Boundary conditions, statics, wave propagation

E8-202 Class 2

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Module 1: Review of EM Principles

- Maxwell's equations
- Applications of Computational Electromagnetics
- Electrostatics and Magnetostatics
- Wave equation and propagation
- Scalar and vector potentials
- Surface equivalence principle
- Greens Function
- Boundary conditions
- Linear algebra for computational EM

References

- C. A. Balanis, Advanced Engineering Electromagnetics, Wiley and Sons: Chapter 1,3,4

Boundary Conditions

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

$$\vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

What about PEC?

Electrostatics

$$\nabla \times \vec{\mathbf{E}} = 0$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

Magnetostatics

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \cdot \vec{\mathbf{J}} = 0$$

Wave Equation

$$\nabla^2 \vec{E} - \frac{\nabla \rho}{\varepsilon} + \omega^2 \mu \varepsilon \vec{E} = 0$$

$$\nabla^2 \vec{H} + \nabla \times \vec{J}_i + \omega^2 \mu \varepsilon \vec{H} = 0$$

Use Identity

$$\nabla \times \nabla \times \vec{F} = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 F$$

Equation Forms

$$\nabla^2 \phi = 0$$

Laplace

$$\nabla^2 \phi = f$$

Poisson

$$\nabla^2 \phi + k^2 \phi = 0$$

Helmholtz

$\nabla^2 \leftrightarrow \Delta$ Laplace Operator Div. of grad of

Laplace Operator

In Cartesian coordinates,

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

In cylindrical coordinates,

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}.$$

In spherical coordinates:

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}.$$

http://en.wikipedia.org/wiki/Laplace_operator

Wave Eqn Soln: 1D Plane Wave

$$\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = 0$$

Source Free Region

$$E_z = Ae^{-jkz} + Be^{+jkz}$$

γ

Propagation constant

k

Wave number

β

Phase constant

α

Attenuation constant

Some Terms for Wave Propagation

Travelling Wave

Phase Velocity

Standing Wave

Group Velocity

Decaying Wave

Wave Eqn Soln: Rectangular Waveguide

$$E_x = [C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)]$$
$$[C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)]$$
$$\left[A e^{-j\beta_z z} + B e^{+j\beta_z z}\right]$$

Wave Eqn Soln: Cylindrical Waveguide

$$E_x = [C_1 J_m(\beta_\rho \rho) + D_1 Y_m(\beta_\rho \rho)]$$
$$[C_2 \cos(m\phi) + D_2 \sin(m\phi)]$$
$$\left[A e^{-j\beta_z z} + B e^{+j\beta_z z}\right]$$

Wave Eqn Soln: Spherical Waveguide

$$E_x = [C_1 j_m(\beta r) + D_1 y_m(\beta r)]$$
$$[C_2 \cos(m\phi) + D_2 \sin(m\phi)]$$
$$\left[A P_n^m \cos(\theta) + B Q_n^m \cos(\theta)\right]$$