PMCHWT: Poggio, Miller, Chew, Harrington, Wu, Tsai

E8-202 Class 15

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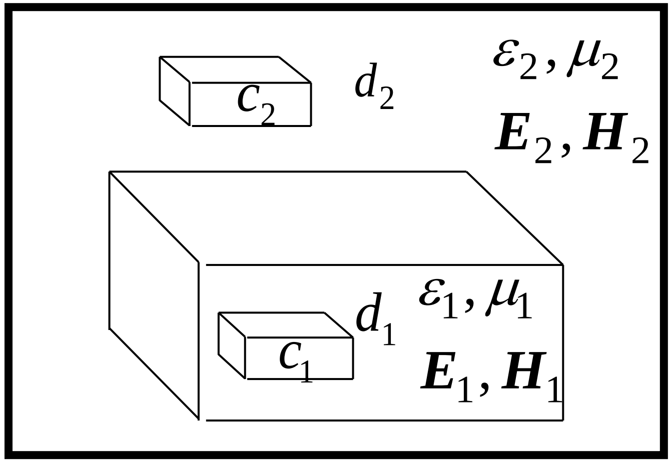
Module 2: Method of Moments

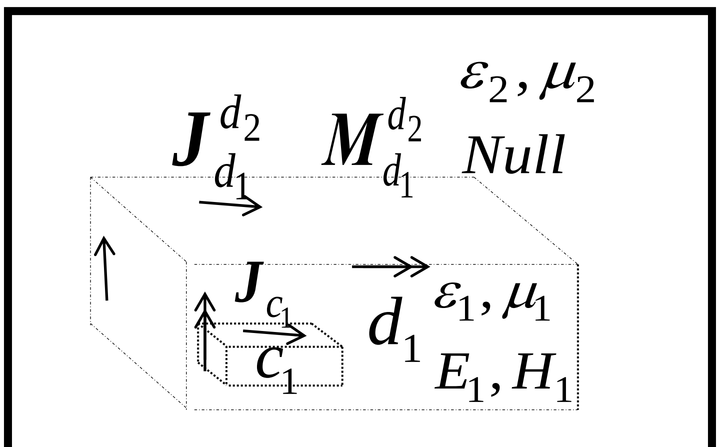
- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques

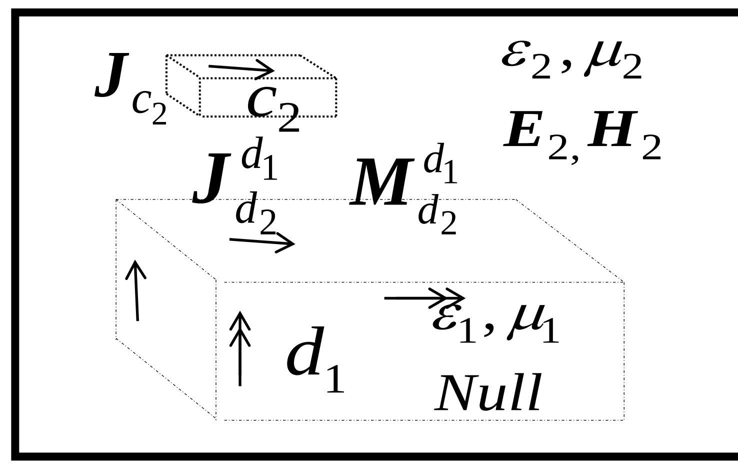
References

- A. Poggio and E. Miller, “Integral equation solutions for three dimensional scattering problems”, Chapter 4 in *Computer Techniques for Electromagnetics*, edited by R. Mittra, Pergamon Press, NY 1973.
- B. M.Kolundzija, “Electromagnetic modeling of composite metallic and dielectric structures,” *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1021–1032, July 1999.

Conductor-Dielectric Problem







$$\boldsymbol{J}_{d_1}^{d_2} = -\boldsymbol{J}_{d_2}^{d_1}$$

$$\boldsymbol{M}_{d_1}^{d_2} = -\boldsymbol{M}_{d_2}^{d_1}$$

Boundary Condition on Dielectric Interface

$$\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2 + \hat{n} \times \boldsymbol{E}_2^{inc}$$

$$\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2$$

Equation on Dielectric Interface

$$\boldsymbol{E}_{1,2}(r_g) = -j\omega \boldsymbol{A}_{1,2}(r_g) - \nabla \phi^{e}_{1,2}(r_g) - \frac{1}{\varepsilon_{1,2}} \nabla \times \boldsymbol{F}_{1,2}(r_g)$$

$$\boldsymbol{H}_{1,2}(\boldsymbol{r}_g) = -j\omega \boldsymbol{F}_{1,2}(\boldsymbol{r}_g) - \nabla \phi^{m}_{1,2}(\boldsymbol{r}_g) + \frac{1}{\mu_{1,2}} \nabla \times \boldsymbol{A}_{1,2}(\boldsymbol{r}_g)$$

$$\boldsymbol{A}_{1,2}(\boldsymbol{r}_g) = \mu_{1,2} \int_S G_{1,2}(\boldsymbol{r}_g, \boldsymbol{r}_{g'})[+,-]\boldsymbol{J}_{d_1}^{d_2}(\boldsymbol{r}_{g'})d\boldsymbol{r}_{g'}$$

$$\boldsymbol{F}_{1,2}(\boldsymbol{r}_g) = \varepsilon_{1,2} \int_S G_{1,2}(\boldsymbol{r}_g, \boldsymbol{r}_{g'})[+,-]\boldsymbol{M}_{d_1}^{d_2}(\boldsymbol{r}_{g'})d\boldsymbol{r}_{g'}$$

$$\phi^{e}_{1,2}(\boldsymbol{r}_g) = \frac{1}{\varepsilon_{1,2}} \int_S G_{1,2}(\boldsymbol{r}_g, \boldsymbol{r}_{g'})[+,-]q^{e}_{1} d\boldsymbol{r}_{g'}$$

$$\phi^{m}_{1,2}(\boldsymbol{r}_g) = \frac{1}{\mu_{1,2}} \int_S G_{1,2}(\boldsymbol{r}_g, \boldsymbol{r}_{g'})[+,-]q^{m}_{1} d\boldsymbol{r}_{g'}$$

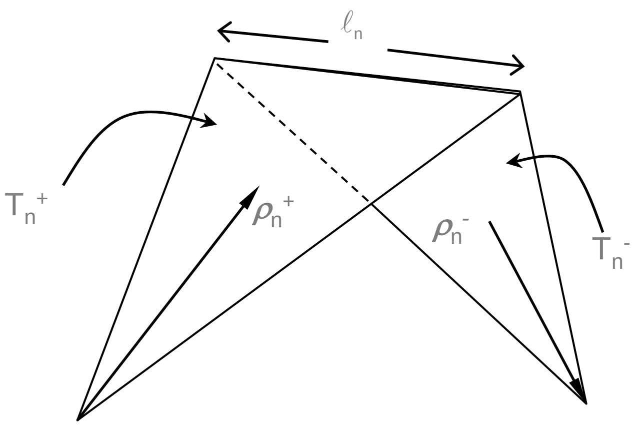
PMCHW Formulation

$$\hat{n}\times \boldsymbol{E}_2^{inc}(r_g) = \hat{n}\times\left(\begin{array}{l} j\omega\left[\boldsymbol{A}_1(r_g)-\boldsymbol{A}_2(r_g)\right]+\nabla\begin{bmatrix}\phi_1^e(r_g)\\ -\phi_2^e(r_g)\end{bmatrix}\\ +\left[\dfrac{1}{\varepsilon_1}\nabla\times\boldsymbol{F}_1(r_g)-\dfrac{1}{\varepsilon_2}\nabla\times\boldsymbol{F}_2(r_g)\right]\end{array}\right)$$

$$0 = \hat{n}\times\left(\begin{array}{l} j\omega\left[\boldsymbol{F}_1(r_g)-\boldsymbol{F}_2(r_g)\right]+\nabla\left[\phi_1^m(r_g)-\phi_2^m(r_g)\right]\\ -\left[\dfrac{1}{\mu_1}\nabla\times\boldsymbol{A}_1(r_g)-\dfrac{1}{\mu_2}\nabla\times\boldsymbol{A}_2(r_g)\right]\end{array}\right)$$

Rao-Wilton-Glisson Basis Function

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \dfrac{l_n}{2A_{n+}}\boldsymbol{\rho}_{n+} & \mathbf{r} \in T_{n+} \\ \dfrac{l_n}{2A_{n-}}\boldsymbol{\rho}_{n-} & \mathbf{r} \in T_{n-} \end{cases}$$



S. M. Rao, D. R. Wilton and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape", *IEEE Trans. Antennas Propagation*, vol. AP-30, pp. 409-418, May 1982.

MoM Matrix Entry: E Field

$$E : \quad T_1 + T_2 + T_3$$

$$T_1 = \frac{j\omega\mu_1}{4\pi}\int f_t \cdot \int G_1 f_s \, ds \, dt - \frac{j\omega\mu_2}{4\pi}\int f_t \cdot \int G_2 f_s \, ds \, dt$$

(J)

$$T_2 = \frac{1}{j\omega 4\pi\epsilon_1}\int \int G_1 \, ds \, dt - \frac{1}{j\omega 4\pi\epsilon_2}\int \int G_2 \, ds \, dt$$

(J)

$$T_3 = \int f_t \cdot \int \nabla \times G_1 f_s \, ds \, dt - \int f_t \cdot \int \nabla \times G_2 f_s \, ds \, dt$$

(M)

MoM Matrix Entry: H Field

$$H : \quad S_1 + S_2 + S_3$$

$$S_1 = \frac{j\omega\epsilon_1}{4\pi}\int f_t \cdot \int G_1 \underbrace{(f_s)}_{\to M} d_s\, dt - \frac{j\omega\epsilon_2}{4\pi}\int f_t \cdot \int G \underbrace{(f_s)}_{\to M} d_s\, dt$$

$$S_2 = \frac{1}{j\omega 4\pi \mu_1}\int\int G_1 \underbrace{(d_s)}_{\to M} dt - \frac{1}{j\omega 4\pi \mu_2}\int\int G_2 \underbrace{(d_s)}_{\to M} dt$$

$$S_3 = \int f_t \cdot \int \nabla \times G_1 \underbrace{(f_s)}_{\to J} d_s\, dt - \int f_t \cdot \int \nabla \times G_2 \underbrace{(f_s)}_{\to J} d_s\, dt$$

MoM Matrix

$$\begin{bmatrix} T_{1a}+T_{2a} & 0 & T_{1a}+T_{2a} & T_{3a} \\ 0 & T_{1b}+T_{2b} & T_{1b}+T_{2b} & T_{3b} \\ T_{1a}+T_{2a} & T_{1b}+T_{2b} & T_{1a}+T_{2a}+T_{1b}+T_{2b} & T_{3a}+T_{3b} \\ S_{3a} & S_{3b} & S_{3a}+S_{3b} & S_{1a}+S_{2a}+S_{1b}+S_{2b} \end{bmatrix} \begin{bmatrix} J_{c_1} \\ J_{c_2} \\ J_d \\ M_d \end{bmatrix} = \begin{bmatrix} \langle E_i \cdot f_t \rangle \\ \langle E_i \cdot f_t \rangle \\ \langle E_i \cdot f_t \rangle \\ 0 \end{bmatrix}$$

MoM matrix for :-