

# Boundary conditions, statics, wave propagation

E8-202 Class 2

Dipanjan Gope



# Module 1: Review of EM Principles

---

- Maxwell's equations
- Applications of Computational Electromagnetics
- Electrostatics and Magnetostatics
- Wave equation and propagation
- Scalar and vector potentials
- Surface equivalence principle
- Greens Function
- Boundary conditions
- Linear algebra for computational EM



# References

---

- C. A. Balanis, Advanced Engineering Electromagnetics, Wiley and Sons: Chapter 1,3,4



# Boundary Conditions

---

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

$$\vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

What about PEC?

# Electrostatics

---

$$\nabla \times \vec{\mathbf{E}} = 0$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$



# Magnetostatics

---

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \cdot \vec{\mathbf{J}} = 0$$



# Wave Equation

---

$$\nabla^2 \vec{E} - \frac{\nabla \rho}{\epsilon} + \omega^2 \mu \epsilon \vec{E} = 0$$

$$\nabla^2 \vec{H} + \nabla \times \vec{J}_i + \omega^2 \mu \epsilon \vec{H} = 0$$

Use Identity

$$\nabla \times \nabla \times \vec{F} = \nabla (\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$$



# Equation Forms

---

$$\nabla^2 \phi = 0$$

Laplace

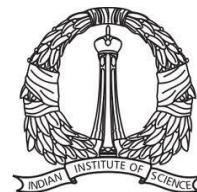
$$\nabla^2 \phi = f$$

Poisson

$$\nabla^2 \phi + k^2 \phi = 0$$

Helmholtz

$\nabla^2 \leftrightarrow \Delta$    Laplace Operator   Div. of grad of



# Laplace Operator

---

In **Cartesian coordinates**,

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

In **cylindrical coordinates**,

$$\Delta f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}.$$

In **spherical coordinates**:

$$\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}.$$

[http://en.wikipedia.org/wiki/Laplace\\_operator](http://en.wikipedia.org/wiki/Laplace_operator)



# Wave Eqn Soln: 1D Plane Wave

---

$$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} = 0$$

Source Free Region

$$E_z = A e^{-jkz} + B e^{+jkz}$$

$\gamma$

$k$

$\beta$

$\alpha$

Propagation constant

Attenuation constant

Wave number

Phase constant

# Some Terms for Wave Propagation

---

Travelling Wave

Phase Velocity

Standing Wave

Group Velocity

Decaying Wave



# Wave Eqn Soln: Rectangular Waveguide

---

$$E_x = [C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)] \\ [C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)] \\ [A e^{-j\beta_z z} + B e^{+j\beta_z z}]$$



# Wave Eqn Soln: Cylindrical Waveguide

---

$$E_x = [C_1 J_m(\beta_\rho \rho) + D_1 Y_m(\beta_\rho \rho)] \\ [C_2 \cos(m\phi) + D_2 \sin(m\phi)] \\ [A e^{-j\beta_z z} + B e^{+j\beta_z z}]$$



# Wave Eqn Soln: Spherical Waveguide

---

$$E_x = [C_1 j_m(\beta r) + D_1 y_m(\beta r)] \\ [C_2 \cos(m\phi) + D_2 \sin(m\phi)] \\ [A P_n^m \cos(\theta) + B Q_n^m \cos(\theta)]$$

