methods for equal gain transmission with finite rate feedback," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 233–245, Jan. 2007.

[24] D. J. Love, R. Heath, V. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[25] C. Masouros, "Improving the diversity of spatial modulation in MISO channels by phase alignment," *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 729–732, May 2014.

[26] M. Maleki, H. R. Bahrami, A. Alizadeh, and N. H. Tran, "On the performance of spatial modulation: Optimal constellation breakdown," *IEEE Trans. Wireless Commun.*, vol. 62, no. 1, pp. 144–157, Jan. 2014.

[27] T. Handte, A. Muller, and J. Speidel, "BER analysis and optimization of generalized spatial modulation in correlated fading channels," in *Proc. IEEE Veh. Technol. Conf. Fall*, Sep. 2009, pp. 1–5.

Mishfad Shaikh Veedu was born in Kerala, India. He received the B. Tech. degree in Electronics and Communication Engineering from the M. G. University, Kerala, India, and the M. E. degree in Telecommunication Engineering from the Indian Institute of Science, Bangalore, India, in 2011 and 2014, respectively. He is currently working at P.E.S. Institute of Technology, Bangalore South Campus, Bangalore. His research interests are broadly in the applications of signal processing to wireless communication systems.

Chandra R. Murthy (S'03–M'06–SM'11) received the B.Tech. degree in Electrical Engineering from the Indian Institute of Technology, Madras in 1998, the M.S. and Ph.D. degrees in Electrical and Computer Engineering from Purdue University and the University of California, San Diego, in 2000 and 2006, respectively.

From 2000 to 2002, he worked as an engineer for Qualcomm Inc., where he worked on WCDMA baseband transceiver design and 802.11b baseband receivers. From Aug. 2006 to Aug. 2007, he worked as a staff engineer at Beceem Communications Inc. on advanced receiver architectures for the 802.16e Mobile WiMAX standard. In Sept. 2007, he joined the Department of Electrical Communication Engineering at the Indian Institute of Science, where he is currently working as an Associate Professor. His research interests are in the areas of Cognitive Radio, Energy Harvesting Wireless Sensors, MIMO systems with channel-state feedback, and sparse signal recovery techniques applied to wireless communications. He is currently serving as the Chair of the IEEE Signal Processing Society, Bangalore Chapter and as an associate editor for the IEEE Signal Processing Letters. He is an elected member of the IEEE SPCOM Technical Committee for the years 2014-16.

Lajos Hanzo (http://www-mobile.ecs.soton.ac.uk) FREng, FIEEE, FIET, Fellow of EURASIP, DSc received his degree in electronics in 1976 and his doctorate in 1983. In 2009 he was awarded the honorary doctorate "Doctor Honoris Causa" by the Technical University of Budapest. During his 38-year career in telecommunications he has held various research and academic posts in Hungary, Germany and the UK. Since 1986 he has been with the School of Electronics and Computer Science, University of Southampton, UK, where he holds the chair in telecommunications. He has successfully supervised about 100 PhD students, co-authored 20 John Wiley/IEEE Press books on mobile radio communications totalling in excess of 10 000 pages, published 1400+ research entries at IEEE Xplore, acted both as TPC and General Chair of IEEE conferences, presented keynote lectures and has been awarded a number of distinctions. His published work has over 20 000 citations. Currently he is directing a 100-strong academic research team, working on a range of research projects in the field of wireless multimedia communications sponsored by industry, the Engineering and Physical Sciences Research Council (EPSRC) UK, the European Research Council's Advanced Fellow Grant and the Royal Society's Wolfson Research Merit Award. He is an enthusiastic supporter of industrial and academic liaison and he offers a range of industrial courses. He is also a Governor of the IEEE VTS. During 2008 - 2012 he was the Editor-in-Chief of the IEEE Press and a Chaired Professor also at Tsinghua University, Beijing. His research is funded by the European Research Council's Senior Research Fellow Grant. Further information on research in progress and associated publications is available at http://www-mobile.ecs.soton.ac.uk.