On the Optimal Tradeoff of Age of Information and Transmission Power for Point-to-Point Links

Sudarsanan A. K., Vineeth B. S., and Chandra R. Murthy

Abstract—**In applications such as remote estimation and monitoring, wireless networks need to be end-to-end optimized for information freshness as well as transmit power efficiency. In this paper, we study the fundamental tradeoff between an information freshness metric, the age of information, and transmit power for a point-to-point link. In contrast to prior work, we model the effect of transmission duration on the energy consumption, and consider control policies that vary the transmission duration. We propose two families of control policies, a threshold based and a fixed transmission time policy, and evaluate their age-power tradeoff. We analytically characterize the tradeoff for the family of fixed transmission time policies, which is also an upper bound on the optimal tradeoff. For small packet generation rates, we also obtain an analytical lower bound on the optimal tradeoff, which shows that fixed transmission time based policies are near-optimal. We provide numerical and simulation results to illustrate and compare the tradeoff offered by the different policies.**

Index Terms—**Age of information, Transmission power, Optimal tradeoff, Semi-Markov decision process**

I. Introduction

Remote estimation and monitoring of relevant system processes are becoming increasingly important in smart cities, internet-of-things (IoT) and industrial IoT for various applications such as environmental monitoring, feedback control and actuation, and security [1]. Wireless networks for such applications have to be end-to-end optimized for information freshness [2] (for instance using age of information) rather than for conventional metrics such as delay or throughput. Transmission power is also a prime concern in these battery-constrained monitoring systems. Understanding the optimal tradeoff between the average age of information (AoI) and transmit power is therefore important for designing energy-efficient remote estimation systems.

Uysal et al. [3] considered the dynamic control of packet transmission times in order to tradeoff delay and transmit power, with longer transmission times requiring lower power (and vice versa). The majority of traffic generated by sensors for freshness-sensitive applications comprises short packets [4], [5]. In order to transmit these short packets over noisy channels, short packet codes (SPC) with *smaller* codeword lengths are employed. In the transmission of short packet codewords, for a given reliability of codeword transmissions, the per-packet transmission time and transmit power can be traded off with one another [6]. Motivated by these, in this paper we investigate transmitter control policies which dynamically choose the duration τ over which each packet is transmitted in order to adapt its per-packet transmission power $P(\tau)$.[1] We characterize the fundamental tradeoff between the average age of information (AoI) and average transmission power for such policies. To the best of our knowledge, the characterization of the AoI-power tradeoff for such policies are not available in prior work.

The tradeoff of average transmit power and AoI has been considered in other contexts. In [7], the authors solve the problem of minimizing a linear combination of the AoI and the total energy consumption by casting the problem as a constrained Markov decision process (CMDP) and solving it using Lagrangian relaxation. The tradeoff between the AoI, quality/distortion, and energy is considered in [8]. An online greedy algorithm is developed to minimize a linear combination of quality metric, AoI, and energy cost. The tradeoff between age and quality/distortion is analyzed in terms of age-dependent distortion constraints in [9]. Energy minimization under a peak AoI constraint is considered in [10], where the packets can be selected/deselected for service and the transmission rate can be chosen based on the current AoI to satisfy the AoI constraint. In [11], an optimal non-preemptive policy that minimizes a linear combination of weighted AoI and total service cost (in a G/G/1 queuing system with a single server) by transmitting a subset of updates is developed. The energy-age tradeoff in a status update system with feedback having packet losses is considered in [12]. A threshold-based retransmission policy with a constraint on the maximum allowed retransmissions of a packet is analyzed, and closed-form expressions for the average AoI and energy consumption are derived. In [13], the age-energy tradeoff for two-hop decode-and-forward relaying networks based on short packets is investigated and the tradeoff is achieved by minimizing the weighted sum of the average AoI and the average energy cost.

Contributions: We formulate the optimal tradeoff problem between AoI and average transmission power as a semi-Markov decision problem. The numerical solution of this problem is useful in determining the performance gap of two practical control policies that we propose. Specifically, we analyze two families of policies (a family of fixed transmission time policies and a family of threshold policies) and obtain

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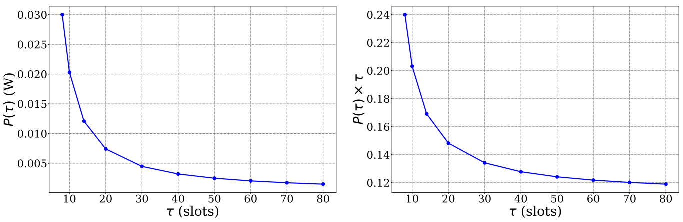
The work of Chandra R. Murthy was supported by a young faculty research fellowship from the Ministry of Electronics & Technology, Govt. of India.

[1] The power $P(\tau)$ can be modelled as a convex non-increasing function of τ, see Section II-A.



(a) τ versus $P(\tau)$. (b) τ versus $P(\tau) \times \tau$.

Fig. 1: Relationship between the transmission time τ and (a) the transmit power $P(\tau)$ and (b) the product $P(\tau) \times \tau$.

upper bounds on the tradeoff. This analytical upper bound is used to design the constant transmission time policy for a given power constraint. We also obtain an approximate analytical lower bound on the optimal tradeoff when the arrival rate into the system is small. Interestingly, this lower bound shows that the family of fixed transmission time policies is a *good* choice when arrival rate is sufficiently small. Thus, we obtain analytical upper and lower bounds to the optimal tradeoff between AoI and average transmission power and provide policy design guidelines.

Notation: $\mathbb{Z}_+$ and $\mathbb{Z}_{++}$ to denote the set of non-negative integers and the set of positive integers, respectively. Random variables and their specific realizations are denoted using capital letters (e.g., X) and corresponding small letters (e.g., x). The variance of a random variable X is denoted by $\mathrm{Var}(X)$.

II. System Model and Problem Statement

A. Model for transmit power $P(\tau)$ as a function of τ

We consider a point-to-point link where the transmission time τ of a packet can be chosen by the transmitter from $\{\tau_{min}, \tau_{min}+1, \ldots, \tau_{max}\}$, where $\tau_{min} < \tau_{max} \in \mathbb{Z}_{++}$. We model $P(\tau)$ as well as $\tau P(\tau)$ as convex non-increasing functions of the packet transmission duration τ motivated by the following discussion.

Consider a packet of length K bits to be sent using a transmission rate ρ. Then, the transmission time $\tau = K/\rho$. From Shannon's channel capacity theorem for AWGN channels, the rate of transmission, ρ, is given by $\rho = W \log_2(1 + \mathsf{SNR})$, where W denotes the bandwidth of communication and $\mathsf{SNR} = P/N$ denotes the signal-to-noise ratio, where P is the transmit power and N is the noise power. Then,

$$\tau = \frac{K}{W \log_2(1 + P/N)} \text{ and } P = N\left(2^{\frac{K}{W\tau}} - 1\right). \quad (1)$$

We note that $P(\tau)$ and $\tau P(\tau)$ are both convex non-increasing functions of τ; the relationship is plotted in Fig. 1.

B. Model for the point-to-point link

We consider a time-slotted model with slots indexed by $t \in \mathbb{Z}_+$. We assume that packets of a fixed length K arrive into the transmitting node according to an independent and identically distributed (IID) Bernoulli random process denoted by $(U[t], t \geq 0)$, with arrival rate $\lambda < 1$ (i.e., $Pr(U[t] = 1) = \lambda$). A packet arrival occurs at the start of a slot t if $U[t] = 1$. Immediately after a new packet arrives, it is transmitted over a controllable number of slots; a minimum of τ_{min} slots are required for packet transmission. We assume that if a new packet arrives while a previous packet is being transmitted, then the new packet *preempts* the earlier transmission. Thus, only the latest packet is stored and transmitted. This model is appropriate when the transmitting node does not buffer the data to be transmitted; we extend the analysis to alternate models without preemption in the full version of this paper. We index packets using $m \in \mathbb{Z}_+$. The inter-arrival time between the mth and $(m+1)$th packet is denoted by T_g^m (a specific realization is t_g^m). Note that the inter-arrival times are IID and geometrically distributed with parameter λ. The slot in which the mth packet arrives at the transmitting node is denoted by $T[m]$; $T[m] = \sum_{n<m} T_g^n$ and $T[0] = 0$.

The transmission time of a packet can be controlled, and the decision about the (possibly random) transmission time of a packet is made upon its arrival. Thus, the arrival slots constitute the decision epochs of the transmitter. The transmission time of the mth packet is denoted by $T_s^m \geq 1$ (a specific realization is t_s^m). Thus, the slot in which the mth packet is received, denoted by $R[m]$, is $R[m] = T[m] + T_s^m$. In this setup, increasing the transmission time allows us to transmit using lower transmit power and energy, but runs the risk of the packet getting preempted by the next arrival. The transmit power corresponding to the choice of $T_s^m = t_s^m$ is $P(t_s^m)$, which satisfies the properties discussed in Section II-A. The energy expended during transmission (if not preempted) is therefore $t_s^m P(t_s^m)$. For simplicity, we assume that the transmissions are lossless since the transmit power and duration are selected to ensure packet delivery with high reliability.

The performance metric that we consider is the average AoI. The AoI process [2] (denoted by $A[t], \forall t$) is defined as the time elapsed at the receiver since the generation time of the latest successfully received packet. So, at time slot t, if $U[t] \leq t$ denotes the time slot at which the latest successfully received packet arrived at the transmitter, then

$$A[t] \triangleq t - U[t].$$

An illustration of the evolution of $A[t]$ is shown in Fig. 2.

A transmission policy π chooses a transmission time T_s^m at every decision epoch, as a (possibly randomized) function of the past evolution of $A[t]$, as well as past decisions. Note that, since the transmissions are lossless, the age evolution $A[t]$ is known at the transmitter. The set of all transmission policies is denoted by Π. We also consider a class of stationary randomized policies Π_s that chooses T_s^m as a randomized function $\tau(\cdot)$ of $A[T[m]]$. For a policy $\pi \in \Pi_s$ we define the average age of information as

$$\overline{A}^\pi = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} A[t]$$

We define $P[t]$ as the transmit power in slot t. We note that $P[t] = P(t_s^m)$ if the mth packet is being transmitted in slot t.

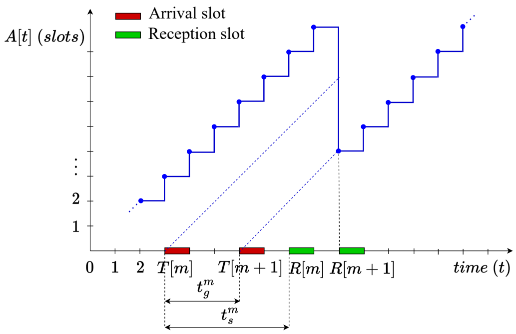


Fig. 2: Illustration of the evolution of AoI $A[t]$. Preemption of a packet under transmission by a new arrival is also shown.

Then, for a policy $\pi \in \Pi_s$, we define the average power as

$$\overline{P}^{\pi} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} P[t]$$

The AoI-power tradeoff problem that we consider in this paper can be written as

$$\min_{\pi\in\Pi_s} \quad \overline{A}^{\pi} \text{ s.t. } \quad \overline{P}^{\pi} \le p_c, \tag{2}$$

where $p_c > 0$ is an average power constraint.[2] The optimal value of the above problem (if it exists) is denoted by $A^*(p_c)$. In the following sections, we characterize $A^*(p_c)$ analytically and numerically. We note that the Pareto points of the above tradeoff can also be obtained by considering the following optimization problem:

$$\min_{\pi\in\Pi_s} \overline{A}^{\pi} + \beta \overline{P}^{\pi}, \tag{3}$$

where $\beta \ge 0$ is a Lagrange multiplier.

III. Semi-Markov decision process formulation

The problem in (3) can be formulated as a semi-Markov decision process (SMDP) with an infinite horizon average cost criterion to obtain an optimal policy.[3] The SMDP is characterized by the tuple $(\mathcal{S}, \mathcal{A}, \mathbb{P}, \tau, \mathcal{C})$, where $\mathcal{S}$ is the set of possible states, $\mathcal{A}$ is a finite set of possible actions, $\mathbb{P}$ is the Markov state transition probability: $\mathbb{P}(s' \mid s, u)$ is the probability that the system will be in state s' at the next decision epoch if the action u is chosen in the present state s. Also, $\tau(s,u)$ is the expected time until the next decision epoch and $\mathcal{C}$ is the expected cost incurred until the next decision epoch, if action u is chosen in the present state s.

The state space $\mathcal{S}$ of the process is the set of all possible age values ($\mathcal{S} = \mathbb{Z}_{++}$). We assume that any transmission time in the action space $\mathcal{A} = \{\tau_{min}, \tau_{min}+1, \cdots, \tau_{max}\}$ can be chosen. We denote $P(\tau_{min})$ by P_{min} and $P(\tau_{max})$ by P_{max}. Since the transmission times are discrete valued, the transmit power takes a set of discrete values in the range $[P_{max}, P_{min}]$. The decision times of the SMDP coincide with the arrival times of the packets. We note that the time between two consecutive decision epochs is $T_g^m \in \mathbb{Z}_{++}$. In the following, we use $G \sim T_g^m$ which is a Geometrically distributed random variable. We denote the state at the mth decision epoch by A_m; $A_m = A[T[m]]$. Then, two cases arise in the transition probability from A_m to A_{m+1}, depending on whether $t_g^m < t_s^m$ or $t_g^m \ge t_s^m$. Based on these cases, the transition probability can be calculated as follows:

$$A_{m+1} = \begin{cases} t_g^m, & t_g^m \ge t_s^m \\ A_m + t_g^m, & t_g^m < t_s^m. \end{cases} \tag{4}$$

Using the conditional distribution of T_g^m, depending on whether $T_g^m \ge t_s^m$ or not, the transition probabilities of the SMDP can be obtained.

In order to optimize the objective function in (3), we define the following single stage cost $c(a,\tau)$ which is the expected cumulative age and power over the time duration between two consecutive decision epochs. Here, a is the age at the decision epoch and τ is the chosen transmission time. If the chosen $\tau \le G$ (where G is the time to the next packet arrival at the transmitter), then we use the random variable $\tilde{G}$ to represent the number of slots counted at the transmitter between the reception of the packet at the receiver and the arrival of the next packet at the transmitter. That is, $\tilde{G} = G - \tau$ conditioned on $G \ge \tau$. We note that $\tilde{G}$ is a geometric random variable with parameter λ that takes values in $\{0, 1, 2, \cdots\}$. We can express $c(a,\tau)$ as

$$\begin{aligned} &\mathbb{E}\Bigg[\mathbb{I}\{\tau \leqslant G\}\left\{a\tau + (\tau-1)\frac{\tau}{2} + \tau\tilde{G} + (\tilde{G}-1)\frac{\tilde{G}}{2}\right\} + \\ &\mathbb{I}\{\tau > G\}\left\{aG + (G-1)\frac{G}{2}\right\}\Bigg| A_m = a, \tau(A_m) = \tau\Bigg] + \\ &\beta\mathbb{E}\Bigg[\mathbb{I}\{\tau \le G\} P(\tau)\tau + \\ &\mathbb{I}\{\tau > G\} P(\tau) G \Bigg| A_m = a, \tau(A_m) = \tau\Bigg] \end{aligned}$$

where $\mathbb{I}\{.\}$ is the indicator function.

We note that a numerical procedure such as value iteration [15] can be used to solve a truncated version of the SMDP (where the state or age values are limited to a_{max}). The optimal policy for this truncated SMDP is denoted by π_{SMDP}. The average AoI and power for π_{SMDP} (denoted by $\overline{A}^{\pi_{SMDP}}$ and $\overline{P}^{\pi_{SMDP}}$ respectively) are useful to evaluate the performance of other practically implementable policies.

IV. Policies for trading off AoI with average power

In this section, we define and analyze two families of policies which can be used to tradeoff AoI with average power.

[2] This constrained optimization problem, but over $\pi \in \Pi$, can be formulated as a constrained Markov decision process (CMDP) [14]. From [14], under some technical assumptions, it can be shown that the class of stationary randomized policies contains an optimal policy. This motivates our restriction to $\pi \in \Pi_s$ in this paper.

[3] In this approach, the policy is obtained numerically for an appropriately state-truncated system. Therefore, it is only approximately optimal for the actual system.

A. Threshold policy

A threshold policy is parameterized by a threshold h on age at a decision epoch and two transmission times τ_a and τ_b ($\tau_a, \tau_b \in \mathcal{A}$ with $\tau_a > \tau_b$). The threshold policy chooses the transmission time as a function $\tau(A_m)$ of the age at a decision epoch. The function

$$\tau(A_m) = \begin{cases} \tau_a \text{ if } A_m \leq h, \\ \tau_b \text{ if } A_m > h. \end{cases} \quad (5)$$

When h is small, the policy uses the smaller service time τ_b to transmit the packets most of the time (i.e., unless the age is below h at the decision epoch); this comes at the cost of a higher average power consumption. When h is large, it uses the larger service time τ_a most of the time; this lowers the average power consumption but could lead to a large average age if many packets get preempted. Thus, by varying the threshold h, we obtain a tradeoff for fixed values of τ_a and τ_b.

B. Fixed transmission time policy

A fixed transmission time (FTT) policy uses a fixed time t_s for every transmission. The parameter t_s can be varied to obtain different $\overline{A}^{\pi}$ and $\overline{P}^{\pi}$. A small t_s is expected to give a larger $\overline{P}^{\pi}$ and a smaller $\overline{A}^{\pi}$ compared to a large t_s.

The tradeoff performance of the FTT policy can be characterized analytically. The average AoI and average power for an FTT policy are obtained by identifying a renewal reward process in the evolution of $A[t]$. Consider the slot just after a packet's transmission is over. Note that the age value at this time slot is t_s. We define a renewal cycle as the duration between two such successive packet service time completions. These durations are IID. By characterizing the expected cumulative age and power over a renewal cycle, we obtain the following result using the renewal reward theorem [16].

Proposition 1. *For an FTT (t_s) policy, π_{t_s}, the AAoI is*

$$\overline{A}^{FTT} = t_s + \frac{\mathbb{E}R^2}{2\mathbb{E}R} - \frac{1}{2},$$

and the average power is $\overline{P}^{FTT} = \frac{P(t_s)\left(E_G\left[\frac{1}{\alpha}-1\right]+t_s\right)}{\mathbb{E}R}$, *where*

$$\begin{aligned}
\alpha &= (1-\lambda)^{t_s-1},\ E_G = \sum_{g=1}^{t_s-1} g\left(\frac{\lambda(1-\lambda)^{g-1}}{1-\alpha}\right), \\
V_G &= \sum_{g=1}^{t_s-1} (g-E_G)^2 \cdot \left(\frac{(1-\lambda)^{g-1}\lambda}{1-\alpha}\right), \\
\mathbb{E}R &= \frac{1-\lambda}{\lambda} + E_G\left[\frac{1}{\alpha}-1\right] + t_s, \\
\mathbb{E}R^2 &= \frac{1-\lambda}{\lambda^2} + V_G\left(\frac{1}{\alpha}-1\right) + E_G^2\left(\frac{1-\alpha}{\alpha^2}\right) + (\mathbb{E}R)^2
\end{aligned}$$

An outline of the proof is presented in Appendix A.

V. A LOWER BOUND FOR SMALL λ

In this section, we obtain an approximate lower bound on the minimum AoI $A^*(p_c)$ for a power constraint p_c under the assumption that λ is small.

Proposition 2. *For sufficiently small λ, for a power constraint $p_c > 0$, we have that*

$$A^*(p_c) \gtrapprox c_l(\tau^*),$$

where τ^* is the smallest $\tau \in [\tau_{min}, \tau_{max}]$ such that $\lambda \tau P(\tau) \leq p_c$ and

$$c_l(\tau) \;=\; \lambda\left(\tau + \frac{\tau(\tau-1)}{2} + \tau\frac{1-\lambda}{\lambda} + \left(\frac{1-\lambda}{\lambda}\right)^2\right).$$

Proof. If λ is sufficiently small, then, for finite τ_{max}, packets under service are not preempted with high probability. We note that without preemption, the age A_m at every such epoch is T_g^{m-1}. We identify a Markov renewal reward process (MRRP) in the evolution of the age as follows. The embedded Markov chain associated with the MRRP is A_m. We associate two cumulative rewards with the MRRP over each renewal cycle. The cumulative age $c_m(A_m, \tau(A_m))$ reward is

$$\mathbb{E}\Bigg[A_m \tau(A_m) + \frac{\tau(A_m)(\tau(A_m))-1)}{2} + \tau(A_m)\tilde{G} + \frac{\tilde{G}(\tilde{G}-1)}{2}\Bigg].$$

The cumulative power reward is $\mathbb{E}\left[P(\tau(A_m)) \cdot \tau(A_m)\right]$. Here, $\tau(\cdot)$ is obtained from a stationary policy. Using the Markov renewal reward theorem[4] (MRRT) [16, Appendix D], we obtain the average AoI and average power as $\mathbb{E}\left[c(A_m, \tau(A_m))\right]/(1/\lambda)$ and $\mathbb{E}\left[P(\tau(A_m))\tau(A_m)\right]/(1/\lambda)$, respectively. Here, the expectation is taken over the stationary distribution of A_m, which is geometric. Also, $\tilde{G}$ is a geometric random variable with parameter λ that takes values in $\{0, 1, 2, \cdots\}$ (therefore, $\mathbb{E}\tilde{G} = (1-\lambda)/\lambda$ and $\mathbb{E}\tilde{G}^2 = (1-\lambda)(2-\lambda)/\lambda^2$). Let us denote the stationary version of A_m by A. We now consider the problem

$$\begin{aligned}
\text{minimize}_{\tau(.)} \quad & \mathbb{E}\left[c(A, \tau(A))\right]/(1/\lambda), \\
\text{such that} \quad & \mathbb{E}\left[P(\tau(A))\tau(A)\right]/(1/\lambda) \leq p_c.
\end{aligned}$$

In order to obtain a lower bound on the above optimization problem, we bound the first term $A\tau(A)$ in $c(A, \tau(A))$ from below by $1 \times \tau(A)$, since $A \geq 1$. The lower bound $c_l(\tau(A))$ is then a function of $\tau(A)$. We note that the average power is also a function of $\tau(A)$. Thus, we have the following optimization problem, where we optimize over all possible choices of the distribution of a random variable $\tau \in [\tau_{min}, \tau_{max}]$. Note that we have relaxed the integer constraint on τ, which is allowed since we seek a lower bound.

$$\begin{aligned}
\text{minimize}_{\tau(.)} \quad & \mathbb{E}\left[c_l(\tau)\right]/(1/\lambda), \\
\text{such that} \quad & \mathbb{E}\left[P(\tau)\tau\right]/(1/\lambda) \leq p_c.
\end{aligned}$$

The objective $c_l(\tau)$ is a convex increasing function in τ,

[4] We can apply MRRT here since the cumulative age and power are independent of the past conditioned on A_m.

while the constraint is convex decreasing in τ. Therefore, by Jensen's inequality, an optimal distribution would assign all the probability mass to the smallest τ^* such that the constraint is satisfied. The approximate lower bound is then $c_l(\tau^*)$ which we denote by $A_l(p_c)$, i.e., $A_l(p_c) = c_l(\tau^*)$. $\square$

Remark 1. *Proposition 2 shows that the FTT policy with a single parameter t_s is optimal. Therefore, the family of FTT policies is a* good *candidate policy for* low *arrival rates.*

VI. Numerical & Simulation Results

We first validate the analytical characterization of $\overline{A}^{\pi}$ and $\overline{P}^{\pi}$ for the FTT policy obtained in Proposition 1 using simulations. In Fig. 3, we plot $\overline{A}^{\pi}$ vs. $\overline{P}^{\pi}$ for FTT policies as t_s is varied in the range $[5, 10]$ in steps of 1. We set probability of packet arrival in each slot, $\lambda = 0.05$, and the number of bits in a packet $K = 800$. The simulation and theoretical results match perfectly, which validates our analytical characterization of the tradeoff for the family of FTT policies.

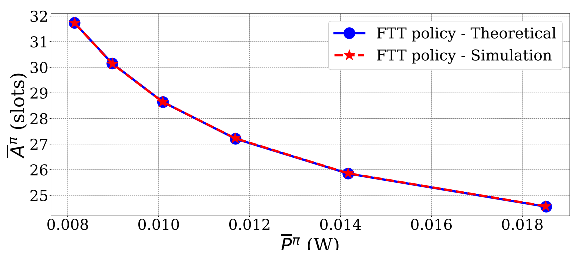
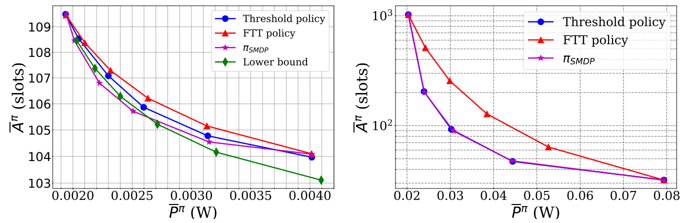


Fig. 3: AAoI-power tradeoff under the FTT policy.

Our characterization of the average AoI-average power tradeoff, for $P(\tau)$ from (1), is shown in Fig. 4a. We consider an arrival rate of 0.01 and compare the AoI-power tradeoff of the FTT policy, the threshold policy, π_{SMDP}, and the lower bound. In order to obtain the tradeoff for the threshold based policies, we fixed the two power levels $P_a = 21$ mW and $P_b = 90$ mW for which the corresponding service times are $\tau_a = 10$ and $\tau_b = 5$, respectively, from (1) with noise power $N = 10$ mW and bandwidth $W = 50$ Hz. We vary the threshold h from 0 to 2000 to get different points in the tradeoff curve. Both FTT and threshold based policies offer a tradeoff that is close to that of π_{SMDP}. We also see that when the power constraint is reduced, the no-preemption assumption used in the approximate lower bound fails. In Fig.4b, we illustrate the tradeoff offered by the FTT policy, threshold policy and π_{SMDP} for a higher arrival rate of $\lambda = 0.5$ using $P(\tau)$ from (1). For large λ, the performance of the threshold policy is close to optimal. This motivates the choice of the family of threshold policies for large λ.

VII. Conclusions

We considered the optimal tradeoff between AoI and average transmit power for a point-to-point link. The dependence of transmit power on the transmit duration for a simplified physical layer model (obtained from Shannon's capacity formula) was considered. By comparing with an optimal policy obtained from an SMDP formulation, we showed that a family of simple fixed transmission time policies which have a single parameter (the transmission time t_s) offers a near-optimal tradeoff when the packet arrival rate is small. The analytical characterization of AoI and average power for fixed transmission time policies can be used to obtain the transmission time to be used for a given average power constraint. Finally, we also proposed and evaluated the performance of threshold based policies, which could be suitable for larger packet generation rates. We plan to investigate the AoI-transmit power tradeoff for unreliable point-to-point links in our future work.



(a) $\lambda = 0.01$ (b) $\lambda = 0.5$

Fig. 4: The tradeoff between AAoI and average power for $P(\tau)$ given by (1). Performance of threshold, FTT and π_{SMDP} policies are compared for $\lambda = 0.01$ and $\lambda = 0.5$. The analytical lower bound is also plotted for $\lambda = 0.01$.

Appendix A
Proof of Proposition 1

We obtain the average AoI and average power for the FTT policy using renewal reward theorem (RRT) [16]. For applying RRT, we first identify a renewal process in the evolution of $A(t)$ under an FTT policy with parameter t_s. We define a renewal epoch as the slot in which the age $A(t)$ drops due to a packet's reception. We note that since FTT uses a fixed service time t_s, the age at a renewal epoch is t_s. Furthermore, the time to the next arrival is geometrically distributed with parameter λ due to the memoryless property of the arrival process. The renewal cycle R is thus composed of the geometric(λ) time till the first arrival, followed by the time taken for service, t_s, if t_s is less than or equal to the time to the second arrival, or the time to preemption otherwise. At every m, there are two cases to consider depending on whether $t_s \le T_g^m$ or not. If the first packet in a renewal cycle is preempted (i.e., if $t_s > T_g^m$), then the renewal cycle extends by a further t_s slots after preemption or till the next preemption. We define the following quantities to describe R. The number of slots for the first packet arrival in a renewal cycle is denoted by $G_0 \sim$ Geometric(λ). We denote the total number of packet arrivals till a complete packet transmission (which is the sum of arrivals that are preempted and the last arrival which gets transmitted without preemption) by X. We denote the number of slots between each preemption by G_i, where $G_i \sim$ Geometric(λ) conditioned on $G_i < t_s$. Then the length R of the renewal cycle is

$$R = G_0 + \sum_{i=1}^{X-1} G_i + t_s$$

The distribution of X (with $\alpha = (1-\lambda)^{t_s-1}$) is

$$P_X(x) = \alpha(1-\alpha)^{x-1}, 1 \le x < \infty. \tag{6}$$

To apply RRT, we first obtain the cumulative age in the renewal cycle as $\overline{\mathcal{A}} = t_s R + \frac{R(R-1)}{2}$. Then, using RRT,

$$\overline{A}^\pi = \frac{\mathbb{E}[\overline{\mathcal{A}}]}{\mathbb{E}[R]} = t_s + \frac{\mathbb{E}\left[R^2\right]}{2\mathbb{E}[R]} - \frac{1}{2}.$$

We note that

$$\mathbb{E}[R] = \mathbb{E}\left[G_0\right] + \mathbb{E}\left[\sum_{i=1}^{X-1} G_i\right] + t_s.$$

Since $G_0 \sim$ Geometric(λ), $\mathbb{E}\left[G_0\right] = \frac{1-\lambda}{\lambda}$. We note that $E_G = \mathbb{E}[G_i]$ is $\mathbb{E}\left[T_g \mid T_g < t_s\right]$, which is

$$\sum_{g=1}^{t_s-1} g\left(\frac{\lambda(1-\lambda)^{g-1}}{1-(1-\lambda)^{t_s-1}}\right).$$

Taking the expectation over X, we can show that $\mathbb{E}\left[\sum_{i=1}^{X-1} G_i\right] = E_G\left[\frac{1}{\alpha}-1\right]$, so that $\mathbb{E}[R] = \frac{1-\lambda}{\lambda} + E_G\left[\frac{1}{\alpha}-1\right] + t_s$. Now, we compute $\mathbb{E}R^2$ as $\mathrm{Var}(R) + (\mathbb{E}[R])^2$. We have that

$$\mathrm{Var}(R) = \mathrm{Var}\left(G_0\right) + \mathrm{Var}\left(\sum_{i=1}^{X-1} G_i\right),$$

where $\mathrm{Var}\left(G_0\right) = \frac{1-\lambda}{\lambda^2}$. We let $S_G = \sum_{i=1}^{X-1} G_i$. Then

$$\mathrm{Var}\left(S_G\right) = \mathbb{E}\left[\mathrm{Var}\left(S_G \mid X\right)\right] + \mathrm{Var}\left(\mathbb{E}\left[S_G \mid X\right]\right).$$

We note that

$$\mathrm{Var}\left(S_G \mid X = x\right) = \mathrm{Var}\left(\sum_{i=1}^{x-1} G_i\right) = (x-1) \cdot \mathrm{Var}\left(G_i\right),$$

so that $\mathbb{E}[\mathrm{Var}(S_G \mid X)]$

$$= \mathbb{E}\left[(X-1)\,\mathrm{Var}\left(G_i\right)\right] = \mathrm{Var}\left(G_i\right) \cdot (\mathbb{E}[X]-1).$$

Also, from (6) we have $\mathbb{E}[X] = \frac{1}{\alpha}$. Further,

$$\mathbb{E}\left[S_G \mid \mathrm{X} = x\right] = \mathbb{E}\left[\sum_{i=1}^{x-1} G_i\right] = (x-1) \cdot \mathbb{E}\left[G_i\right].$$

We denote $\mathrm{Var}(G_i)$ by V_G:

$$V_G = \sum_{g=1}^{t_s-1} \left(g - E_G\right)^2 \cdot \left(\frac{(1-\lambda)^{g-1}\lambda}{1-\alpha}\right).$$

Therefore, $\mathrm{Var}\left(\mathbb{E}\left[S_G \mid X\right]\right) =$

$$\mathrm{Var}\left((X-1)\mathbb{E}\left[G_i\right]\right) = \left(\mathbb{E}\left[G_i\right]\right)^2 \mathrm{Var}(X),$$

where $\mathrm{Var}(X) = \frac{1-\alpha}{\alpha^2}$. Finally, we have that

$$\begin{aligned} \mathrm{Var}\left(\sum_{i=1}^{X-1} G_i\right) &= V_G \cdot (E[X]-1) + E_G^2\left(\frac{1-\alpha}{\alpha^2}\right), \text{ and,} \\ \mathrm{Var}(R) &= \frac{1-\lambda}{\lambda^2} + V_G\left(E[X]-1\right) + E_G^2\left(\frac{1-\alpha}{\alpha^2}\right). \end{aligned}$$

Then, we obtain

$$\mathbb{E}\left[R^2\right] = \frac{1-\lambda}{\lambda^2} + V_G\left(E[X]-1\right) + E_G^2\left(\frac{1-\alpha}{\alpha^2}\right) + (\mathbb{E}[R])^2$$

Similarly, we obtain the average power using RRT. The power consumed for transmitting is $P(t_s)$, which is fixed. This fixed power is consumed over a duration with expected value:

$$\mathbb{E}[R] - \mathbb{E}[G_0] = E_G\left[\frac{1}{\alpha} - 1\right] + t_s$$

Therefore, applying RRT, we have that the average power is

$$\overline{P}^\pi = \frac{P(t_s)\left(E_G\left[\frac{1}{\alpha}-1\right] + t_s\right)}{\mathbb{E}R}.$$

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