Scalar vector potentials, Green's Functions, surface equivalence principle

E8-202 Class 3

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Module 1: Review of EM Principles

- Maxwell's equations
- Applications of Computational Electromagnetics
- Electrostatics and Magnetostatics
- Wave equation and propagation
- Scalar and vector potentials
- Surface equivalence principle
- Greens Function
- Boundary conditions
- Linear algebra for computational EM

References

- Walton C. Gibson: The Method of Moments in Electromagnetics, 1st Ed., Chapman and Hall, Chapter 2

Equations Review

$$\nabla^2 \phi + k^2 \phi = 0$$

$$\nabla^2 \phi = 0$$

$$\nabla^2 \phi = f$$

Helmholtz Theorem

$$F = -\nabla\phi + \nabla \times A$$

Curl Free Part: irrotational

Divergence Free Part: Solenoidal

Scalar and Vector Potential

$$\vec{E} = -j\omega\vec{A} - \nabla\phi$$

$\vec{A}$ Magnetic Vector Potential

ϕ Electric Scalar Potential

$$\nabla \times \vec{A} = \vec{B}$$

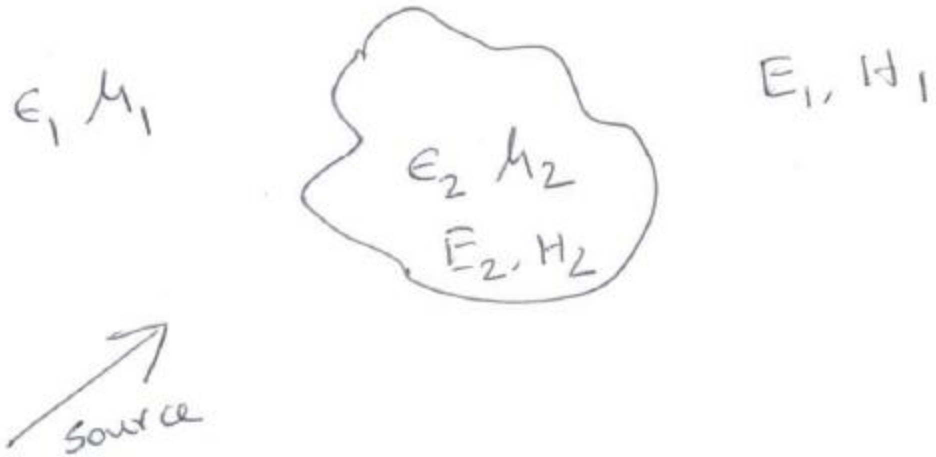
$$\nabla \cdot \vec{A} = -jw\mu\varepsilon\phi$$ Lorentz Gauge

Scalar and Vector Potential

$$\left(\nabla^2 + k^2\right)A = -\mu J$$

$$\left(\nabla^2 + k^2\right)\phi = -\frac{\rho}{\varepsilon}$$

Surface Equivalence Principle



Surface Equivalence Principle

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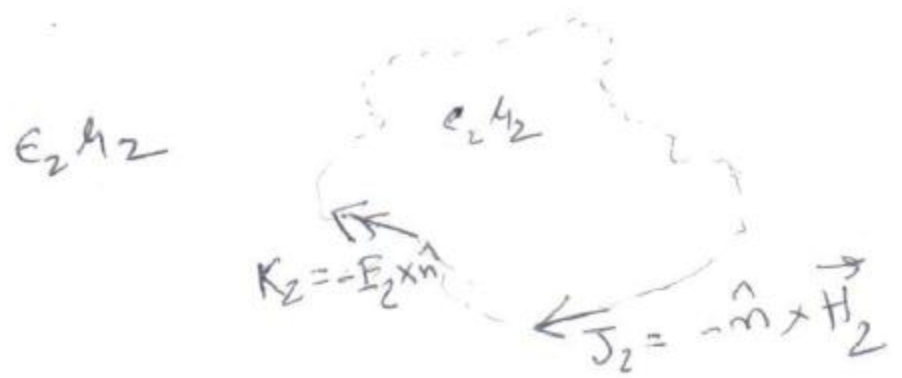
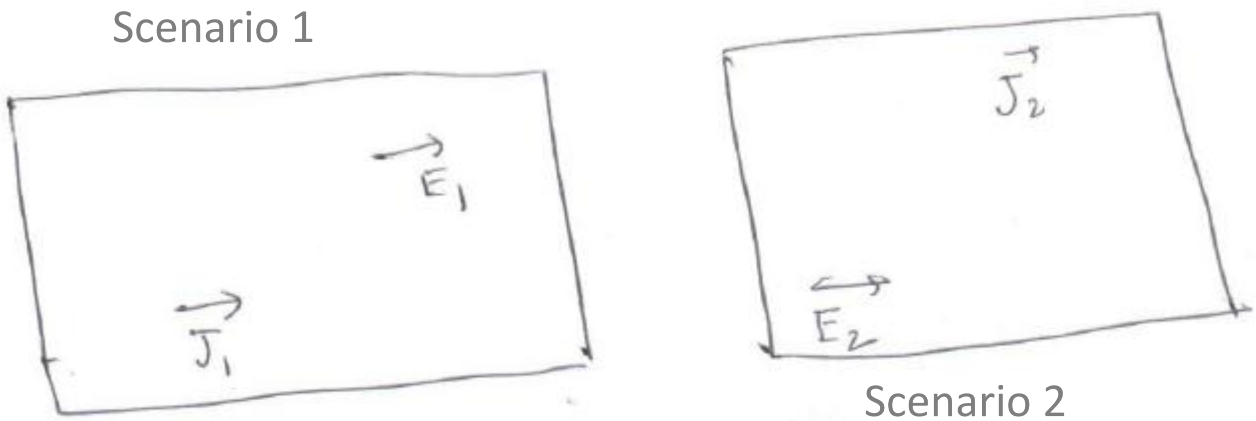


Null Fields

Reciprocity Theorem



$$\langle E_1, J_2 \rangle = \langle E_2, J_1 \rangle$$

Green's Function: Electrostatics

$$\left(\nabla^2 + k^2\right)\phi = -\frac{\rho}{\varepsilon}$$

k=0 for static

$$\nabla^2 g(r,r') = -\frac{\partial(r-r')}{\varepsilon}$$

$$g(r,r') = \frac{C_1}{|r-r'|} + C_2$$

$$\phi(r) = \int dr' \rho(r') g(r,r')$$

Find C1 using Gauss Divergence Theorem

$$\oiint_S \vec{\mathbf{A}} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{\mathbf{A}} dv$$

Green’s Function: Magnetostatics

$$\left(\nabla^2 + k^2\right)A = -\mu J$$ k=0 for static

$$\nabla^2 g(r,r') = -\mu\partial(r-r')$$

$$g(r,r') = \frac{C_1}{|r-r'|} + C_2$$

$$\vec{A}(r) = \mu\varepsilon\int dr'\bar{I}\cdot g(r,r')J(r')$$

Green's Function: Full wave

$$\left(\nabla^2 + k^2\right)\phi = -\frac{\rho}{\varepsilon}$$

$$\left[\nabla^2 + k^2\right]g(r,r') = -\frac{\partial(r-r')}{\varepsilon}$$

$$g(r,r') = C_1\frac{e^{-jk|r-r'|}}{|r-r'|} + C_2\frac{e^{+jk|r-r'|}}{|r-r'|}$$

$$g(r,r') = \frac{1}{4\pi\varepsilon}\frac{e^{-jk|r-r'|}}{|r-r'|}$$

Electric Field Green's Function

$$G_E = g(r,r')\left(\bar{I} + \frac{\nabla\nabla'}{k^2}\right)$$

$$E(r) = \int dr' G_E(r,r') \cdot \vec{J}(r')$$

Linear Algebra

- Direct Solver
- Iterative Solver

LU Decomposition

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

http://en.wikipedia.org/wiki/LU_decomposition