



# EFIE-based MoM (Part V)

- Volumetric Dielectric Formulation

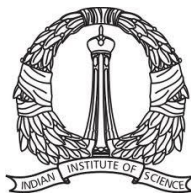
E8-202 Class 11  
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# Module 2: Method of Moments

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- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques



# References

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- D. H. Schaubert, D. R. Wilton, and A. W. Glisson, “A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies,” *IEEE Trans. Antennas Propag.*, vol. AP-32, pp. 77–85, Jan. 1984.



# Volumetric Formulation (Preferred)

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- Equation:

$$J = j\omega(\epsilon_r - \epsilon_0)E = j\omega\gamma D$$

$$E_s + E_i = E_T = \frac{J}{j\omega(\epsilon_r - \epsilon_0)}$$

$$E_i = \frac{J}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$

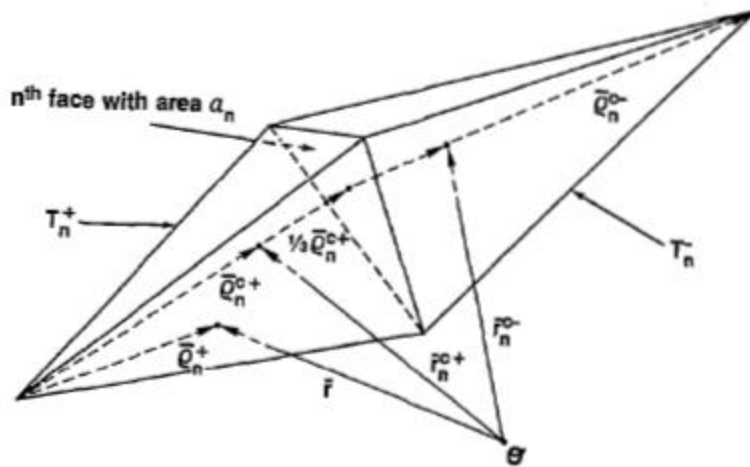
$$E_i = \frac{j\omega\gamma D}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$



# Volumetric Formulation

- Basis Function:

$$f_s \rightarrow j\omega D$$



$$f_n(\mathbf{r}) = \begin{cases} \frac{a_n}{3V_n^+} \rho_n^+, & \mathbf{r} \in T_n^+ \\ \frac{a_n}{3V_n^-} \rho_n^-, & \mathbf{r} \in T_n^- \\ 0, & \text{otherwise} \end{cases}$$

$$\nabla \cdot \mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{a_n}{V_n^+}, & \mathbf{r} \in T_n^+ \\ -\frac{a_n}{V_n^-}, & \mathbf{r} \in T_n^- \\ 0, & \text{otherwise} \end{cases}$$

D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," IEEE Trans. Antennas Propag., vol. AP-32, pp. 77–85, Jan. 1984.



# Term 1

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$$E_i = \frac{j\omega\gamma D}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$

$$\left\langle \frac{j\omega\gamma D}{j\omega(\epsilon_r - \epsilon_0)}, f_t \right\rangle$$

$$\left\langle \frac{f_s}{j\omega\epsilon_r}, f_t \right\rangle$$

$$\frac{1}{j\omega\epsilon_r} \int_v f_s \cdot f_t dv$$



# Term 2

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$$\langle j\omega A, f_t \rangle$$

$$j\omega \int_t f_t \cdot \int_s G J ds$$

$$j\omega \int_t f_t \cdot \int_s G j\omega \gamma D ds$$

$$\frac{j\omega\gamma\mu}{4\pi} \int_t f_t \cdot \int_s \frac{e^{-jk|r-r'|}}{|r-r'|} f_s ds$$



# Term 3

$$\langle \nabla \phi, f_t \rangle$$

$$\int_t \overset{3b}{\phi f_m \cdot \hat{n}} dt - \int_t \overset{3a}{\phi \nabla \cdot f_t} dt = \int_t f \cdot \nabla \phi dt$$

$$\rho = \frac{\gamma \nabla \cdot f_s}{-j\omega} + \frac{f_s \cdot \nabla \gamma}{-j\omega}$$

$$T_{3a} = \frac{A_t A_s}{V_t V_s} \frac{\gamma}{j\omega 4\pi \epsilon_0} \iint_t \iint_s \frac{e^{-jk|r-r'|}}{|r-r'|} ds dt + \frac{A_t}{V_t} \frac{1}{j\omega 4\pi \epsilon_0} (\gamma^+ - \gamma^-) \iint_t \iint_s \frac{e^{-jk|r-r'|}}{|r-r'|} ds dt$$

Surface

++ term only mentioned

$$T_{3b} = \frac{A_s}{V_s} \frac{\gamma}{j\omega 4\pi \epsilon_0} \iint_t \iint_s \frac{e^{-jk|r-r'|}}{|r-r'|} ds dt + \frac{1}{j\omega 4\pi \epsilon_0} (\gamma^+ - \gamma^-) \iint_t \iint_s \frac{e^{-jk|r-r'|}}{|r-r'|} ds dt$$

Surface

