



EFIE-based MoM (Part IV)

- Dielectric Formulations

E8-202 Class 10

Dipanjan Gope



Module 2: Method of Moments

- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques



References

- D. H. Schaubert, D. R. Wilton, and A. W. Glisson, “A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies,” *IEEE Trans. Antennas Propag.*, vol. AP-32, pp. 77–85, Jan. 1984.



Data Structures

- N2XYZ
- P2N
- E2P
- E2N
- P2E



Dielectric Handling

- Multilayered Green's Functions (Not Full 3D)
- PMCHW
- Approximate quasi-static formulation
- Volumetric formulation (3D Maxwell Accurate)



Approximate Quasi-Static Formulation

$$\begin{aligned}\bar{\mathbf{Z}}_{ij} &= \frac{j\omega\mu}{4\pi} \int_{T_i^{+,-}} \int_{T_j^{+,-}} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|} \mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \cdot \mathbf{f}_i(\mathbf{r}) ds \\ &+ \frac{1}{4j\omega\pi\epsilon} \int_{T_i^{+,-}} \int_{T_j^{+,-}} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|} \nabla' \cdot \mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \nabla \cdot \mathbf{f}_i(\mathbf{r}) ds\end{aligned}$$

- Is vector potential affected by permittivity?
- Is scalar potential affected by permittivity?



Approximate Quasi-Static Formulation

Boundary Condition

$$n \cdot D_2 - n \cdot D_1 = \rho_s = 0$$

Equation

$$E = -\nabla \phi$$

$$E^{+,-} = \frac{1}{4\pi\epsilon_0} \int_{s'} \sigma_T(r') \frac{r - r'}{|r - r'|^3} ds' \pm n \frac{\sigma_T(r)}{2\epsilon_0}$$



Approximate Quasi-Static Formulation

- Current and related charge unknowns (RWG) on conductors
- Charge unknowns (Piecewise constant) on dielectrics

$$\begin{pmatrix} Z_{cc} & Z_{cd} \\ Z_{dc} & Z_{dd} \end{pmatrix} \begin{bmatrix} J_c \\ \sigma_d \end{bmatrix} = \begin{bmatrix} \langle E, f \rangle \\ 0 \end{bmatrix}$$



Volumetric Formulation (Preferred)

- Equation:

$$J = j\omega(\epsilon_r - \epsilon_0)E = j\omega\gamma D$$

$$E_s + E_i = E_T = \frac{J}{j\omega(\epsilon_r - \epsilon_0)}$$

$$E_i = \frac{J}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$

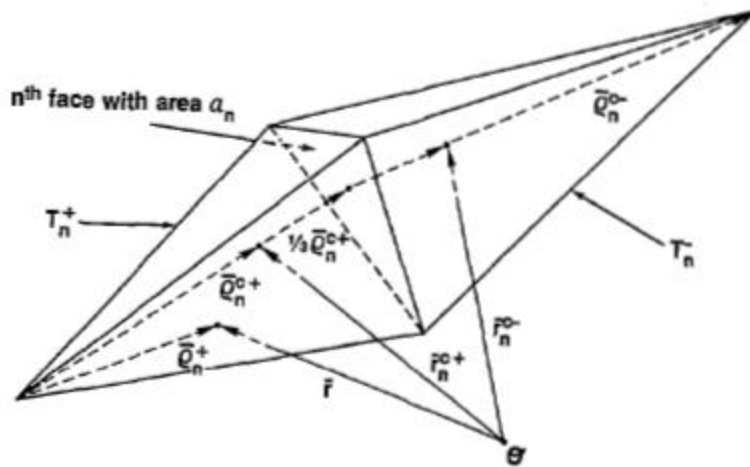
$$E_i = \frac{j\omega\gamma D}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$



Volumetric Formulation

- Basis Function:

$$f_s \rightarrow j\omega D$$



$$f_n(\mathbf{r}) = \begin{cases} \frac{a_n}{3V_n^+} \rho_n^+, & \mathbf{r} \in T_n^+ \\ \frac{a_n}{3V_n^-} \rho_n^-, & \mathbf{r} \in T_n^- \\ 0, & \text{otherwise} \end{cases}$$

$$\nabla \cdot \mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{a_n}{V_n^+}, & \mathbf{r} \in T_n^+ \\ -\frac{a_n}{V_n^-}, & \mathbf{r} \in T_n^- \\ 0, & \text{otherwise} \end{cases}$$

D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," IEEE Trans. Antennas Propag., vol. AP-32, pp. 77–85, Jan. 1984.

