Maxwell's equations and properties of media

E8-202 Class 1

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Module 1: Review of EM Principles

- Maxwell's equations
- Applications of Computational Electromagnetics
- Electrostatics and Magnetostatics
- Wave equation and propagation
- Scalar and vector potentials
- Surface equivalence principle
- Greens Function
- Boundary conditions
- Linear algebra for computational EM

References

- C. A. Balanis, Advanced Engineering Electromagnetics, Wiley and Sons: Chapter 1

Maxwell's Equations

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{H}} = \frac{\partial \vec{\mathbf{D}}}{\partial t} + \vec{\mathbf{J}}$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

Time Domain – Frequency Domain

Differential Form – Integral Form

Maxwell's Equations

4 Forms

Time/Differential | Frequency/Differential | Time/Integral | Frequency/Integral

$$\frac{\partial}{\partial t} \leftrightarrow j\omega$$

Stoke's Theorem

$$\oint_C \vec{\mathbf{A}} \cdot d\vec{l} = \iint_S \left(\nabla \times \vec{\mathbf{A}}\right) \cdot d\vec{s}$$

$$\frac{\partial}{\partial t} \leftrightarrow j\omega$$

Gauss' Divergence Theorem

$$\oiint_S \vec{\mathbf{A}} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{\mathbf{A}} dv$$

Gradient Identity: not useful here

$$\oiint_S \phi \hat{n} dS = \oiiint_V \nabla \phi dV$$

Constitutive Parameters

$$\vec{\mathbf{D}} = \overline{\overline{\varepsilon}}\vec{\mathbf{E}}$$

$$\varepsilon, \mu, \sigma = f(r, |E \, or \, H|, dir(E or H), \omega)$$

$$\vec{\mathbf{B}} = \overline{\overline{\mu}}\vec{\mathbf{H}}$$

$$\vec{\mathbf{J}}_c = \overline{\overline{\sigma}}\vec{\mathbf{E}}$$

Classification based on parameters:

- Homogenous - Inhomogenous
- Linear – Nonlinear
- Isotropic – Nonisotropic
- Nondispersive - dispersive

Continuity Equation

Derive Continuity Equation from Maxwell's Equation

Vector Identities

$$\nabla \cdot \nabla \times V = 0$$

$$\nabla \times \nabla s = 0$$

Continuity Equation

$$\nabla \cdot \vec{\mathbf{J}} = -j\omega\rho$$

Complex Permittivity

Displacement Current (real eps) + Conduction Current (imag eps)

$$\tan\delta = \frac{\varepsilon_{imag}}{\varepsilon_{real}}$$

Can imaginary permittivity be negative?

Derivation for Silicon

Circuit Equivalents

- KVL – Maxwell's first equation
- KCL – Continuity equation
- Time-of-flight delay: Full-wave