Supplemental Material for "Novel Receiver Designs for Low-resolution ADC-Based Uplink MU-MIMO Systems with Unknown Channel Covariance Information"

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I. Computation of Relevant Expectations

We describe the computation of the expectation of $\mathbf{R}_k^{-1}$, $\mathbf{R}_k^{*-1} \otimes \mathbf{R}_k^{-1}$, $\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$, $\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$ and $\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}$, which are used in the Algorithm 1 and the proof of Lemma 1. We use the notation $\mathbf{E}_{ij} = \mathbf{e}_i\mathbf{e}_j^T$, where $\mathbf{e}_i$ denotes the ith canonical basis vector.

1) Expectation of $\mathbf{R}_k^{-1}$ *and* $\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$*:* Consider the function $Q(\mathbf{S}) = \mathbb{E}(\exp(-\mathrm{tr}(\mathbf{S}^T\mathbf{R}_k^{-1})))$, and note that $\mathbb{E}(\mathbf{R}_k^{-1}) = -\left.\frac{\partial Q(\mathbf{S})}{\partial \mathbf{S}}\right|_{\mathbf{S}=\mathbf{0}}$. A closed form expression for $Q(\mathbf{S})$ can be easily computed for the complex inverse Wishart distribution, and is given by $Q(\mathbf{S}) =$

$$\int \exp(-\mathrm{tr}((\mathbf{S}^T + \boldsymbol{\Psi})\mathbf{R}_k^{-1}))\frac{|\boldsymbol{\Psi}|^{\nu}}{\mathcal{C}\Gamma_{N_r}(\nu)}|\mathbf{R}_k|^{-(\nu+p)}d\mathbf{R}_k,$$
$$= \frac{|\boldsymbol{\Psi}|^{\nu}}{|\boldsymbol{\Psi} + \mathbf{S}^T|^{\nu}}. \quad (41)$$

Using (41), the derivative of $Q(\mathbf{S})$ is given by

$$\frac{\partial Q(\mathbf{S})}{\partial \mathbf{S}} = -\nu\frac{|\boldsymbol{\Psi}|^{\nu}}{|\boldsymbol{\Psi} + \mathbf{S}^T|^{\nu}}(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}. \quad (42)$$

Substituting $\mathbf{S} = \mathbf{0}$ in the above expression, we obtain

$$\mathbb{E}(\mathbf{R}_k^{-1}) = \nu\boldsymbol{\Psi}^{-1}. \quad (43)$$

Next, from (41), it can be seen that to compute the lth column of $\mathbb{E}\big[\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}\big]$, we must use the following steps:

- Compute $\mathbf{q}_i(\mathbf{S})$, the partial derivative of $Q(\mathbf{S})$ with respect to the ith column of $\mathbf{S}$.
- Compute the partial derivative of $\mathbf{q}_i(\mathbf{S})$ with respect to the (j, l)th element of $\mathbf{S}$.
- Evaluate the expression at $\mathbf{S} = \mathbf{0}$.

The partial derivative of $Q(\mathbf{S})$ with respect to the ith column of $\mathbf{S}$ is given by

$$\mathbf{q}_i(\mathbf{S}) = -\nu\,\frac{|\boldsymbol{\Psi}|^{\nu}}{|\boldsymbol{\Psi} + \mathbf{S}^T|^{\nu}}\,(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}\mathbf{e}_i. \quad (44)$$

Next, the partial derivative of $\mathbf{q}_i(\mathbf{S})$ with respect to $[\mathbf{S}]_{jl}$ is

$$\mathbf{q}_{ijl}(\mathbf{S}) = \nu\,\frac{|\boldsymbol{\Psi}|^{\nu}}{|\boldsymbol{\Psi} + \mathbf{S}^T|^{\nu}}\,(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}\mathbf{E}_{lj}(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}\mathbf{e}_i$$
$$+\,\nu^2\,\frac{|\boldsymbol{\Psi}|^{\nu}}{|\boldsymbol{\Psi} + \mathbf{S}^T|^{\nu}}\,\mathbf{e}_j^T(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}\mathbf{e}_l\,(\boldsymbol{\Psi} + \mathbf{S}^T)^{-1}\mathbf{e}_i. \quad (45)$$

Thus, the lth column of $\mathbb{E}\big[\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}\big]$ is

$$\big[\mathbb{E}\big(\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}\big)\big]_{.l} = \mathbf{q}_{ijl}(\mathbf{S})\big|_{\mathbf{S}=\mathbf{0}}$$
$$= \nu\,\boldsymbol{\Psi}^{-1}\mathbf{e}_l\mathbf{e}_j^T\boldsymbol{\Psi}^{-1}\mathbf{e}_i + \nu^2\,\boldsymbol{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\boldsymbol{\Psi}^{-1}\mathbf{e}_l.$$

Collecting all columns $\{[\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})]_{.l}\}_{l=1}^{N_r}$ into a matrix yields the final expression

$$\mathbb{E}\big(\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}\big) = \nu\,(\mathbf{e}_j^T\boldsymbol{\Psi}^{-1}\mathbf{e}_i)\boldsymbol{\Psi}^{-1} + \nu^2\,\boldsymbol{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\boldsymbol{\Psi}^{-1}. \quad (46)$$

2) ***Expectation of*** $\mathbf{R}_k^{*^{-1}} \otimes \mathbf{R}_k^{-1}$*:* We use the expression for $\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})$ derived in the previous subsection to compute the expectation of the required quantity, $\mathbf{R}_k^{*^{-1}} \otimes \mathbf{R}_k^{-1}$. Note that the $(i + (j-1)N_r)$th column of $\mathbf{R}_k^{*^{-1}} \otimes \mathbf{R}_k^{-1}$ is $\mathrm{vec}(\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})$. Hence,

$$\mathbf{R}_k^{*-1} \otimes \mathbf{R}_k^{-1} = [\mathrm{vec}(\mathbf{R}_k^{-1}\mathbf{E}_{11}\mathbf{R}_k^{-1}), \mathrm{vec}(\mathbf{R}_k^{-1}\mathbf{E}_{21}\mathbf{R}_k^{-1})$$
$$\cdots, \mathrm{vec}(\mathbf{R}_k^{-1}\mathbf{E}_{N_rN_r}\mathbf{R}_k^{-1})]$$

Therefore, using (46), we get

$$\mathbb{E}(\mathbf{R}_k^{*-1} \otimes \mathbf{R}_k^{-1}) = [\mathrm{vec}(\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{11}\mathbf{R}_k^{-1})),$$
$$\cdots, \mathrm{vec}(\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{N_rN_r}\mathbf{R}_k^{-1}))]$$
$$= \nu\mathrm{vec}(\boldsymbol{\Psi}^{-1})\mathrm{vec}(\boldsymbol{\Psi}^{-1})^H + \nu^2(\boldsymbol{\Psi}^{-1^T} \otimes \boldsymbol{\Psi}^{-1}).$$

3) ***Expectation of*** $\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$*:* The computation of $\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$ is also essential to obtain the closed form expression of MB-CRLB for the estimation of the channel covariance matrix. Computing $\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}$ involves the following steps:

- Compute $\mathbf{Q}_{rs}(\mathbf{S})$ as given by (48) later in this section, and note that $\mathbf{Q}_{rs}(\mathbf{0}) = \mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1})$, which can be computed using (46).
- Compute $\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})$ using the partial derivatives of the ith column of $\mathbf{Q}_{rs}(\mathbf{S})$ with respect to the elements of the jth row of the matrix $\mathbf{S}$.
- Finally, compute $\mathbb{E}(\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})$ using

$$\mathbb{E}(\mathbf{R}_k^{-1}\boldsymbol{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}) =$$
$$\sum_{r,s=1}^{N_r}\mathbf{e}_r^T\boldsymbol{\Psi}\mathbf{e}_s\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}). \quad (47)$$

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Similar to (44), (45), the partial derivative of $Q(\mathbf{S})$ with respect to the rth column, followed by the partial derivative with respect to $[\mathbf{S}]_{sl}$ of $\mathbf{S}$ yield the lth column of $\mathbf{Q}_{rs}(\mathbf{S})$. Arranging these vectors in a matrix, we obtain

$$\mathbf{Q}_{rs}(\mathbf{S}) = \nu \frac{|\mathbf{\Psi}|^{\nu}}{|\mathbf{\Psi}+\mathbf{S}^T|^{\nu}}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}(\mathbf{e}_s^T(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_r) + \nu^2 \frac{|\mathbf{\Psi}|^{\nu}}{|\mathbf{\Psi}+\mathbf{S}^T|^{\nu}}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{rs}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}. \quad (48)$$

Note that $\mathbf{Q}_{rs}(\mathbf{S})|_{\mathbf{S}=\mathbf{0}}$ is $\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1})$. From (48), the ith column of $\mathbf{Q}_{rs}(\mathbf{S})$ can be expressed as

$$\begin{aligned}\mathbf{q}_{rsi}(\mathbf{S}) =& \nu \frac{|\mathbf{\Psi}|^{\nu}}{|\mathbf{\Psi}+\mathbf{S}^T|^{\nu}}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_i\mathbf{e}_s^T(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_r \\ &+ \nu^2 \frac{|\mathbf{\Psi}|^{\nu}}{|\mathbf{\Psi}+\mathbf{S}^T|^{\nu}}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{rs}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_i \\ =& \mathbf{q}_{1,rsi}(\mathbf{S}) + \mathbf{q}_{2,rsi}(\mathbf{S}).\end{aligned}$$

Now, we compute the derivative of $\mathbf{q}_{1,rsi}(\mathbf{S})$ with respect to $[\mathbf{S}]_{jl}$, and this is given by

$$\begin{aligned}&\mathbf{g}_{1,rsilj}(\mathbf{S}) = \frac{\partial \mathbf{q}_{1,rsi}(\mathbf{S})}{\partial [\mathbf{S}]_{jl}} = \\ &-\nu^2 d_s \mathbf{e}_j^T(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_l(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{is}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_r \\ &-\nu d_s(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{lj}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{is}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_r \\ &-\nu d_s(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{is}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{lj}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_r, \\ &\text{where } d_s = \frac{|\mathbf{\Psi}|^{\nu}}{|\mathbf{\Psi}+\mathbf{S}^T|^{\nu}}.\end{aligned}$$

Note that

$$\begin{aligned}&\mathbf{g}_{1,rsilj}(\mathbf{0}) = -\nu^2(\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_l)\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r \\ &-\nu\mathbf{\Psi}^{-1}\mathbf{E}_{lj}\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r - \nu\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{E}_{lj}\mathbf{\Psi}^{-1}\mathbf{e}_r.\end{aligned}$$

Arranging the vectors $\{\mathbf{g}_{1,rsilj}(\mathbf{0})\}_{l=1}^{N_r}$ into a matrix yields

$$\begin{aligned}\mathbf{G}_{1,rsij}(\mathbf{0}) =& -\nu^2\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &-\nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r \\ &-\nu\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_r.\end{aligned} \quad (49)$$

Next, we differentiate $\mathbf{q}_{2,rsi}(\mathbf{S})$ with respect to $[\mathbf{S}]_{jl}$:

$$\begin{aligned}&\mathbf{g}_{2,rsilj}(\mathbf{S}) = \frac{\partial \mathbf{q}_{2,rsi}(\mathbf{S})}{\partial [\mathbf{S}]_{jl}} = \\ &-\nu^3 d_s \mathbf{e}_j^T(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_l(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{rs}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_i \\ &-\nu^2 d_s(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{lj}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{rs}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_i \\ &-\nu^2 d_s(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{rs}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{E}_{lj}(\mathbf{\Psi}+\mathbf{S}^T)^{-1}\mathbf{e}_i.\end{aligned}$$

Arranging $\{\mathbf{g}_{2,rsilj}(\mathbf{0})\}_{l=1}^{N_r}$ into a matrix, we obtain

$$\begin{aligned}\mathbf{G}_{2,rsij}(\mathbf{0}) =& -\nu^3\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &-\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_i \\ &-\nu^2\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_i.\end{aligned} \quad (50)$$

We can compute $\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1})$ using (49) and (50) as follows:

$$\begin{aligned}&\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{E}_{rs}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}) = -(\mathbf{G}_{1,rsi}(\mathbf{0}) + \mathbf{G}_{2,rsi}(\mathbf{0})) \\ &= \nu^2\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_j^T\mathbf{\Psi}^{-1} + \nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r \\ &+ \nu\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_j^T(\mathbf{\Psi})^{-1}\mathbf{e}_r + \nu^3\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &+ \nu^2\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_i + \nu^2\mathbf{\Psi}^{-1}\mathbf{E}_{rs}\mathbf{\Psi}^{-1}\mathbf{e}_j^T(\mathbf{\Psi})^{-1}\mathbf{e}_i.\end{aligned}$$

Using (47), we can compute

$$\begin{aligned}&\mathbb{E}(\mathbf{R}_k^{-1}\mathbf{\Psi}\mathbf{R}_k^{-1}\mathbf{E}_{ij}\mathbf{R}_k^{-1}) \\ &= \sum_{r,s=1}^{N_r}\{\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &\quad + \nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s \\ &\quad + \nu\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s \\ &\quad + \nu^3\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &\quad + \nu^2\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_i \\ &\quad + \nu^2\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_i\}, \\ &= (N_r\nu^2 + \nu + \nu^3)\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &\quad + (\nu N_r + 2\nu^2)\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_i.\end{aligned} \quad (51)$$

The simplifications performed in each term of (51) to obtain the final expression are described below.

$$\begin{aligned}&\sum_{r,s=1}^{N_r}\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &= \sum_{s=1}^{N_r}\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_s^T\mathbf{\Psi}^{-1}(\sum_{r=1}^{N_r}\mathbf{e}_r\mathbf{e}_r^T)\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &= \sum_{s=1}^{N_r}\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_s^T\mathbf{e}_s\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &= N_r\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1}\end{aligned}$$

$$\begin{aligned}&\sum_{r,s=1}^{N_r}\nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s \\ &= \sum_{s=1}^{N_r}\nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{E}_{is}\mathbf{\Psi}^{-1}(\sum_{r=1}^{N_r}\mathbf{e}_r\mathbf{e}_r^T)\mathbf{\Psi}\mathbf{e}_s \\ &= \sum_{s=1}^{N_r}\nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_s^T\mathbf{e}_s \\ &= N_r\nu\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_i\end{aligned}$$

$$\begin{aligned}&\sum_{r,s=1}^{N_r}\nu^3\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &= \nu^3\mathbf{\Psi}^{-1}\sum_{r=1}^{N_r}(\mathbf{e}_r\mathbf{e}_r^T)\mathbf{\Psi}(\sum_{s=1}^{N_r}\mathbf{e}_s\mathbf{e}_s^T)\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1} \\ &= \nu^3\mathbf{\Psi}^{-1}\mathbf{e}_i\mathbf{e}_j^T\mathbf{\Psi}^{-1}\end{aligned}$$

$$\begin{aligned}&\sum_{r,s=1}^{N_r}\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}\mathbf{e}_r\mathbf{e}_r^T\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_i \\ &= \sum_{s=1}^{N_r}\nu^2\mathbf{\Psi}^{-1}\mathbf{e}_j^T\mathbf{\Psi}^{-1}(\sum_{r=1}^{N_r}\mathbf{e}_r\mathbf{e}_r^T)\mathbf{\Psi}\mathbf{e}_s\mathbf{e}_s^T\mathbf{\Psi}^{-1}\mathbf{e}_i\end{aligned}$$

$$= \nu^2 \mathbf{\Psi}^{-1} \mathbf{e}_j^T (\sum_{s=1}^{N_r} \mathbf{e}_s \mathbf{e}_s^T) \mathbf{\Psi}^{-1} \mathbf{e}_i$$
$$= \nu^2 \mathbf{\Psi}^{-1} \mathbf{e}_j^T \mathbf{\Psi}^{-1} \mathbf{e}_i$$

$$\sum_{r,s=1}^{N_r} \nu^2 \mathbf{\Psi}^{-1} \mathbf{e}_r \mathbf{e}_r^T \mathbf{\Psi} \mathbf{e}_s \mathbf{e}_s^T \mathbf{\Psi}^{-1} \mathbf{e}_j^T \mathbf{\Psi}^{-1} \mathbf{e}_i$$
$$= \nu^2 \mathbf{\Psi}^{-1} (\sum_{r=1}^{N_r} \mathbf{e}_r \mathbf{e}_r^T) \mathbf{\Psi} (\sum_{s=1}^{N_r} \mathbf{e}_s \mathbf{e}_s^T) \mathbf{\Psi}^{-1} \mathbf{e}_j^T \mathbf{\Psi}^{-1} \mathbf{e}_i$$
$$= \nu^2 \mathbf{\Psi}^{-1} \mathbf{e}_j^T \mathbf{\Psi}^{-1} \mathbf{e}_i$$

In this derivation, we use the following facts:

$$\sum_{l=1}^{N_r} \mathbf{e}_l \mathbf{e}_l^T = \mathbb{I},$$

$$\mathbf{e}_l^T \mathbf{e}_j = \begin{cases} 1, & \text{if } l = j, \\ 0, & \text{otherwise.} \end{cases}$$

4) ***Expectation of*** $\mathbf{R}_k^{-1} \mathbf{E}_{ij} \mathbf{R}_k^{-1} \mathbf{\Psi} \mathbf{R}_k^{-1}$ *:* The computation of $\mathbb{E}(\mathbf{R}_k^{-1} \mathbf{E}_{ij} \mathbf{R}_k^{-1} \mathbf{\Psi} \mathbf{R}_k^{-1})$ is similar to (51). We get

$$\mathbb{E}(\mathbf{R}_k^{-1} \mathbf{E}_{ij} \mathbf{R}_k^{-1} \mathbf{\Psi} \mathbf{R}_k^{-1}) =$$
$$(N_r \nu^2 + \nu + \nu^3) \mathbf{\Psi}^{-1} \mathbf{E}_{ij} \mathbf{\Psi}^{-1} + (\nu N_r + 2\nu^2) \mathbf{\Psi}^{-1} \mathbf{e}_j^T \mathbf{\Psi}^{-1} \mathbf{e}_i,$$

which is the same as (51).

II. Concavity of $(\mathbf{Z}^{-1} + 2\sigma_w^2 \mathbf{Z}^{-1/2} + \sigma_w^4 \mathbf{I})^{-1}$

In this subsection, we show that the function

$$\mathbf{G}(\mathbf{Z}) = \left(\mathbf{Z}^{-1} + 2\sigma_w^2 \mathbf{Z}^{-1/2} + \sigma_w^4 \mathbf{I} \right)^{-1}$$

is concave with respect to the matrix inequality. In order to prove concavity, we consider the scalar function $f(\mathbf{Z}) = \mathbf{w}^H \mathbf{G}(\mathbf{Z}) \mathbf{w}$, for an arbitrary nonzero vector $\mathbf{w}$, and show that $f(\mathbf{Z})$ is concave. First, we consider the eigenvalue decomposition of the matrix $\mathbf{Z} \in \mathbb{C}^{P \times P}$ as $\mathbf{Z} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$, where $\mathbf{U}$ is unitary and $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_P)$ contains the eigenvalues of $\mathbf{Z}$. Then, $f(\mathbf{Z})$ is given by

$$f(\mathbf{Z}) = \mathbf{w}^H \mathbf{U} \left(\mathbf{\Lambda}^{-1} + 2\sigma_w^2 \mathbf{\Lambda}^{-1/2} + \sigma_w^4 \mathbf{I} \right)^{-1} \mathbf{U}^H \mathbf{w}.$$

Now, $f(\mathbf{Z})$ is concave with respect to the matrix inequality if and only if the function $g(\alpha) = f(\mathbf{Z}_0 + \alpha \mathbf{Z})$ is concave in α. Let $\mathbf{v} = \mathbf{U}^H \mathbf{w}$ and let $[v]_i$ denote its ith entry. Consider the function $g_1(\alpha) = f(\alpha \mathbf{Z})$, which can be simplified to

$$g_1(\alpha) = \sum_{i=1}^{P} \frac{\frac{|[v]_i|^2}{\sigma_w^4} \alpha}{\frac{\sigma_w^4}{\lambda_i} + \frac{2}{\sigma_w^2} \sqrt{\frac{\alpha}{\lambda_i}} + \alpha} \tag{52}$$

It is easy to see that the function $\tilde{g}_1(\alpha) = \frac{\alpha}{\alpha + \sqrt{\alpha} c_1 + c_2}$ with $c_1 > 0$ and $c_2 > 0$ is concave for $\alpha > 0$. Since a non-negative weighted sum of concave functions remains concave, $g_1(\alpha)$ in (52) is concave for any $\mathbf{w}$. Because affine transformations preserve concavity, $f(\mathbf{Z}_0 + \alpha \mathbf{Z})$ is concave. Therefore, the function $\mathbf{G}(\mathbf{Z})$ is concave with respect to the matrix inequality.

III. Lower-bound of M-FIM

Denote the eigenvalue decomposition of $\mathbf{A}_p^{t_n} \mathbf{R} {\mathbf{A}_p^{t_n}}^H$ by $\mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$, where $\mathbf{U}$ is a unitary eigenvector matrix, $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_P)$ contains the eigenvalues of $\mathbf{A}_p^{t_n} \mathbf{R} {\mathbf{A}_p^{t_n}}^H$, and P is the matrix dimension. Then, we have $\mathbf{G}(\mathbf{R}) \triangleq ({\mathbf{A}_p^{t_n}}^* \mathbf{R}^* {\mathbf{A}_p^{t_n}}^T + \sigma_w^2 \mathbb{I})^{-1} \otimes (\mathbf{A}_p^{t_n} \mathbf{R} {\mathbf{A}_p^{t_n}}^H + \sigma_w^2 \mathbb{I})^{-1} =$

$$(\mathbf{U}^* \otimes \mathbf{U}) \Big(\big(\mathbf{\Lambda} + \sigma_w^2 \mathbb{I} \big)^{-1} \otimes \big(\mathbf{\Lambda} + \sigma_w^2 \mathbb{I} \big)^{-1} \Big) (\mathbf{U}^* \otimes \mathbf{U})^H. \tag{53}$$

Consider the scalar function $f(\mathbf{R}) = \mathbf{w}^H \mathbf{G}(\mathbf{R}) \mathbf{w}$. Using (53), $g_1(\alpha) = f(\alpha \mathbf{R})$ can be represented as

$$g_1(\alpha) = \mathbf{v}^H ((\alpha \mathbf{\Lambda} + \sigma_w^2 \mathbb{I}))^{-1} \otimes (\alpha \mathbf{\Lambda} + \sigma_w^2 \mathbb{I})^{-1}) \mathbf{v},$$

where the vector $\mathbf{v} = (\mathbf{U}^* \otimes \mathbf{U})^H \mathbf{w}$. Since the second derivative of $g_1(\alpha)$ is non-negative, the function $g_1(\alpha)$ is convex. As affine transformations preserve convexity, $f(\mathbf{R}_0 + \alpha \mathbf{R}_1)$, and hence $f(\mathbf{R})$, are convex. Consequently, $\mathbf{G}(\mathbf{R})$ is convex with respect to the matrix inequality. By Jensen's inequality, we have

$$\mathbb{E}(({\mathbf{A}_p^{t_n}}^* \mathbf{R}^* {\mathbf{A}_p^{t_n}}^T + \sigma_w^2 \mathbb{I})^{-1} \otimes (\mathbf{A}_p^{t_n} \mathbf{R} {\mathbf{A}_p^{t_n}}^H + \sigma_w^2 \mathbb{I})^{-1}) \succeq$$
$$({\mathbf{A}_p^{t_n}}^* \mathbb{E}(\mathbf{R})^* {\mathbf{A}_p^{t_n}}^T + \sigma_w^2 \mathbb{I})^{-1} \otimes (\mathbf{A}_p^{t_n} \mathbb{E}(\mathbf{R}) {\mathbf{A}_p^{t_n}}^H + \sigma_w^2 \mathbb{I})^{-1}.$$

Using the fact that $\mathbb{E}(\mathbf{R}) = \frac{\mathbf{\Psi}_c}{\tilde{\nu}}$ and $\tilde{\nu} = \nu - N_r$, the lower bound (RHS) can be expressed as

$$(\frac{1}{\tilde{\nu}} {\mathbf{A}_p^{t_n}}^* \mathbf{\Psi}_c^* {\mathbf{A}_p^{t_n}}^T + \sigma_w^2 \mathbb{I})^{-1} \otimes (\frac{1}{\tilde{\nu}} \mathbf{A}_p^{t_n} \mathbf{\Psi}_c {\mathbf{A}_p^{t_n}}^H + \sigma_w^2 \mathbb{I})^{-1}.$$

IV. Proof of (10)

Using the conditional independence between the random variables in the graphical model in Fig. 1, the joint pdf can be expressed as

$$p(\mathbf{Z}_d^{t_N}, \mathbf{Z}_p^{t_N}, \mathbf{X}_d^{t_N}, \mathbf{H}^{t_N}, \mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N})$$
$$= p(\mathbf{Y}_p^{t_N}, \mathbf{Y}_d^{t_N}, \mathbf{Z}_p^{t_N}, \mathbf{Z}_d^{t_N}, \mathbf{X}_d^{t_N} |$$
$$\mathbf{H}^{t_N}, \mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N}, \sigma_w^2)$$
$$\times p(\mathbf{H}^{t_N}, \mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N}, \sigma_w^2)$$

$$= p(\mathbf{Y}_p^{t_N} | \mathbf{Z}_p^{t_N}) p(\mathbf{Y}_d^{t_N} | \mathbf{Z}_d^{t_N}) p(\mathbf{Z}_p^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_p^{t_N}, \sigma_w^2)$$
$$\times p(\mathbf{Z}_d^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_d^{t_N}, \sigma_w^2) p(\mathbf{X}_d^{t_N})$$
$$\times p(\mathbf{H}^{t_N}, \mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N-1}, \sigma_w^2)$$

$$= p(\mathbf{Y}_p^{t_N} | \mathbf{Z}_p^{t_N}) p(\mathbf{Y}_d^{t_N} | \mathbf{Z}_d^{t_N}) p(\mathbf{Z}_p^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_p^{t_N}, \sigma_w^2)$$
$$\times p(\mathbf{Z}_d^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_d^{t_N}, \sigma_w^2) p(\mathbf{X}_d^{t_N})$$
$$\times p(\mathbf{H}^{t_N} | \mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N-1}, \sigma_w^2)$$
$$\times p(\mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N-1}, \sigma_w^2)$$

$$= p(\mathbf{Y}_p^{t_N} | \mathbf{Z}_p^{t_N}) p(\mathbf{Y}_d^{t_N} | \mathbf{Z}_d^{t_N}) p(\mathbf{Z}_p^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_p^{t_N}, \sigma_w^2)$$
$$\times p(\mathbf{Z}_d^{t_N} | \mathbf{H}^{t_N}; \mathbf{X}_d^{t_N}, \sigma_w^2) p(\mathbf{X}_d^{t_N})$$
$$\times p(\mathbf{H}^{t_N} | \mathbf{R})$$
$$\times p(\mathbf{R}, \{\mathbf{Y}_p^{t_n}, \mathbf{Y}_d^{t_n}\}_{n=1}^{N-1}; \{\mathbf{X}_p^{t_n}\}_{n=1}^{N-1}, \sigma_w^2).$$