

# Design of Communication Systems with Energy Harvesting Transmitters and Receivers

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# Outline

- ▶ Introduction
- ▶ Retransmission-based multi-hop links
  1. PDP analysis
  2. Energy management policies
- ▶ Uncoordinated EH links
  1. Upper bounds
  2. Energy management policies
- ▶ Conclusions

# Introduction

- ▶ 5G vision: connectivity to massive number of sensors
- ▶ Limited lifetime due to pre-charged batteries
- ▶ Potential solution: energy harvesting nodes (EHN)

- ▶ Harvesting sources
  - ▶ Solar, thermal, RF etc.



Figure: Forest fire monitoring

(<http://news.mit.edu/2008/trees-0923>)

# Challenges

- ▶ Energy neutrality constraint (ENC)

$$\sum_{n=1}^N e_n \leq \sum_{n=1}^N \mathcal{E}_n, \text{ for all } N$$

$e_n$  : energy consumed at  $n^{\text{th}}$  slot

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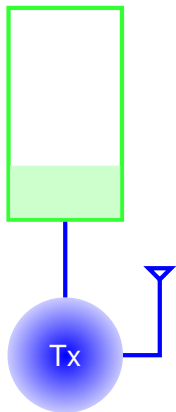
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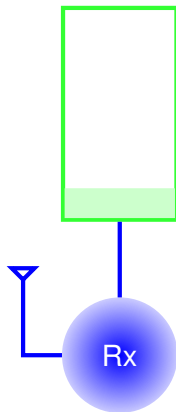
- ▶ Goal shifts from energy conservation to judicious energy consumption
- ▶ Random and sporadic nature of the harvested energy
  - ▶ Necessitates the design of energy management policies
- ▶ Measurement of accurate state-of-charge (SoC) is difficult
  - ▶ SoC-independent policies

# Challenges: EH Receivers

Battery size :  $B_{\max}^t$



Battery size :  $B_{\max}^r$



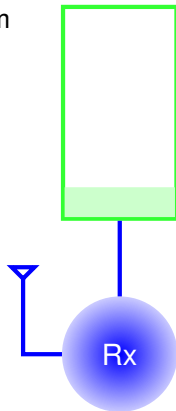
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Battery size :  $B_{\max}^t$



- Nodes harvest energy from distinct sources

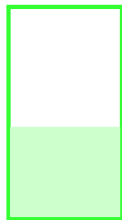
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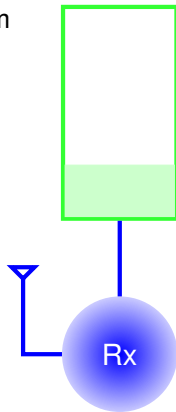
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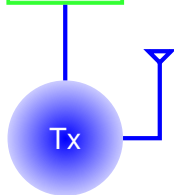
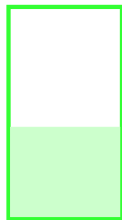
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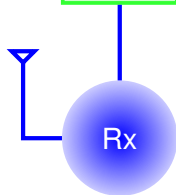
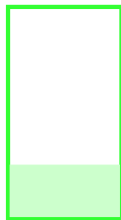
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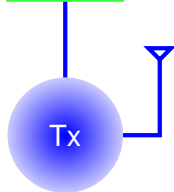
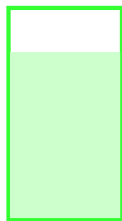
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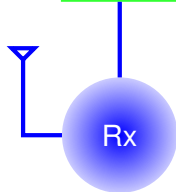
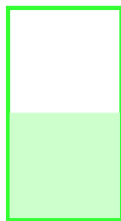
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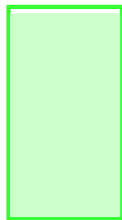
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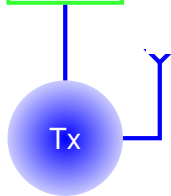


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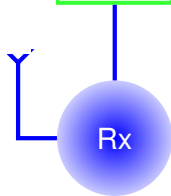
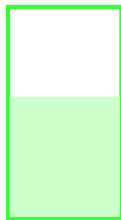
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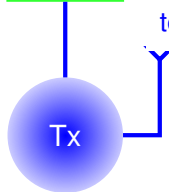
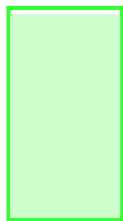


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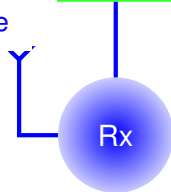
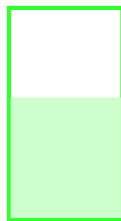
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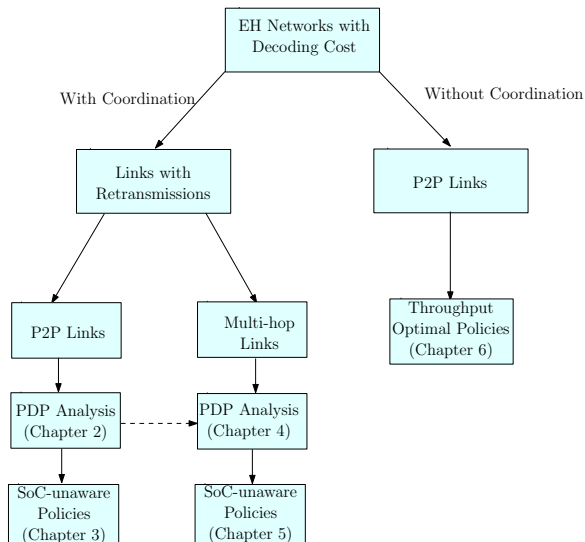


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- **Uncertainty about the battery state of the other node**

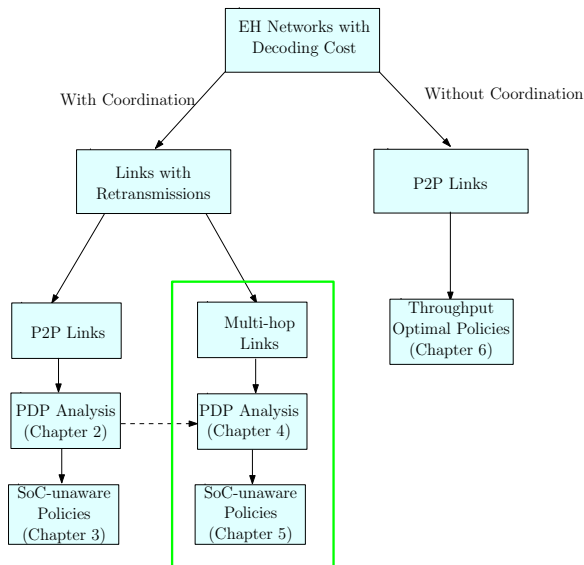
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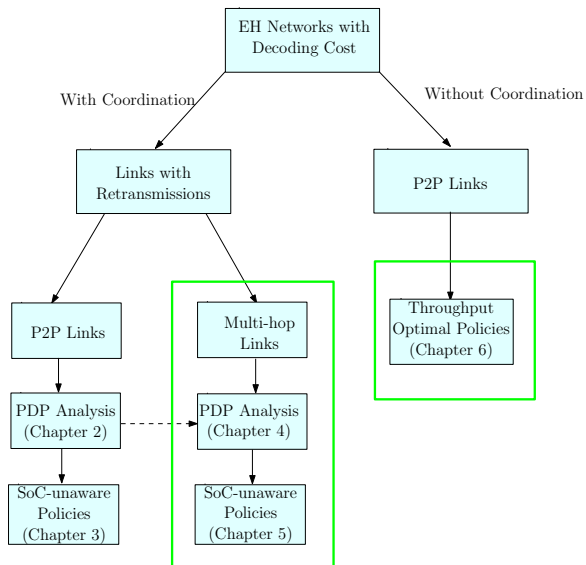
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## Journal Publications:

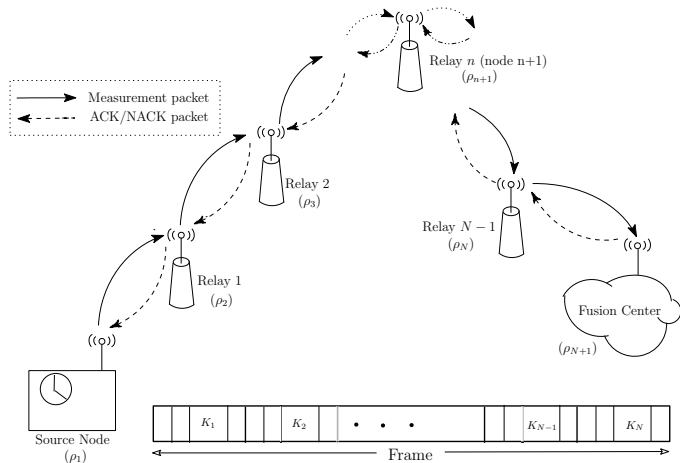
- ▶ M. Sharma and C. R. Murthy, "Packet Drop Probability Analysis of Dual Energy Harvesting Links with Retransmission," *IEEE J. Sel. Areas in Commun.*, vol. 34, no. 12, pp. 3646 - 3660, Dec. 2016.
- ▶ M. Sharma and C. R. Murthy, "On Design of Dual Energy Harvesting Communication Links With Retransmission," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 4079 - 4093, Jun. 2017.
- ▶ M. Sharma and C. R. Murthy, "Distributed Power Control for Multi-hop Energy Harvesting Links with Retransmission," to appear in *IEEE Trans. Wireless Commun.*, Mar. 2018.
- ▶ M. Sharma, C. R. Murthy and R. Vaze, "Asymptotically Optimal Uncoordinated Power Control Policies for Energy Harvesting Multiple Access Channels with Decoding Costs," *submitted to IEEE Trans. Commun.*, Apr. 2018.

## Conference Publications:

- ▶ M. Sharma and C. R. Murthy, "Packet Drop Probability Analysis of ARQ and HARQ-CC with Energy Harvesting Transmitters and Receivers," in *Proc. IEEE GlobalSIP*, Dec. 2014, pp. 148-152.
- ▶ A. Devraj, M. Sharma and C. R. Murthy, "Power Allocation in Energy Harvesting Sensors with ARQ: A Convex Optimization Approach," in *Proc. IEEE GlobalSIP*, Dec. 2014, pp. 208-212.
- ▶ M. Sharma, C. R. Murthy and R. Vaze, "On Distributed Power Control for Uncoordinated Dual Energy Harvesting Links: Performance Bounds and Near-Optimal Policies," in *Proc. WiOpt*, May 2017.
- ▶ M. Sharma and C. R. Murthy, "Near-Optimal Distributed Power Control for ARQ Based Multihop Links with Decoding Costs," in *Proc. IEEE ICC*, May 2017.

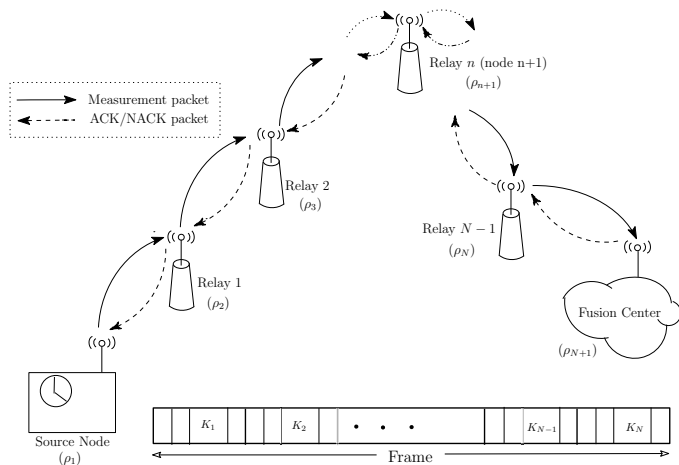
## ARQ-based Multi-hop EH links

# System Model



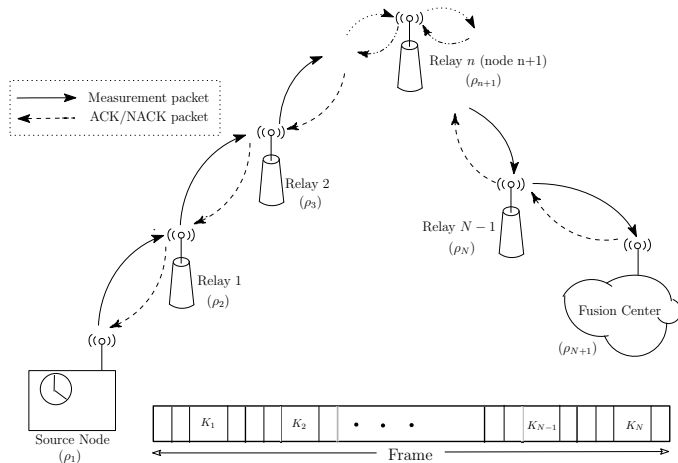
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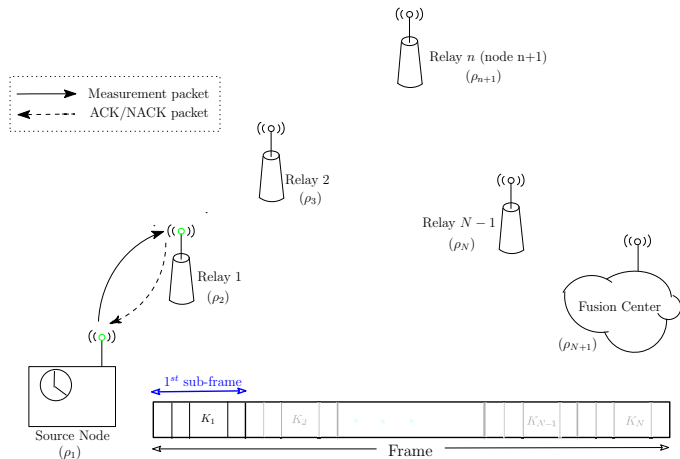
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- Packet is generated at the start of the frame
- Dropped if not delivered by the end of the frame



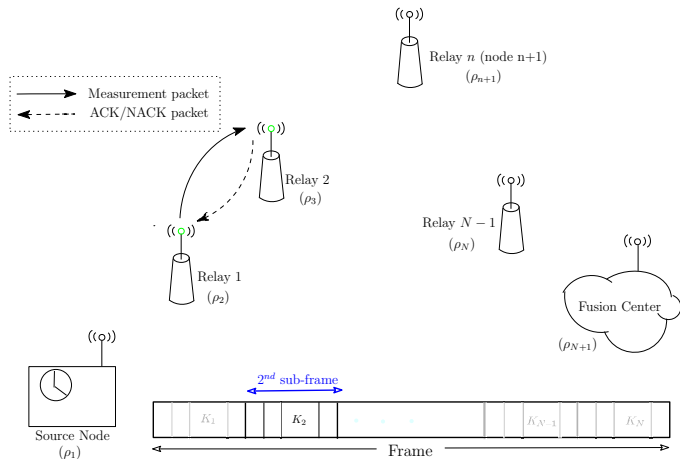
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- Forwarded using half-duplex relays



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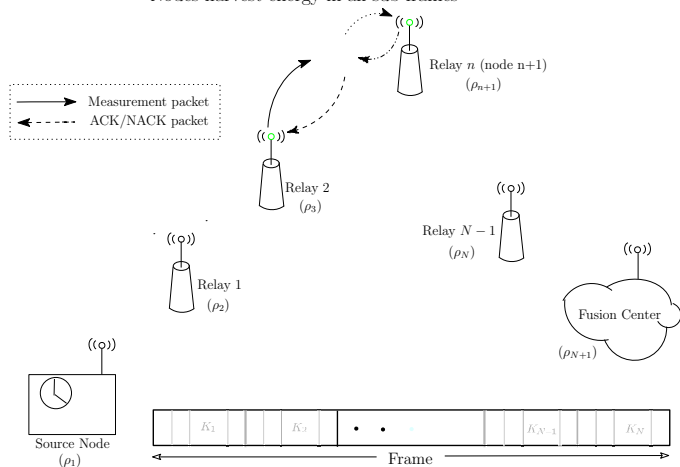
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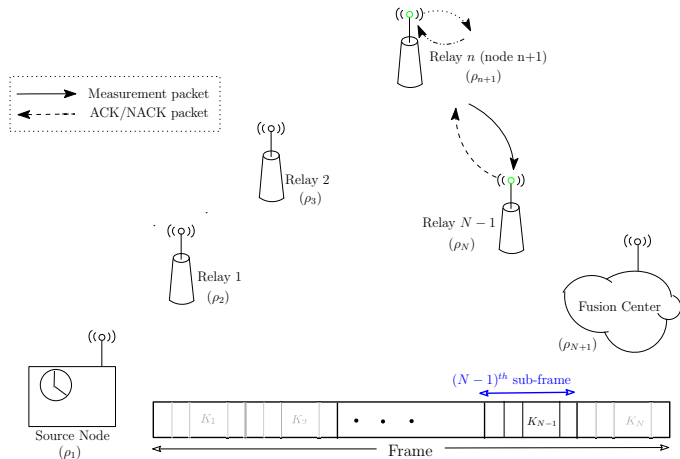
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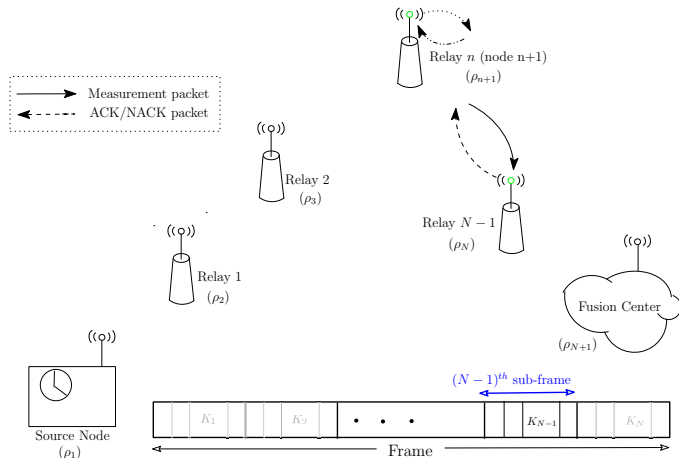
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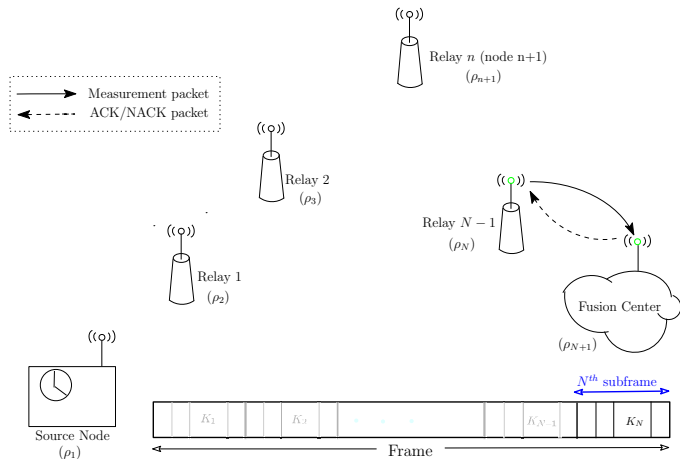
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- Division of slots in sub-frames is fixed over time
- Packet drop: transmission failed at intermediate hop



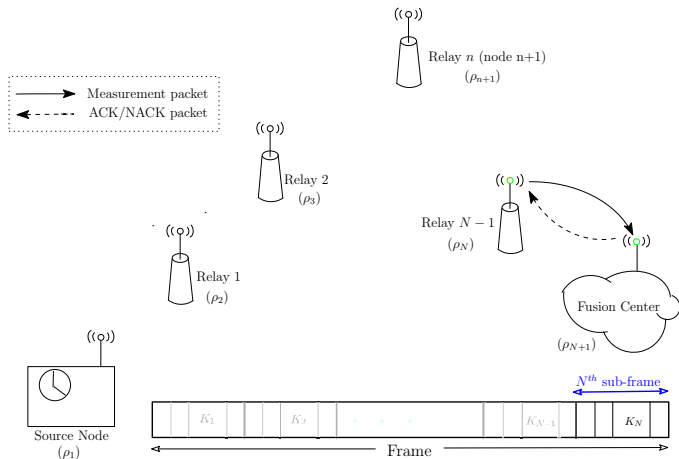
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- Transmission at each hop follows the ARQ protocol



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- ▶ Energy required for decoding:  $R$
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- ▶ Packet failure probability is

$$P_e(E_\ell^n, \gamma) = \exp\left(-\frac{E_\ell^n \gamma}{N_0}\right)$$

# Goal & Contributions

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## Contributions

- ▶ Closed-form expressions for packet drop probability
- ▶ Near-optimal distributed policies
  - ▶ For  $R \approx 0$ : closed-form expressions
  - ▶ For  $R > 0$ : iterative GP based solution
- ▶ Both slow and fast fading channels

## Prior Work: Multi-hop EH links

1. Gatzianas et al. [TWC 2010], maximize the long-term rate
2. Lai et al. [TCOM 2016] use reliability as the metric
3. Mao et al. [TAC 2012], develop a near-optimal power and rate control policy to maximize long-term average sensing rate
4. Joseph et al. [ICUMTW 2009], propose joint power control, scheduling and routing scheme to maximize the throughput

# System Evolution

- ▶ Modeled by the discrete-time Markov chain with state  $(\mathbf{B}_s, \mathbf{U}_s, s)$ <sup>1</sup>
- ▶ Battery Evolution at  $n^{\text{th}}$  Node:

$$B_{s+1}^n = \min \left( \left( B_s^n + \mathbb{1}_{\{\mathcal{H}_s^n\}} - E_\ell^n \mathbb{1}_{\{\mathcal{E}_{t,s}^n\}} - R \mathbb{1}_{\{\mathcal{E}_{r,s}^n\}} \right)^+, B_n^{\max} \right)$$

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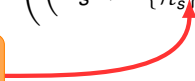
<sup>1</sup>B. Medepally, N. B. Mehta, and C. Murthy, *Implications of energy profile and storage on energy harvesting sensor link performance*, in Proc. IEEE Globecom., Dec. 2009.

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nonzero if node  
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# System Evolution

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nonzero if node harvests energy

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- ▶ Local transmission index,  $U_s$

$$U_s^n \triangleq \begin{cases} -1 & \text{ACK received,} \\ \ell & \ell - 1 \text{ NACKs received, } \ell \in \{1, \dots, K_n\}. \end{cases}$$

$U_s^n$  is reset to zero at the start of the frame

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# Packet Drop Probability

For a given set of policies  $\mathcal{P} \triangleq \{\mathcal{P}^n\}_{n=1}^N$

$$P_D = \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_{\gamma} \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{1}, \gamma, \mathbf{s} = \mathbf{0})\}$$

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- ▶  $\pi$  : stationary distribution of battery states at the start of the frame

$$\pi = (\mathbb{E} [\mathbf{G}'(\gamma)] - \mathbf{I} + \mathbf{A})^{-1} \mathbf{1}$$

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$$\Pr [\mathbf{B}_{(M+1)K} = \mathbf{B}_2 | \mathbf{B}_{MK} = \mathbf{B}_1, \gamma]$$

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- ▶ Conditional PDP can be found in closed-form

# PDP Minimization & Bounds

$$\min_{\{\mathcal{P}^n\}_{n=1}^N} P_D = \min_{\{\mathcal{P}^n\}_{n=1}^N} \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_{\gamma} \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{1}, \gamma)\},$$

**subject to:**  $0 \leq E_{\ell}^n \leq E_{\max}$  for all  $1 \leq \ell \leq K_n$  and  $1 \leq n \leq N$ .



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**subject to:**  $0 \leq E_{\ell}^n \leq E_{\max}$  for all  $1 \leq \ell \leq K_n$  and  $1 \leq n \leq N$ .

Difficult to Solve!

# PDP Minimization & Bounds

$$\min_{\{\mathcal{P}^n\}_{n=1}^N} P_D = \min_{\{\mathcal{P}^n\}_{n=1}^N} \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{1}, \gamma)\},$$

**subject to:**  $0 \leq E_\ell^n \leq E_{\max}$  for all  $1 \leq \ell \leq K_n$  and  $1 \leq n \leq N$ .

## Bounds:

For a multi-hop EH link operating using policies  $\mathcal{P}$ ,

$$P_{D_\infty}^* \leq \min_{\{\mathcal{P}^n\}_{n=1}^N} P_D \leq P_{D_\infty}^* + \sum_{\mathbf{B} \in \mathcal{I}_A^c} \pi(\mathbf{B}) \Big|_{\mathcal{P}^*}$$

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$P_{D\infty}^* \triangleq \min_{\{\mathcal{P}^n\}_{n=1}^N} \mathbb{E}_\gamma \{P_D(K|\cdot)\}$  and  $\mathcal{P}^* \triangleq \arg \min_{\{\mathcal{P}^n\}_{n=1}^N} \mathbb{E}_\gamma \{P_D(K|\cdot)\}$

for any  $\mathbf{B} \in \mathcal{I}_A$ , the set of “GOOD” battery states.

# Tightness of the Bounds

- ▶ **Lower bound:** Optimum PDP under average power constraint, with infinite battery<sup>2</sup>
- ▶ **Difference between lower and upper bound:** upper bound on the penalty due to finite batteries under the EUR

---

<sup>2</sup>V. Sharma, U. Mukherji, V. Josheph, and S. Gupta, *Optimal energy management policies for energy harvesting sensor nodes*, IEEE Trans. Wireless Commun., Apr. 2010.

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## Theorem (MSharma-TWC-June2017)

For a multi-hop EH link operating under EUR

$$\sum_{\mathbf{B} \in \mathcal{I}_A^c} \pi(\mathbf{B}) = \sum_{n=1}^{N+1} \Theta(e^{r_n^* B_n^{\max}})$$

$r_n^*$ : negative root of the asymp. log MGF of battery drift process

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- ▶ **Large battery regime:** use lower bound as objective and replace ENC by EUR constraints

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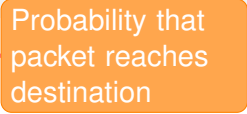
<sup>2</sup>V. Sharma, U. Mukherji, V. Josheph, and S. Gupta, *Optimal energy management policies for energy harvesting sensor nodes*, IEEE Trans. Wireless Commun., Apr. 2010.

# Reformulation

$$\begin{aligned} & \max_{\{\mathcal{P}^n\}_{n=1}^N} \Pr[N + 1] \\ & \text{subject to } \bar{T}_n + \bar{R}_n \leq K\rho_n, \\ & \quad 0 \leq E_\ell^n \leq E_{\max}, \end{aligned}$$

# Reformulation

Probability that  
packet reaches  
destination



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# Reformulation

Probability that packet reaches destination

Average energy used for transmission

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Average energy used for reception

$$\Pr[n] = \prod_{m=1}^{n-1} \left( 1 - \frac{1}{1 + \sum_{\ell=1}^{K_m} E_\ell^m} \right)$$

# Reformulation

Probability that  
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Average energy  
used for trans-  
mission

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Average energy  
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$$\begin{aligned} \Pr[n] &= \prod_{m=1}^{n-1} \left( 1 - \frac{1}{1 + \sum_{\ell=1}^{K_m} E_\ell^m} \right) \\ \bar{T}_n &= \Pr[n] \left( \sum_{\ell=1}^{K_n} \frac{E_\ell^n}{1 + \sum_{i=1}^{\ell-1} E_i^n} \right), \end{aligned}$$

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Probability that packet reaches destination

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Nonconvex mixed integer program with  $2^K$  subproblems

# Reformulation: Negligible Cost of Reception

$$\max_{\{\mathcal{P}^n\}_{n=1}^N} \Pr[N + 1]$$

$$\text{subject to } \bar{T}_n + \bar{R}_n \leq K\rho_n,$$
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$$\Pr[n] = \prod_{m=1}^{n-1} \left( 1 - \frac{1}{1 + \sum_{\ell=1}^{K_m} E_\ell^m} \right)$$

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- ▶ Optimal policy obtained using the solution for P2P links

# Optimal Policy without Peak Power Constraint

## Theorem: slow fading

The unique optimal policy for  $n^{\text{th}}$ -hop is given by

$$E_k^{n*} = \frac{\rho_n K}{K_n \text{Pr}[n]} \left( 1 + \frac{\rho_n K}{K_n N_0 \text{Pr}[n]} \right)^{k-1}, \quad k = 1, 2, \dots, K_n.$$

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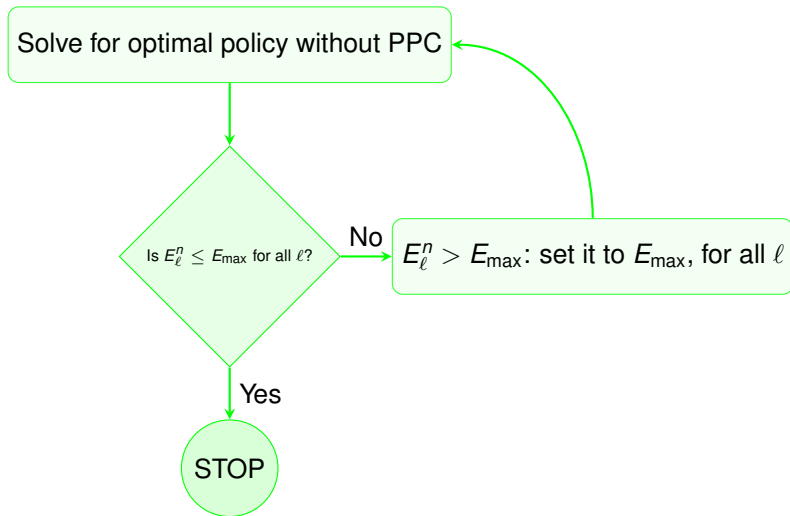
## Theorem: fast fading

The successive power levels of the optimal policy at  $n^{\text{th}}$ -hop satisfies

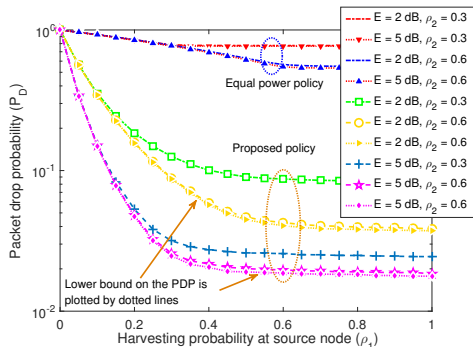
$$E_{\ell+1}^{n*} = \frac{E_{\ell}^{n*} (E_{\ell}^{n*} + 2)}{2},$$

for all  $1 \leq \ell \leq K_n$ .

# Optimal Policy with Peak Power Constraint



# Performance of the Closed-form Policy



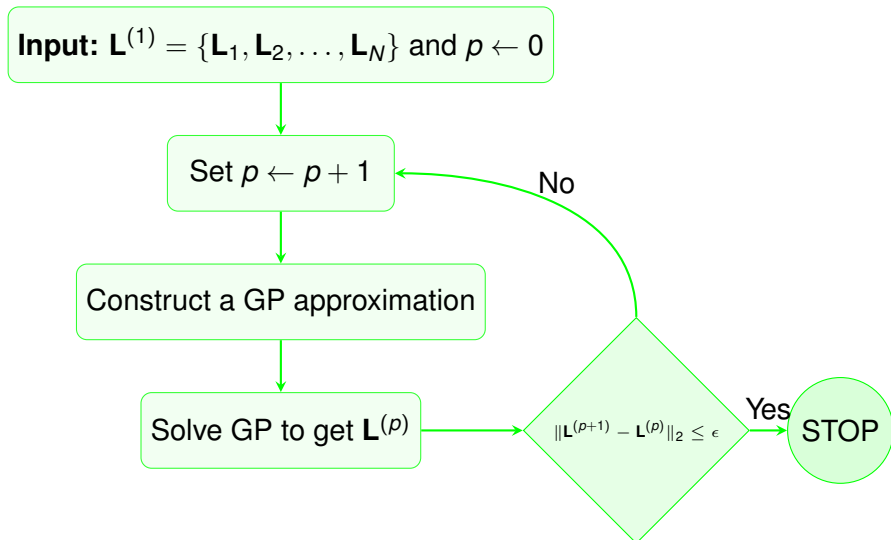
No. of hops = 2

$E_{\max} = 10E_s$

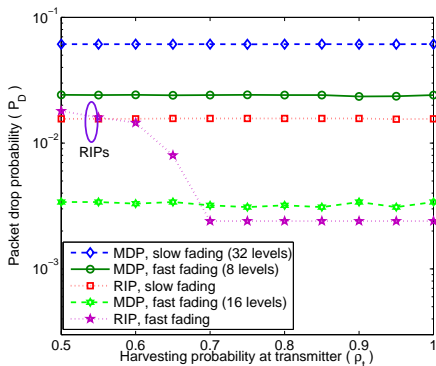
$K_1 = K_2 = 4$

The policy obtained using proposed approach achieves the lower bound and outperforms the EPP [ $E_{\max} E_{\max} E_{\max} E_{\max}$ ].

# General Case: Integer Constraints ( $2^K \rightarrow \prod_{n=1}^N K_n$ ) & Solving CGP



# RIPs Vs MDP based policy



No. of hops = 1

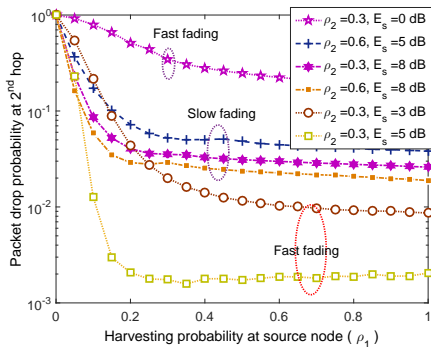
$R = 1$

$E_{\max} = 4E_s$  and  $2E_s$

$K_1 = 4$

For slow fading channels, the RIP uniformly outperforms the MDP, while in the fast fading case, for  $\rho_t > 0.7$ , the RIP outperforms the corresponding MDP based policies.

# PDP at 2<sup>nd</sup> Hop



No. of hops = 2

$R = 1$

$E_{\max} = 10E_s$

$K_1 = K_2 = 4$

PDP at the second hop improves with increase in the harvesting rate at the source node.



# Summary

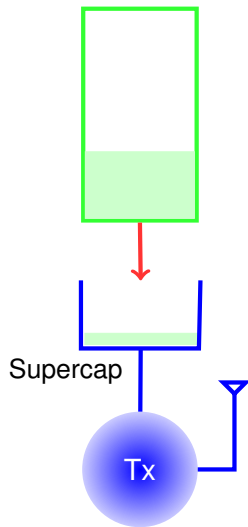
- ▶ Presented closed-form expressions for the PDP
- ▶ Characterized the dependence of the PDP on size of the batteries
  - ▶ Can design policies under EUR if battery capacity is sufficiently large
- ▶ Obtained closed-form expressions for near-optimal RIPs when  $R \approx 0$
- ▶ Near optimal policies when  $R > 0$
- ▶ The proposed policy outperforms the EPP and MDP based policies

# Uncoordinated Dual EH Links

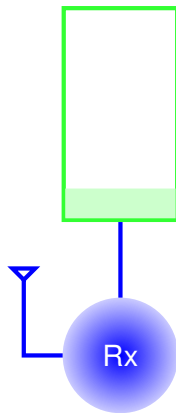
(Joint work with Prof. Rahul Vaze, TIFR)

# System Model

Battery size :  $B_{\max}^t$

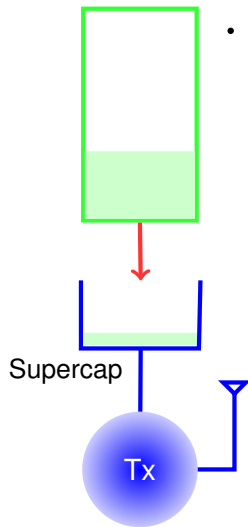


Battery size :  $B_{\max}^r$



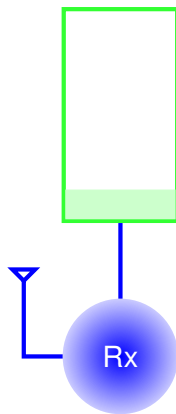
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Battery size :  $B_{\max}^t$



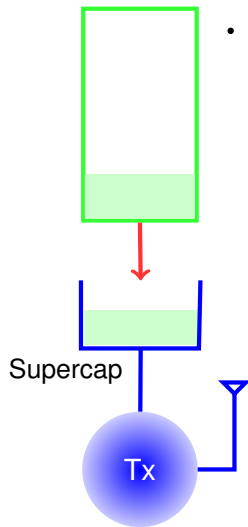
- Transmit energy is transferred to super-capacitor

Battery size :  $B_{\max}^r$



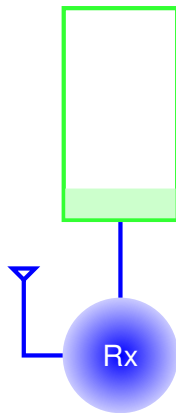
# System Model

Battery size :  $B_{\max}^t$



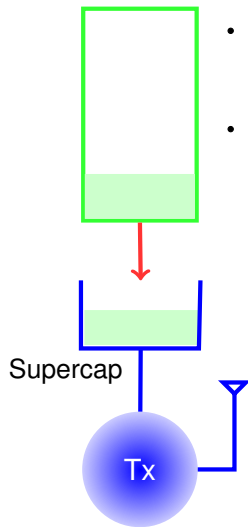
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Battery size :  $B_{\max}^r$



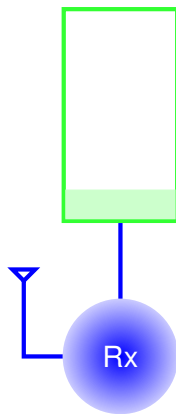
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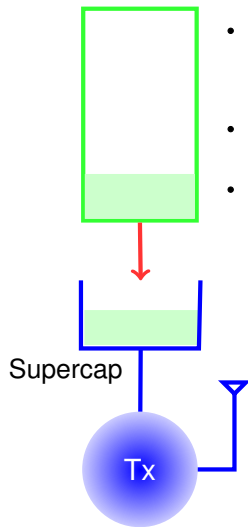
- Transmit energy is transferred to super-capacitor
- AWGN channel

Battery size :  $B_{\max}^r$



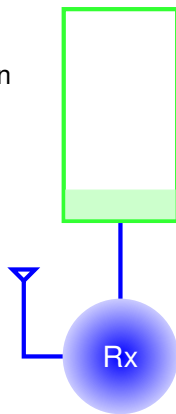
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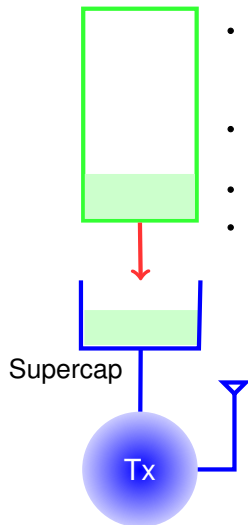
- Transmit energy is transferred to super-capacitor
- AWGN channel
- $R$  units used for reception

Battery size :  $B_{\max}^r$



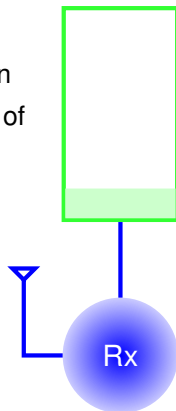
# System Model

Battery size :  $B_{\max}^t$



- Transmit energy is transferred to super-capacitor
- AWGN channel
- $R$  units used for reception
- Only mean and variance of EH process are known

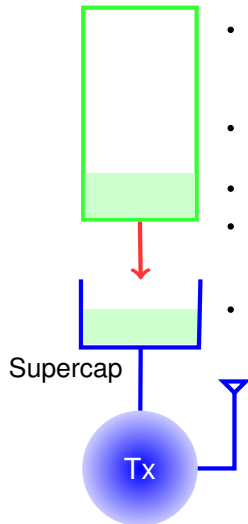
Battery size :  $B_{\max}^r$





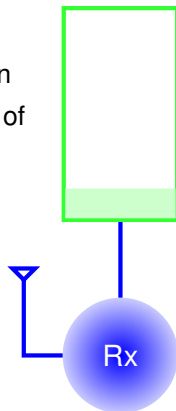
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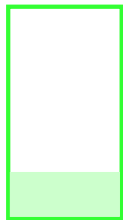
- Transmit energy is transferred to super-capacitor
- AWGN channel
- $R$  units used for reception
- Only mean and variance of EH process are known
- Slot level synchronization

Battery size :  $B_{\max}^r$

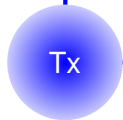


# System Model

Battery size :  $B_{\max}^t$

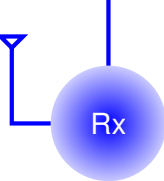
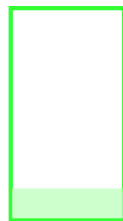


Supercap



- Transmit energy is transferred to super-capacitor
- AWGN channel
- $R$  units used for reception
- Only mean and variance of EH process are known
- Slot level synchronization

Battery size :  $B_{\max}^r$



No coordination

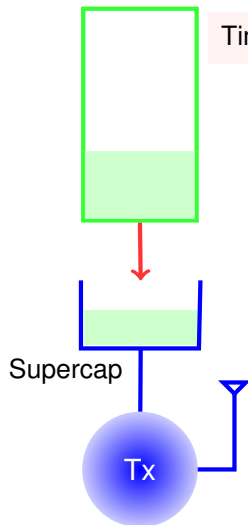


## Prior Work: Dual EH Links

1. Arafa and Ulukus [JSAC 2015], maximize the coordinated throughput
  - ▶ Non-causal knowledge of energy arrivals at both nodes
2. Zhou et al. [JSAC 2015] consider retransmission-based dual EH links
  - ▶ Coordinated sleep-wake protocol
3. Sharma and Murthy [TWC 2017], optimize packet drop probability of retransmission-based dual EH links
  - ▶ Use ACK/NACK messages to achieve perfect coordination
  - ▶ One bit feedback facilitates coordination
4. Doshi and Vaze [ICSS 2014], analyze throughput of uncoordinated links with unit sized batteries

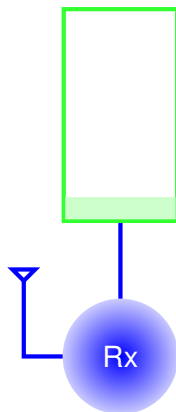
# Impact of Uncoordination

Battery size :  $B_{\max}^t$

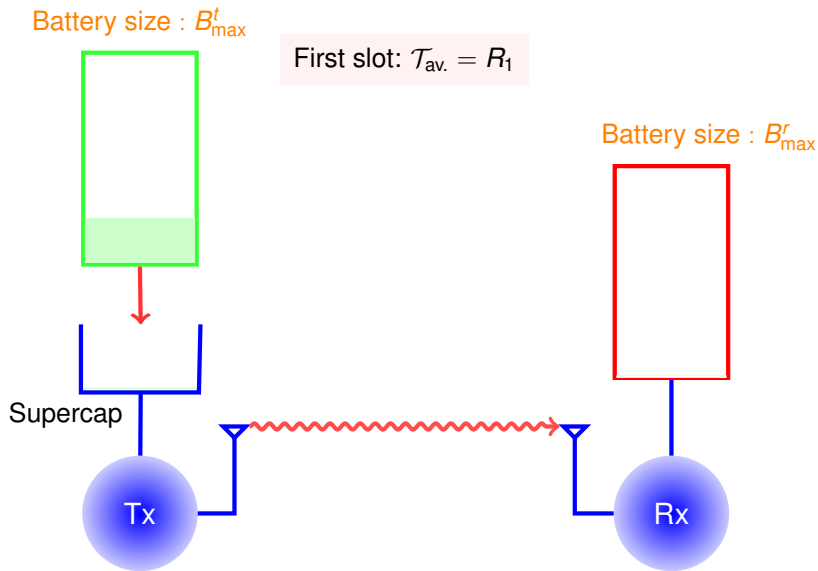


Time-averaged throughput:  $\mathcal{T}_{\text{av}}$ .

Battery size :  $B_{\max}^r$



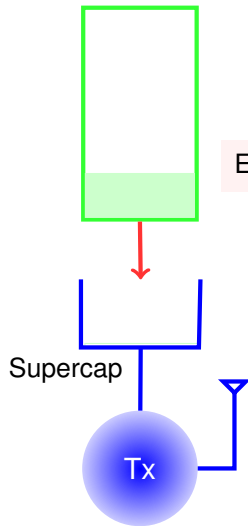
# Impact of Uncoordination



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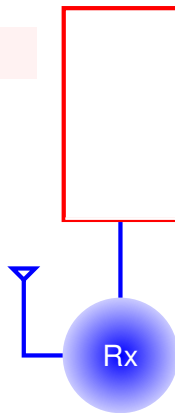
Battery size :  $B_{\max}^t$

First slot:  $\mathcal{T}_{\text{av.}} = R_1$



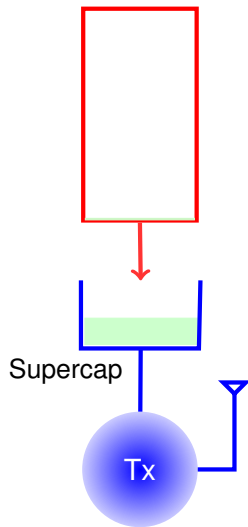
Empty battery at Receiver

Battery size :  $B_{\max}^r$



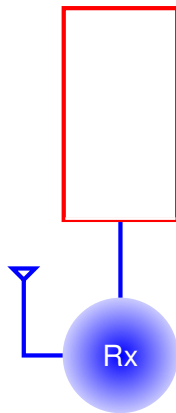
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Battery size :  $B_{\max}^t$



First slot:  $\mathcal{T}_{\text{av.}} = R_1$

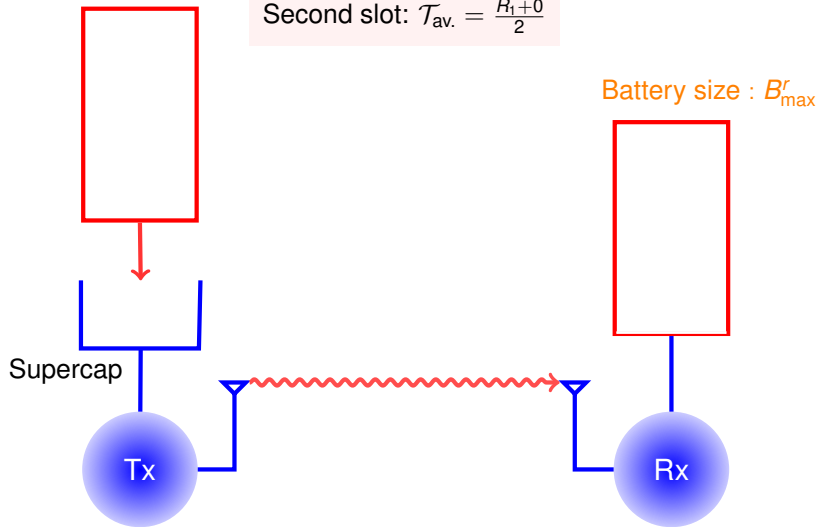
Battery size :  $B_{\max}^r$



# Impact of Uncoordination

Battery size :  $B_{\max}^t$

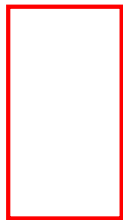
Second slot:  $\mathcal{T}_{\text{av.}} = \frac{R_1 + 0}{2}$





# Impact of Uncoordination

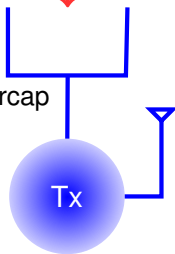
Battery size :  $B_{\max}^t$



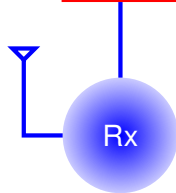
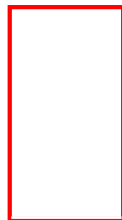
Second slot:  $\mathcal{T}_{av.} = \frac{R_1 + 0}{2}$

Energy wasted due to lack of coordination

Supercap



Battery size :  $B_{\max}^r$



# Impact of Uncoordination

Battery size :  $B_{\max}^t$

Second slot:  $\mathcal{T}_{av.} = \frac{R_1 + 0}{2}$

Energy wasted due to lack of coordination

Empty battery at transmitter

Battery size :  $B_{\max}^r$

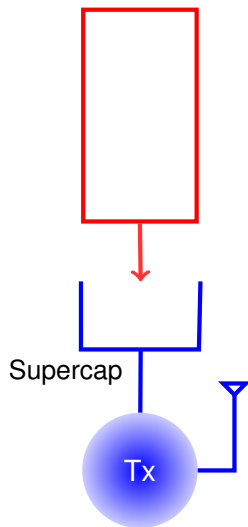
Supercap

Tx

Rx

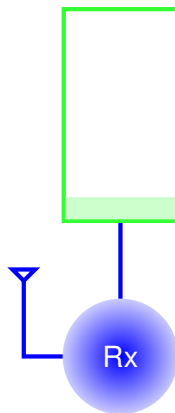
# Impact of Uncoordination

Battery size :  $B_{\max}^t$



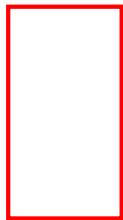
Second slot:  $\mathcal{T}_{av.} = \frac{R_1+0}{2}$

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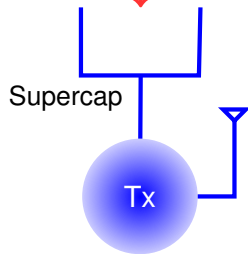
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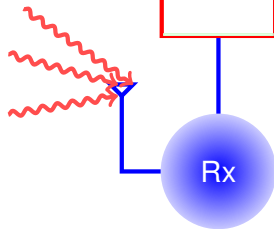
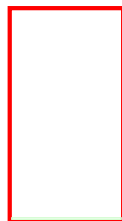


Third slot:  $\mathcal{T}_{\text{av.}} = \frac{R_1+0+0}{3}$

Energy wasted due to lack of coordination



Battery size :  $B_{\max}^r$



# Impact of Uncoordination

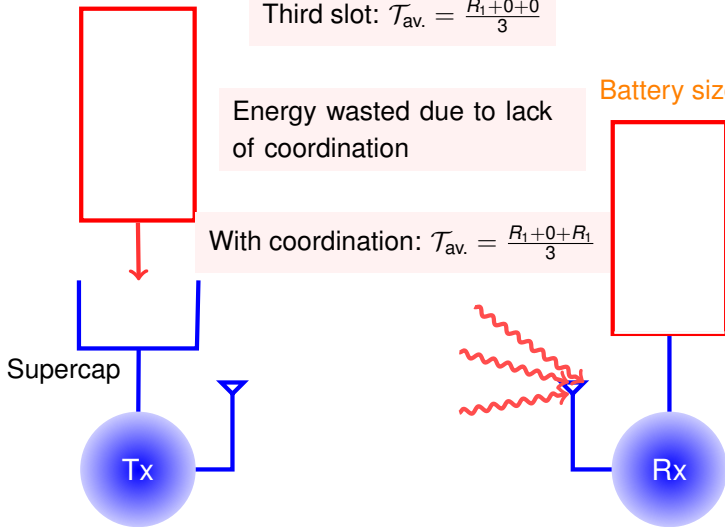
Battery size :  $B_{\max}^t$

Third slot:  $\mathcal{T}_{\text{av.}} = \frac{R_1+0+0}{3}$

Energy wasted due to lack of coordination

With coordination:  $\mathcal{T}_{\text{av.}} = \frac{R_1+0+R_1}{3}$

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# Goal & Contributions

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  - ▶ Energy unconstrained receiver ( $\mu_r \geq R$ )
  - ▶ Energy constrained receiver ( $\mu_r < R$ )
    - ▶ Policy with *occasional one bit feedback* from receiver
    - ▶ Policy with time-dilation at receiver: asymptotically no feedback
    - ▶ Fully uncoordinated policy



# Mathematical Formulation

Objective:


$$\max_{\substack{p_t(n), \\ p_r(n), n \geq 1}} \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbb{1}_{\{p_r(n) \neq 0\}} \log(1 + p_t(n))$$

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Constraints:

1. Energy used by a node can not exceed the energy in its battery, i.e.,

$$B_{n+1}^t = \min \{ \max \{ 0, B_n^t + \mathcal{E}_t(n) - p_t(n) \}, B_{\max}^t \}$$

2. Receiver can consume either 0 or  $R$  units of energy

# Upper Bound

## Lemma

The long-term time-averaged throughput of a dual EH link satisfies:

$$\mathcal{T} \leq \begin{cases} \log(1 + \mu_t) & \text{if } \mu_r > R, \\ \left(\frac{\mu_r}{R}\right) \log\left(1 + \frac{R\mu_t}{\mu_r}\right) & \text{if } \mu_r \leq R \end{cases}$$

where  $\mathcal{T} \triangleq \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbb{1}_{\{p_r(n) \neq 0\}} \log(1 + p_t(n))$

$\mu_t$  and  $\mu_r$ : rate of harvesting at the transmitter and receiver

# Upper Bound

## Lemma

The long-term time-averaged throughput  $\mathcal{T}$  of a system with EHN satisfies:

Equivalent to when only Tx is EHN

$$\mathcal{T} \leq \begin{cases} \log(1 + \mu_t) & \text{if } \mu_r > R, \\ \left(\frac{\mu_r}{R}\right) \log\left(1 + \frac{R\mu_t}{\mu_r}\right) & \text{if } \mu_r \leq R \end{cases}$$

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# Proof Sketch

Unconstrained Receiver:  $\mu_r > R$

$$\begin{aligned}\mathcal{T} &\leq \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \log(1 + p_t(n)) \\ &= \mathbb{E} \{ \log(1 + p_t(n)) \} \leq \log(1 + \mathbb{E} [p_t(n)]) \\ &\leq \log(1 + \mu_t)\end{aligned}$$

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Constrained Receiver:  $\mu_r < R$

1. A genie-aided system which has non-causal information about energy arrivals at both the nodes
2. Number of slots receiver can be on  $= \frac{N\mu_r}{R}$
3. Transmitter uses equal power  $p_t(n) = \frac{R\mu_t}{\mu_r}$  across these slots
4.  $\mathcal{T} \leq \left(\frac{\mu_r}{R}\right) \log\left(1 + \frac{R\mu_t}{\mu_r}\right)$



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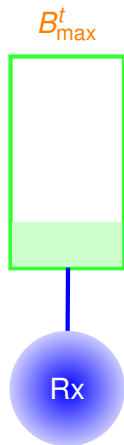
$\frac{R}{\mu_r}$  is time taken to harvest  $R$  units of energy

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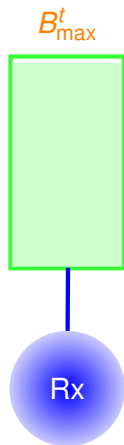
- Receiver remains ON in every slot



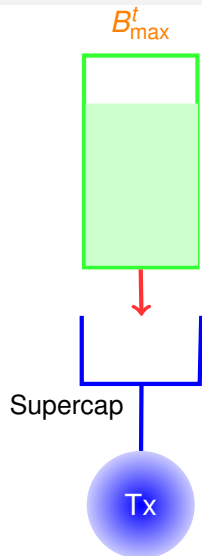
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- Receiver remains ON in every slot
- Since  $\mu_r > R$ , harvesting rate is more than the energy required for receiving the data
- Probability that receiver runs out of energy decays exponentially with receiver battery size
- Equivalent to the Tx-only EHN case

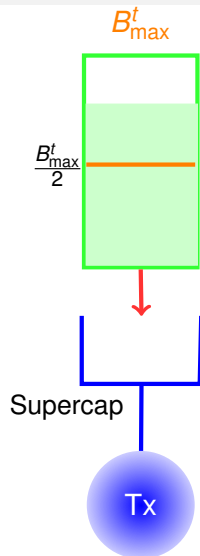


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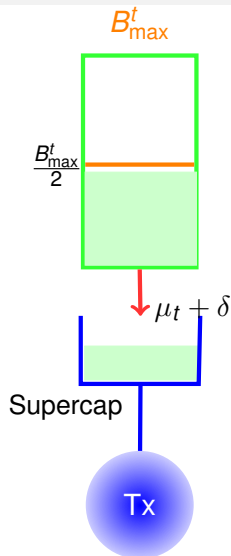
<sup>1</sup>R. Srivastava and C. E. Koksal, *Basic performance limits and tradeoffs in energy-harvesting sensor nodes with finite data and energy storage*, IEEE/ACM Trans. Netw., vol. 21, pp. 1049-1062, Aug. 2013.

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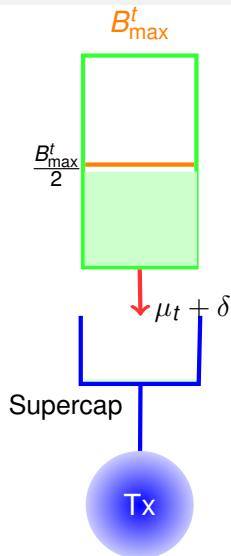
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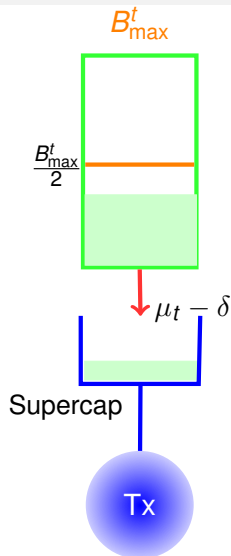
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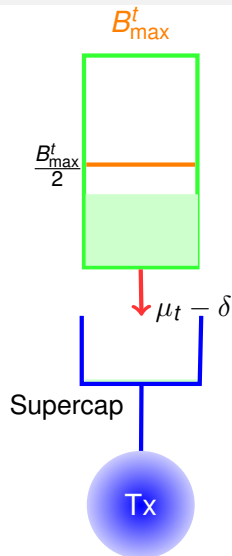


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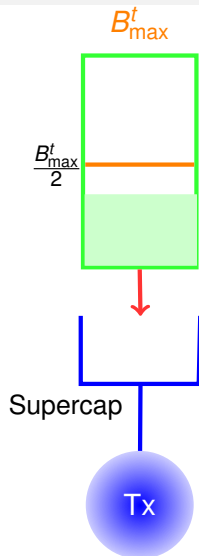
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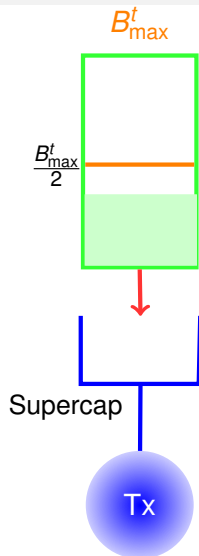


$$p_t(n) = \begin{cases} \mu_t + \delta & \text{if } B_n^t \geq \frac{B_{\max}^t}{2} \\ \min \{B_n^t, \mu_t - \delta\} & \text{if } B_n^t < \frac{B_{\max}^t}{2} \end{cases}$$

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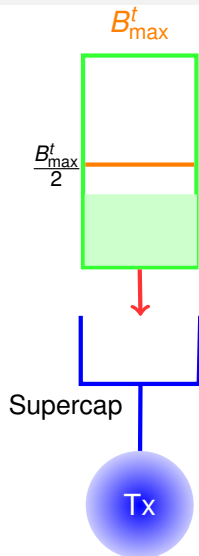


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- Throughput achieved by policy converges to upper bound at rate  $\Theta \left( \left( \frac{\log B_{\max}^t}{B_{\max}^t} \right)^2 \right)$
- Fully uncoordinated policy!

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# Performance: Policy for Unconstrained Receiver

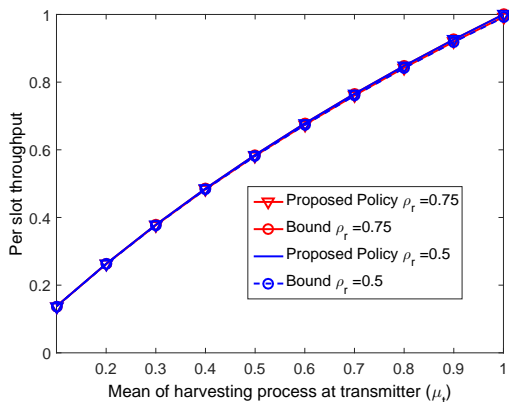
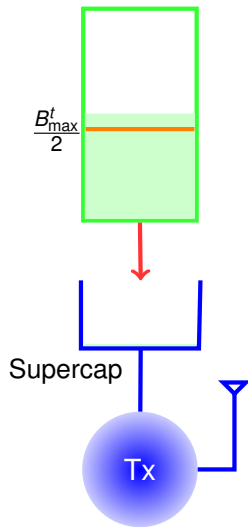


Figure: Parameters:  $R = 0.5$  and  $B_{\max}^t = B_{\max}^r = 50$ .

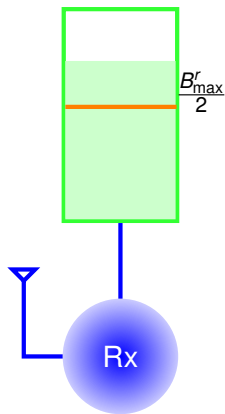
# Optimal Policy Example: $R = 1.25\mu_r$

Battery size :  $B_{\max}^t$

$$\lfloor \frac{R}{\mu_r} \rfloor = 1 \text{ and } \lceil \frac{R}{\mu_r} \rceil = 2$$



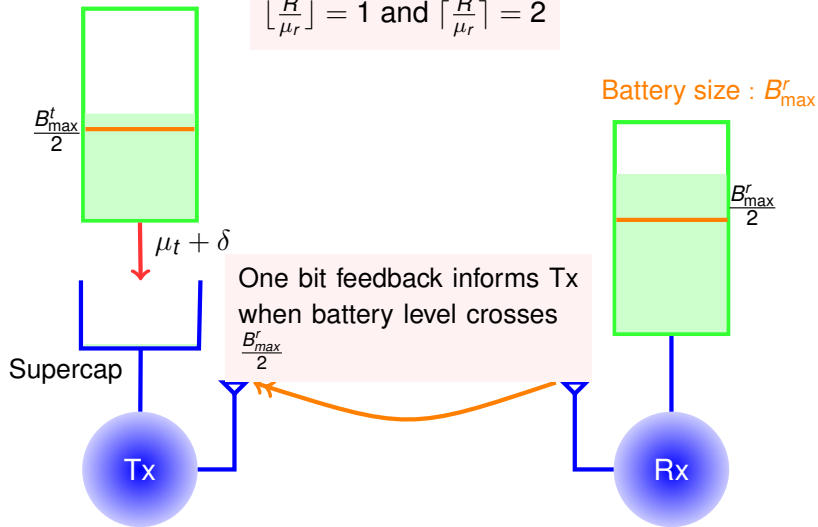
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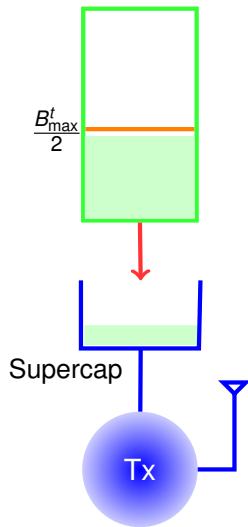




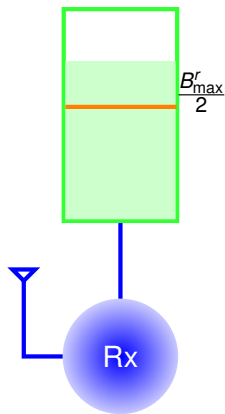
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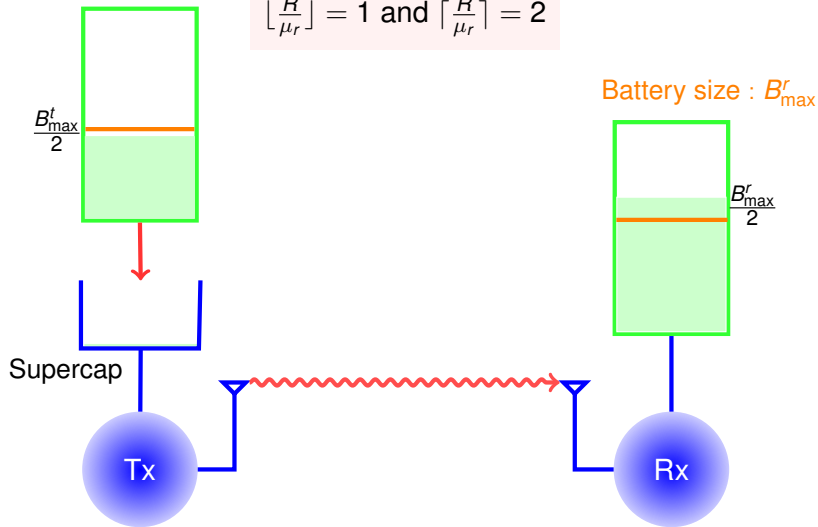
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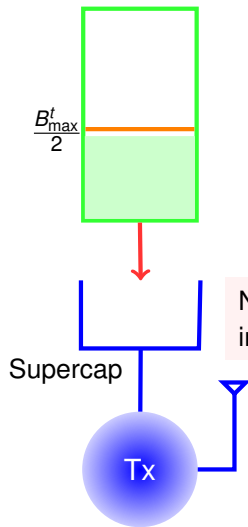
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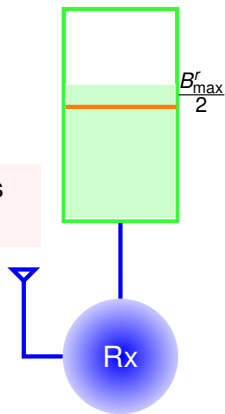
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No feedback, as battery is in the same half

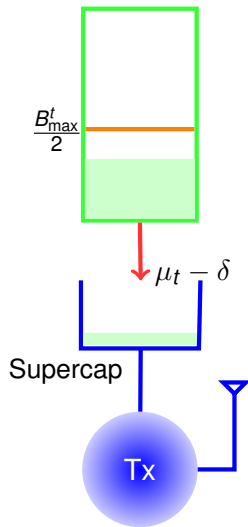
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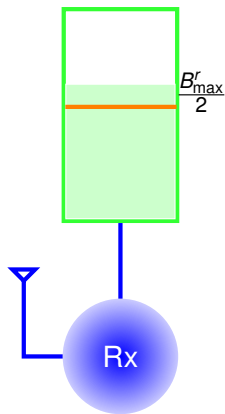
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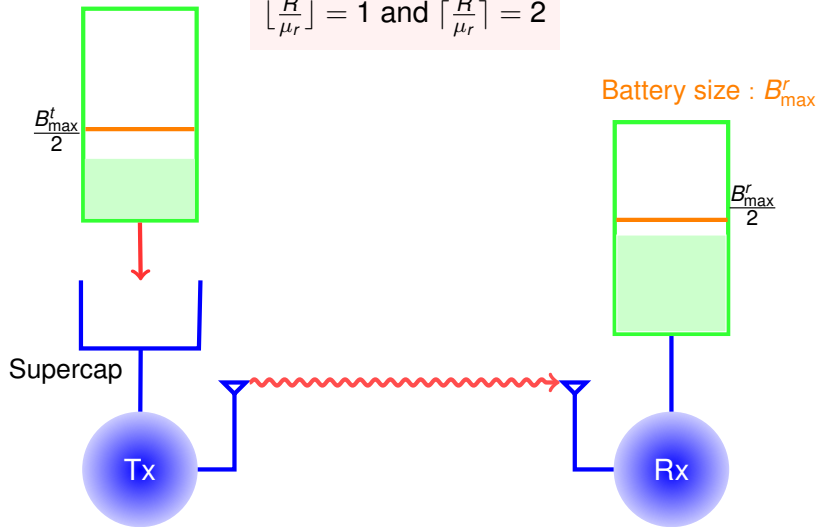
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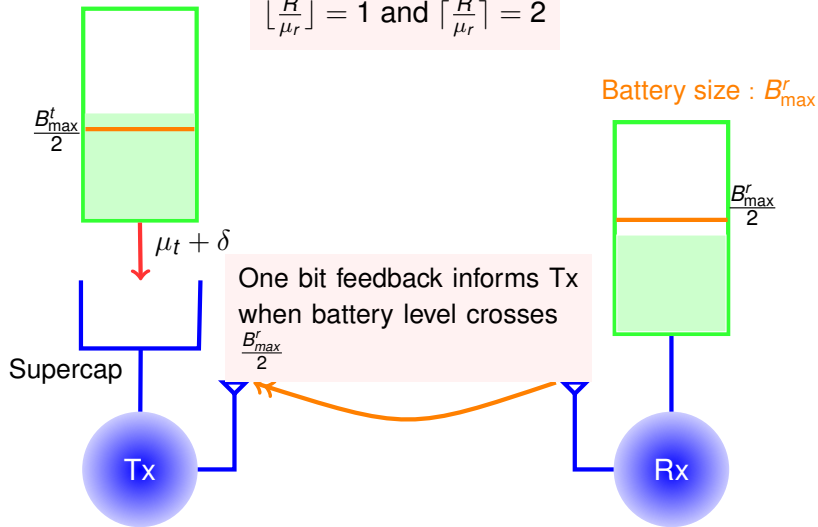
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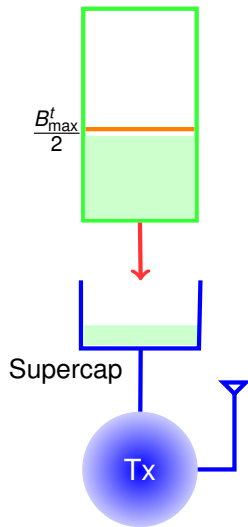
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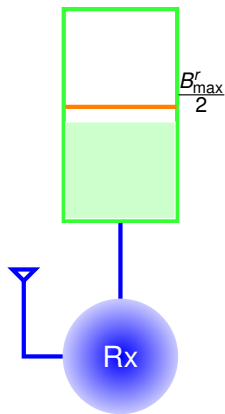
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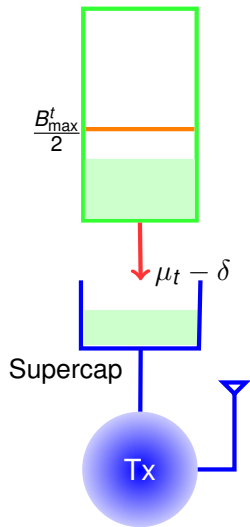
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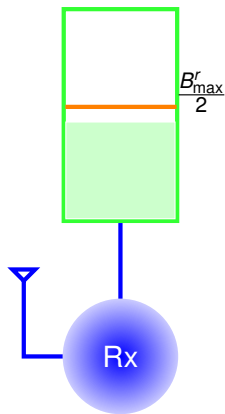
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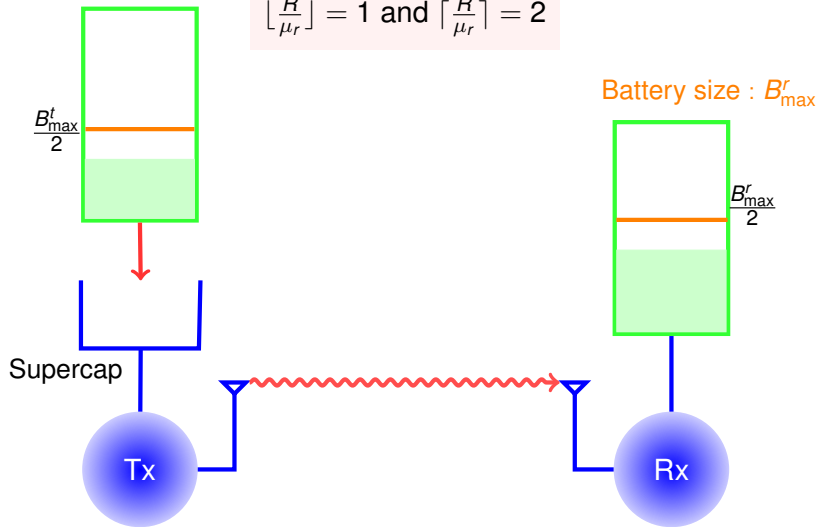




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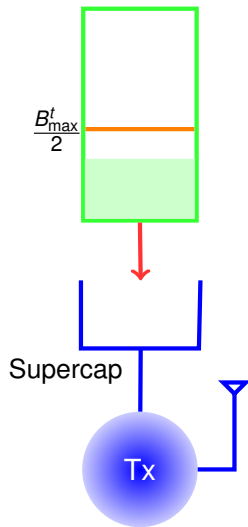
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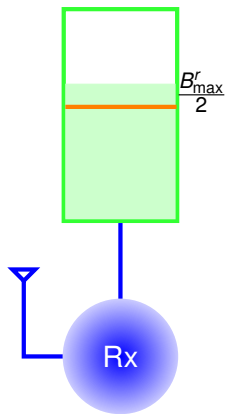
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- ▶ Turns on after  $N_{\text{on}} = \begin{cases} N_r & \text{if } B_n^r \geq \frac{B_{\text{max}}^r}{2} \\ N_r + 1 & \text{if } B_n^r < \frac{B_{\text{max}}^r}{2} \end{cases}$  slots where,

$$N_r = \lfloor \frac{R}{\mu_r} \rfloor$$

## Policy at transmitter

- ▶ Transmit only in the slots when receiver is on



# Optimal Policy: Energy Constrained Receiver

- ▶  $\mu_r < R \implies$  Receiver can only turn on intermittently

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## Policy at transmitter

- ▶ Transmit only in the slots when receiver is on
- ▶ Otherwise accumulate the energy in super-capacitor

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- ▶ Specifically,

$$\left(\frac{\mu_r}{R}\right) \log \left(1 + \frac{R\mu_t}{\mu_r}\right) - \mathcal{T}^c = O\left(\frac{\log B_{\max}^t}{B_{\max}^t}\right) + O(\delta_r^+) + O(\delta_r^-).$$

where  $\delta_r^+ = \frac{R}{\mu_r} - \lfloor \frac{R}{\mu_r} \rfloor$  and  $\delta_r^- = \lceil \frac{R}{\mu_r} \rceil - \frac{R}{\mu_r}$

# Time-dilation to Achieve Upper Bound

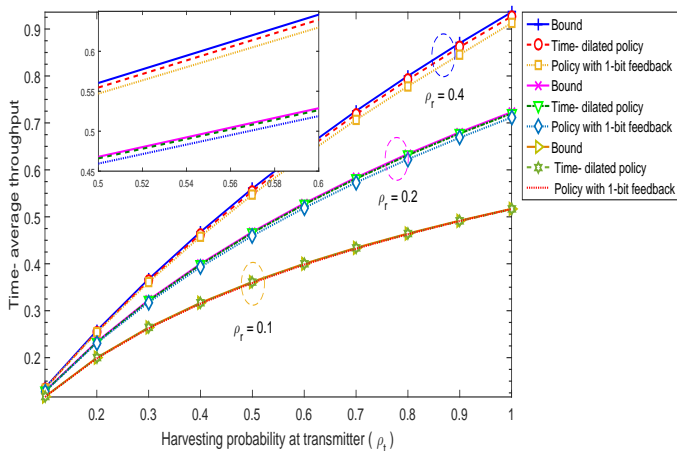
## Policy at transmitter:

- ▶ Same as for 1-bit feedback policy

## Policy at receiver:

- ▶ Receiver turn ON in last  $f(\cdot)$  slots of
  - ▶  $\lfloor \frac{Rf(\cdot)}{\mu_r} \rfloor$  slots if battery is more than half full
  - ▶  $\lceil \frac{Rf(\cdot)}{\mu_r} \rceil$  slots if battery is less than half full
- ▶ Effective drift goes to zero as  $f(\cdot)$  scales

# Performance: Policy for Constrained Receiver



**Figure:** The result corresponds to time-dilation  $f(\cdot) = 100$ . Other parameters are  $R = 0.5$  and  $B_{\max}^t = B_{\max}^r = 1000$ .

# Fully Uncoordinated Policy

- ▶ **Aim:** to prescribe a deterministic pattern for the receiver
- ▶ Match the ratio of  $N_r \triangleq \lfloor \frac{R}{\mu_r} \rfloor$  and  $N_r + 1$  transmissions of 1-bit feedback policy

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## Deterministic Policy $\mathcal{P}^{uc}$

- ▶ Compute  $\frac{n^+}{n^-} = \frac{\sum_{n=1}^N \mathbb{1}_{\{B_n^t \geq B_{\max}^r\}}}{\sum_{n=1}^N \mathbb{1}_{\{B_n^t < B_{\max}^r\}}}$ , for 1-bit feedback policy



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- ▶ Transmitter also follows this deterministic pattern
- ▶  $\mathcal{T}^c - \mathcal{T}^{uc} = O(\pi_0^{uc})$ , where  $\pi_0^{uc}$  denotes the stationary probability that battery at either node is empty, under policy  $\mathcal{P}^{uc}$

# Fully Uncoordinated Policy

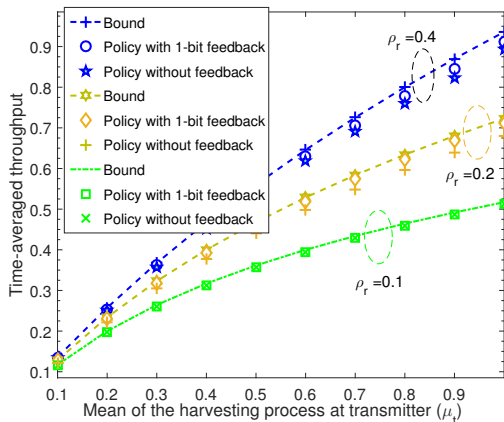


Figure: For  $\mathcal{P}^{UC}$ , the values of  $(n^+, n^-)$  are  $(5, 1)$ ,  $(1, 1)$  and  $(2, 1)$  for  $\rho_r = 0.1, 0.2$  and  $0.4$ , respectively. Other parameters:  $B_{\max}^t = B_{\max}^r = 50$ ,  $R = 0.5$ .

# Numerical Results: Impact of battery size

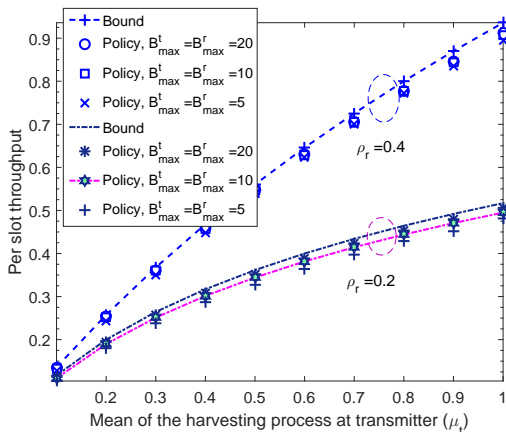


Figure: Impact of battery size on the throughput of policy  $\mathcal{P}^c$ , for  $R = 0.5$ .

# Summary

- ▶ Derived an upper-bound on the throughput of uncoordinated dual EH links
- ▶ Designed fully uncoordinated power control policies which achieve the upper-bound for unconstrained receiver
- ▶ Asymptotically optimal policies were proposed which require occasional 1-bit feedback
- ▶ Proposed a fully uncoordinated policy for a constrained receiver

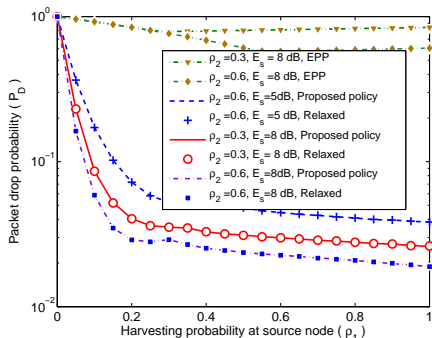


## Other Contributions of the Thesis

- ▶ Proposed a general framework to analyze the PDP of retransmission-based dual EH links
- ▶ Optimal policies for dual EH links with ARQ as well as HARQ-CC
- ▶ Optimal policies for spatio-temporally correlated EH processes

Thank You!

# Performance of the Proposed Policy



No. of hops = 2

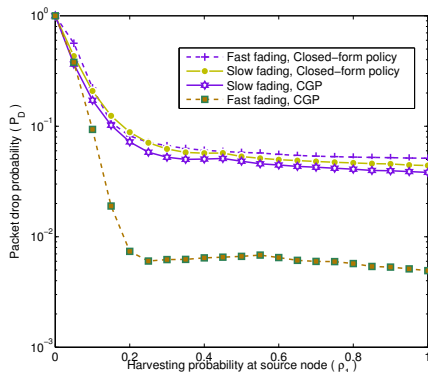
$R = 1$

$E_{\max} = 10E_s$

$K_1 = K_2 = 4$

The policy obtained using proposed approach outperforms the equal power policy [ $P_{\max} P_{\max} P_{\max} P_{\max}$ ].

# Performance of the Closed-form Policy for General Case



No. of hops = 2

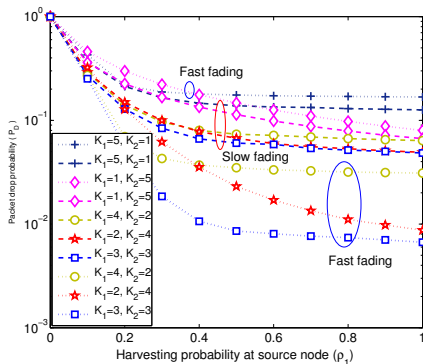
$R = 1$

$E_{\max} = 10E_s$

$K_1 = K_2 = 4$

Performance of optimal policy designed by ignoring the energy cost of packet reception, compared to the near-optimal policy for the general case.

# Impact of Slot Allocation Pattern



No. of hops = 2

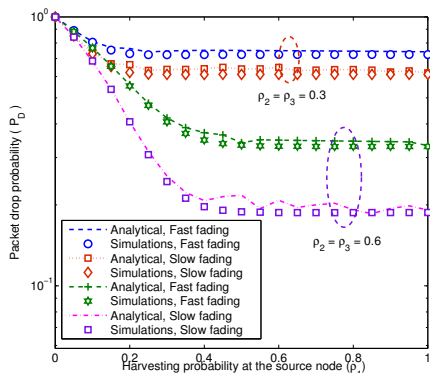
$R = 1$

$E_{\max} = 10E_s$

$K_1 = K_2 = 4$

Impact of slot allocation on the PDP: equal slot allocation performs the best.

# Accuracy of PDP Expressions



Accuracy of the closed-form PDP expressions. Parameters used:  $K_1 = K_2 = 2$ ,  $R = 1$ , and  $B^{\max} = 3$  for all the nodes. The RIP is  $[1 \ 1]$  at both source and relay nodes.