Electrostatic Method of Moments: Part 2

E8-202 Class 5

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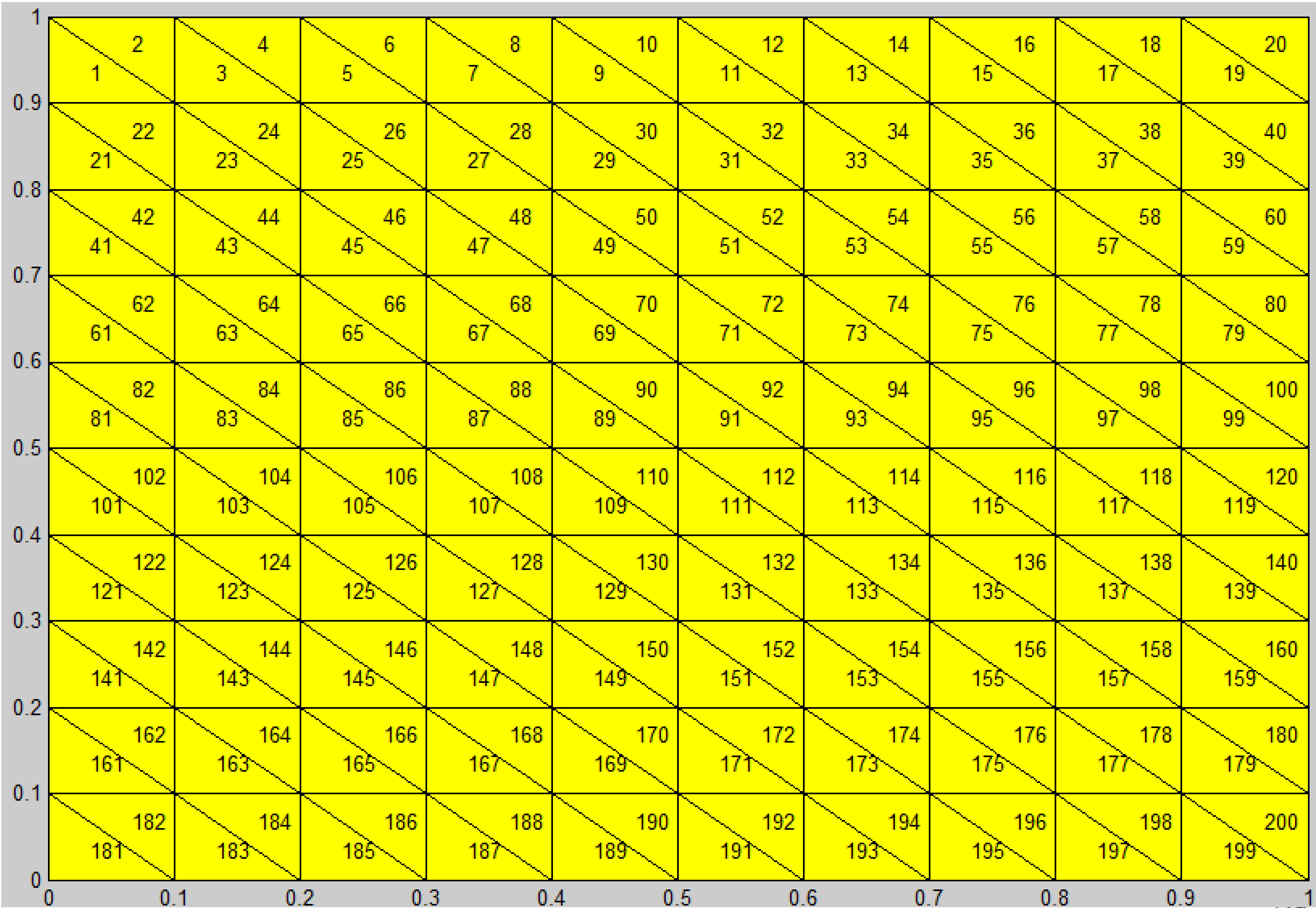
Module 2: Method of Moments

- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques

References

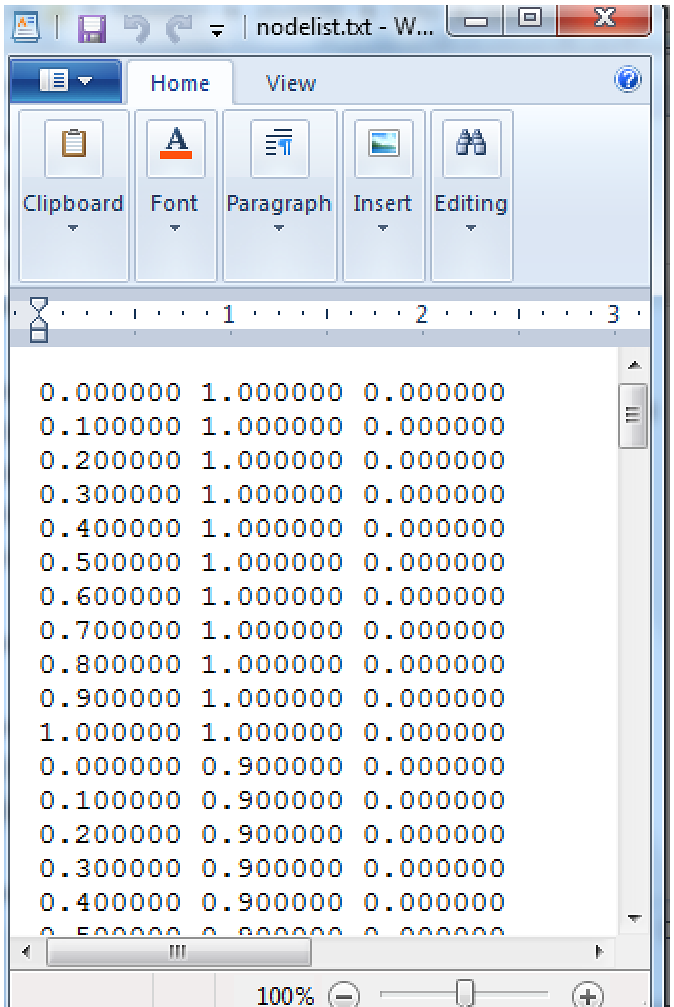
- Walton C. Gibson: The Method of Moments in Electromagnetics, 1st Ed., Chapman and Hall, Chapter 3
- Roger F. Harrington: Field Computation by Moment Methods, 1993, Wiley-IEEE Press, Chapter 2
- S. Rao, T.K. Sarkar and R.F. Harrington, "The Electrostatic Field of Conducting Bodies in Multiple Dielectric Media", *IEEE Transaction on Microwave Theory and Techniques*, vol. 32, issue 11, Nov 1984 pp. 1441-1448.

Mesh



0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140
141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160
161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180
181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200

Mesh




```
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0.100000 1.000000 0.000000
0.200000 1.000000 0.000000
0.300000 1.000000 0.000000
0.400000 1.000000 0.000000
0.500000 1.000000 0.000000
0.600000 1.000000 0.000000
0.700000 1.000000 0.000000
0.800000 1.000000 0.000000
0.900000 1.000000 0.000000
1.000000 1.000000 0.000000
0.000000 0.900000 0.000000
0.100000 0.900000 0.000000
0.200000 0.900000 0.000000
0.300000 0.900000 0.000000
0.400000 0.900000 0.000000
```

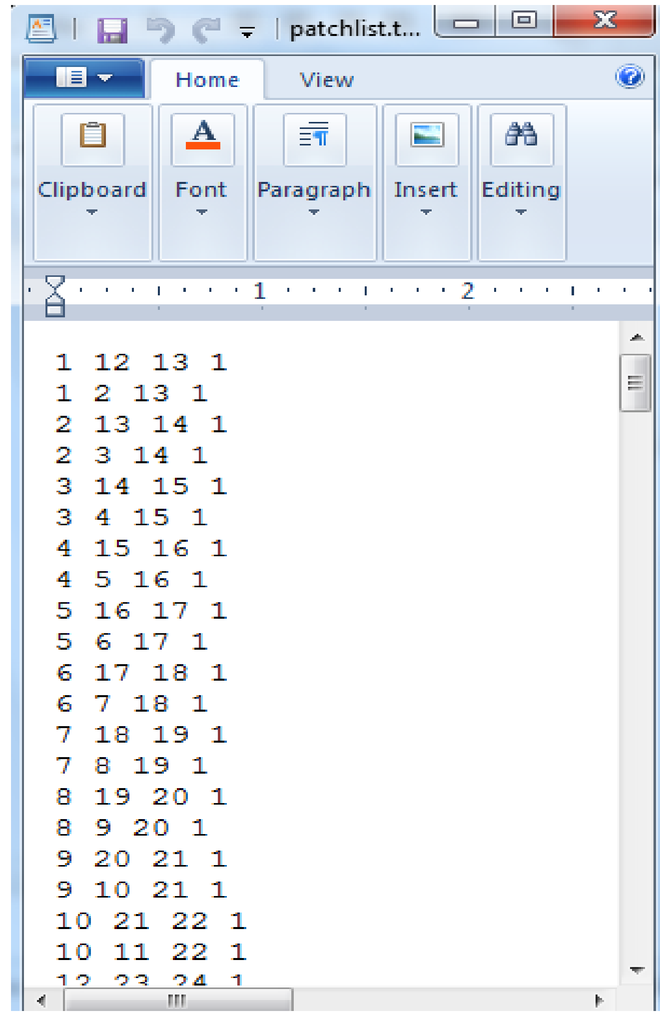


```
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1 2 13 1
2 13 14 1
2 3 14 1
3 14 15 1
3 4 15 1
4 15 16 1
4 5 16 1
5 16 17 1
5 6 17 1
6 17 18 1
6 7 18 1
7 18 19 1
7 8 19 1
8 19 20 1
8 9 20 1
9 20 21 1
9 10 21 1
10 21 22 1
10 11 22 1
```
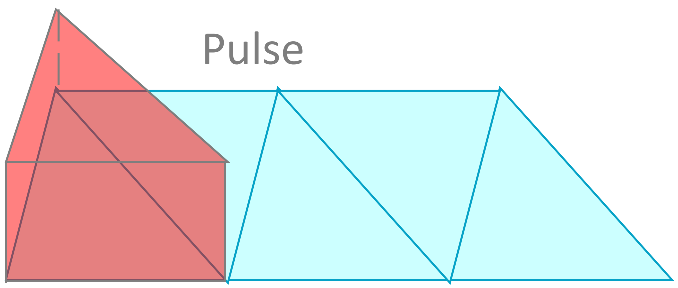

Basis Function

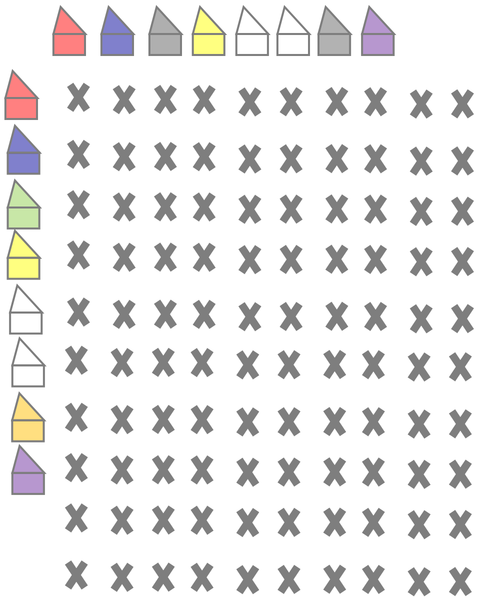
- Piecewise Constant Basis Function

$$f_n(r) = \begin{matrix} 1 & \forall r \in T_n \\ 0 & \forall r \notin T_n \end{matrix}$$



Matrix Equation

- Zx=b
  - Z: Method of Moments matrix
  - x: Charge density
  - b: Voltage or potential



Matrix Entry

$$Z(i,j) = \left\langle f_i, \int G(r,r') f_j(r') ds \right\rangle$$

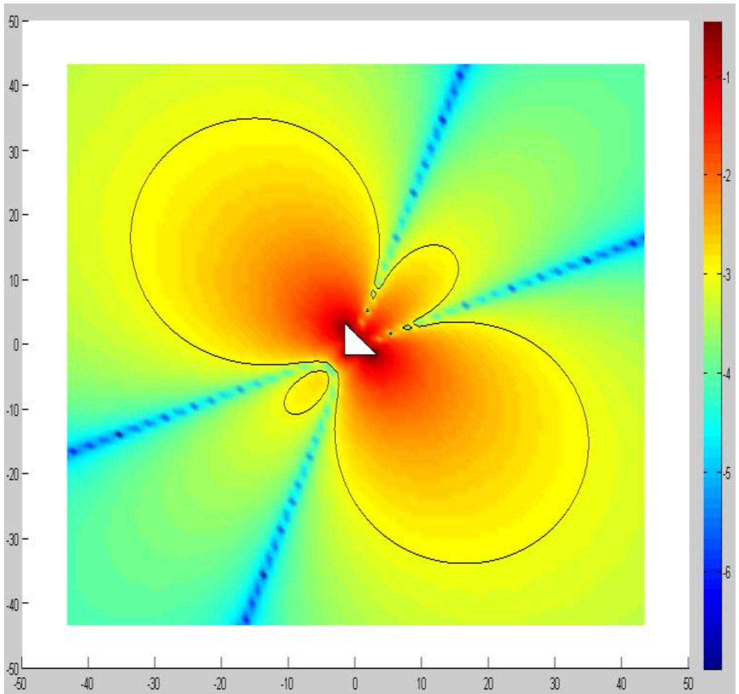
Integration Rules (Also see Appendix)

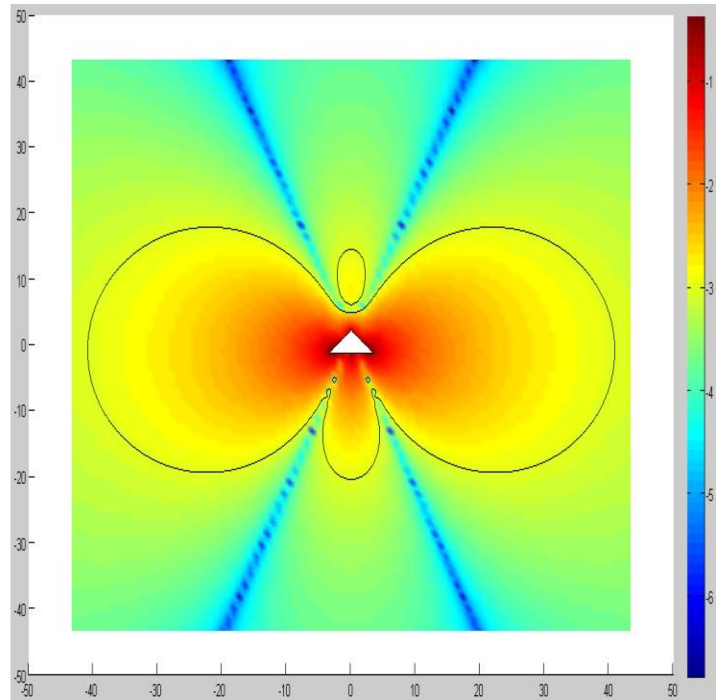
Gaussian Quadrature Integration:

- 1 point integration
- 7 point integration

Analytic Integration:

Integration Rules





1e-3 Error Contour for Different Triangles

Courtesy: Arkaprovo Das

Direct Solver: LU decomposition

for $k = 1$ **to** $n - 1$
    for $i = k + 1$ **to** n
        $\ell_{ik} = a_{ik}/a_{kk}$
    end
    for $j = k + 1$ **to** n
        for $i = k + 1$ **to** n
            $a_{ij} = a_{ij} - \ell_{ik}\, a_{kj}$
        end
    end
end

http://www.cse.uiuc.edu/courses/cs554/notes/06_lu.pdf

Iterative Solver

[GMRES algorithm]

Input: choose x_0, compute $r_0 = b - Ax_0$ and $v_1 = r_0/\|r_0\|$;
Output: solution of linear system $Ax = b$.
Iterate $j = 1, 2, \cdots, k$,
compute $h_{ij} = (Av_j, v_i)$ for $i = 1, 2, \cdots, j$,
$\tilde{v}_{j+1} = Av_j - \sum_{i=1}^{j} h_{ij} v_i$,
$h_{j+1,j} = \|\tilde{v}_{j+1}\|_2$,
$v_{j+1} = \tilde{v}_{j+1}/h_{j+1,j}$.
End;
Form the solution:
$x_k = x_0 + V_k y_k$, where y_k minimizes $J(y)$ in (8).

http://www.math.ntu.edu.tw/~wwang/mtxcomp2010/download/s04e_GMRES.pdf

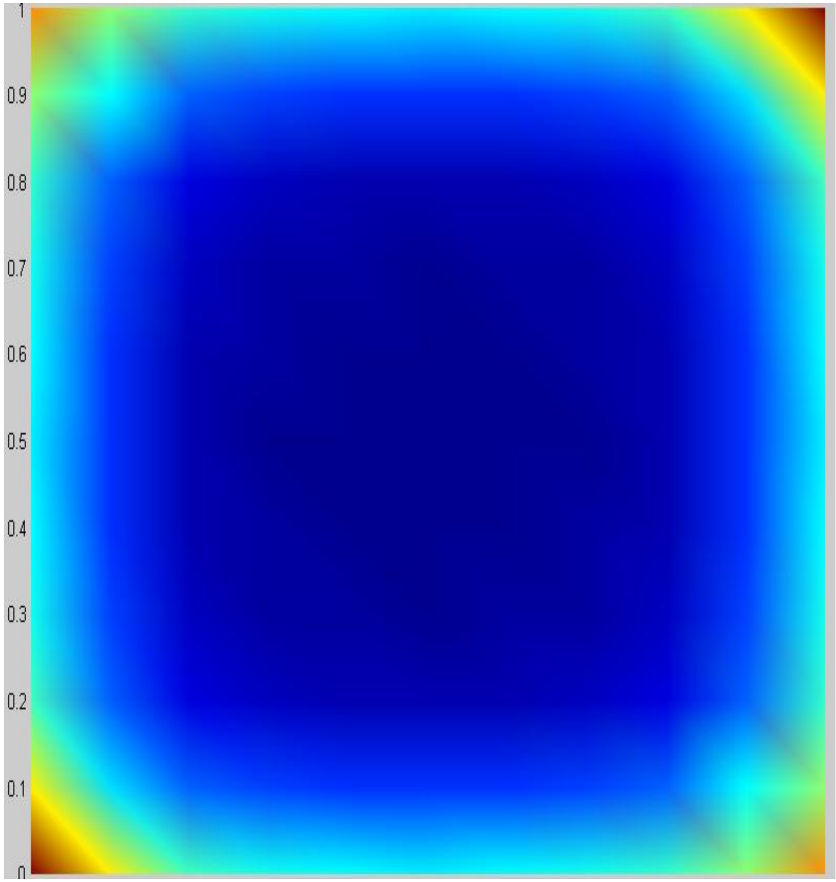
Time and Memory Complexity

Scheme	Setup	Solve
Direct Solver	$O(N^2)$	$O(N^3)+rO(N^2)$
Iterative Solver	$O(N^2)$	$rpO(N^2)$

N= Matrix size = Number of basis = Number of triangles
r = Number of RHS vectors
p = Number of iterations for convergence

Matrix Solution

- Charge Density

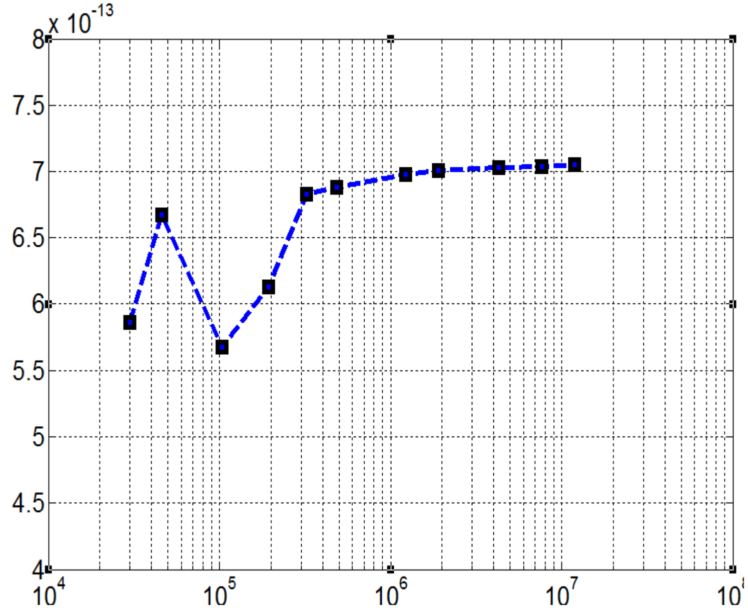


Capacitance Extraction

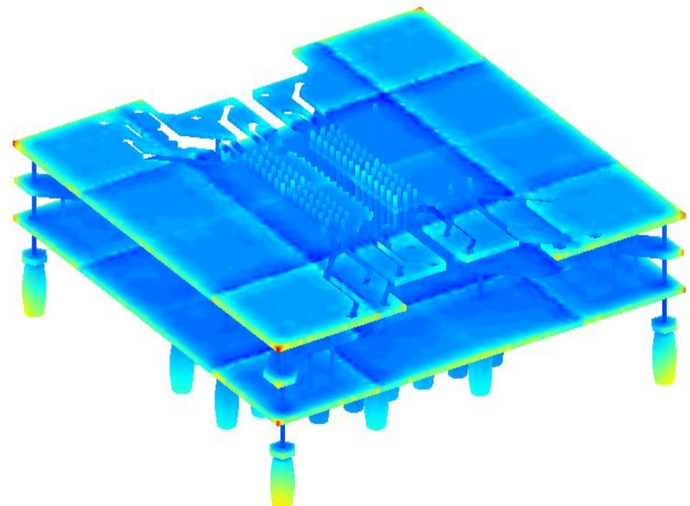
$$\begin{bmatrix} q_1 \\ q_2 \\ \\ q_n \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & & C_{1n} \\ & & & \\ & & & \\ & & & C_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \\ V_n \end{bmatrix}$$

Mesh Convergence

Capacitance Value



Number of mesh elements



Appendix: Integration

- Analytic Integration:

  Potential integrals for uniform and linear source distributions on polygonal and polyhedral domains
  Wilton, D.; Rao, S.; Glisson, A.; Schaubert, D.; Al-Bundak, O.; Butler, C.
  Antennas and Propagation, IEEE Transactions on
  Volume: 32 , Issue: 3 Publication Year: 1984 , Page(s): 276 - 281

- 7 point Gaussian Quadrature Integration:

```
weight=[0.225 0.13239415278851 0.13239415278851 0.13239415278851 0.12593918054483 0.12593918054483 0.12593918054483];
xsi=[0.3333333 0.0597158 0.470142 0.470142 0.7974269 0.1012865 0.1012865];
eta=[0.3333333 0.470142 0.0597158 0.470142 0.1012865 0.7974269 0.1012865];
```