



# Maxwell's equations and properties of media

E8-202 Class 1

Dipanjan Gope



# Module 1: Review of EM Principles

---

- Maxwell's equations
- Applications of Computational Electromagnetics
- Electrostatics and Magnetostatics
- Wave equation and propagation
- Scalar and vector potentials
- Surface equivalence principle
- Greens Function
- Boundary conditions
- Linear algebra for computational EM



# References

---

- C. A. Balanis, Advanced Engineering Electromagnetics, Wiley and Sons: Chapter 1



# Maxwell's Equations

---

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{H}} = \frac{\partial \vec{\mathbf{D}}}{\partial t} + \vec{\mathbf{J}}$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

Time Domain – Frequency Domain

Differential Form – Integral Form



# Maxwell's Equations

## 4 Forms

Time/Differential

Frequency/Differential

Time/Integral

Frequency/Integral

$$\frac{\partial}{\partial t} \leftrightarrow j\omega$$

Stoke's Theorem

$$\frac{\partial}{\partial t} \leftrightarrow j\omega$$

$$\oint_C \vec{A} \cdot d\vec{l} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

Gauss' Divergence Theorem

$$\oiint_S \vec{A} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{A} dv$$

Gradient Identity: not useful here

$$\oiint_S \phi \hat{n} dS = \iiint_V \nabla \phi dV$$



# Constitutive Parameters

---

$$\vec{\mathbf{D}} = \overline{\overline{\boldsymbol{\varepsilon}}}\vec{\mathbf{E}}$$

$$\varepsilon, \mu, \sigma = f(r, |E| \text{ or } |H|, \text{dir}(E \text{ or } H), \omega)$$

$$\vec{\mathbf{B}} = \overline{\overline{\boldsymbol{\mu}}}\vec{\mathbf{H}}$$

$$\vec{\mathbf{J}}_c = \overline{\overline{\boldsymbol{\sigma}}}\vec{\mathbf{E}}$$

Classification based on parameters:

- Homogenous - Inhomogenous
- Linear – Nonlinear
- Isotropic – Nonisotropic
- Nondispersive - dispersive



# Continuity Equation

---

Derive Continuity Equation from Maxwell's Equation

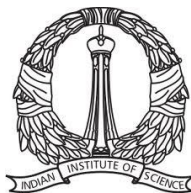
Vector Identities

$$\nabla \cdot \nabla \times V = 0$$

$$\nabla \times \nabla s = 0$$

Continuity Equation

$$\nabla \cdot \vec{\mathbf{J}} = -j\omega\rho$$



# Complex Permittivity

---

Displacement Current (real eps) + Conduction Current (imag eps)

$$\tan \delta = \frac{\epsilon_{imag}}{\epsilon_{real}}$$

Can imaginary permittivity be negative?

Derivation for Silicon





# Circuit Equivalents

---

- KVL – Maxwell's first equation
- KCL – Continuity equation
- Time-of-flight delay: Full-wave

