Electrostatic MoM: Part 3
Electric Field Integral Equation (EFIE) MoM

E8-202 Class 6

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Module 2: Method of Moments

- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques

References

- S. Rao, T.K. Sarkar and R.F. Harrington, "The Electrostatic Field of Conducting Bodies in Multiple Dielectric Media", *IEEE Transaction on Microwave Theory and Techniques*, vol. 32, issue 11, Nov 1984 pp. 1441-1448.

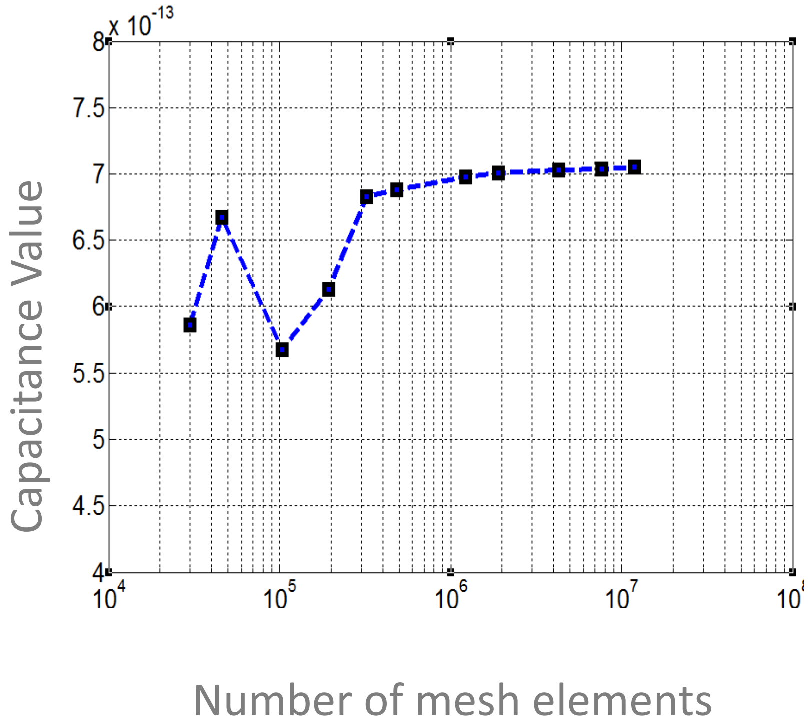
Capacitance Extraction

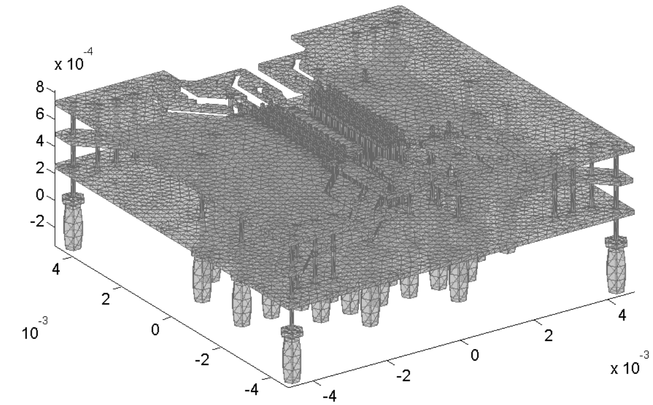
$$\begin{bmatrix} q_1 \\ q_2 \\ \\ q_n \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & & C_{1n} \\ & & & \\ & & & \\ & & & C_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \\ V_n \end{bmatrix}$$

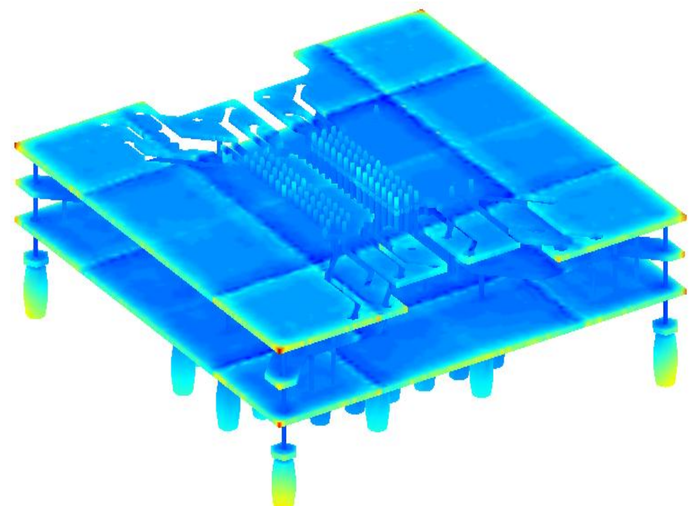
Distinguish between Capacitance matrix and MoM matrix

- How many RHS vectors do we have to solve for?
- What is a better method LU or Gaussian Elimination?

Mesh Convergence







Time and Memory Complexity

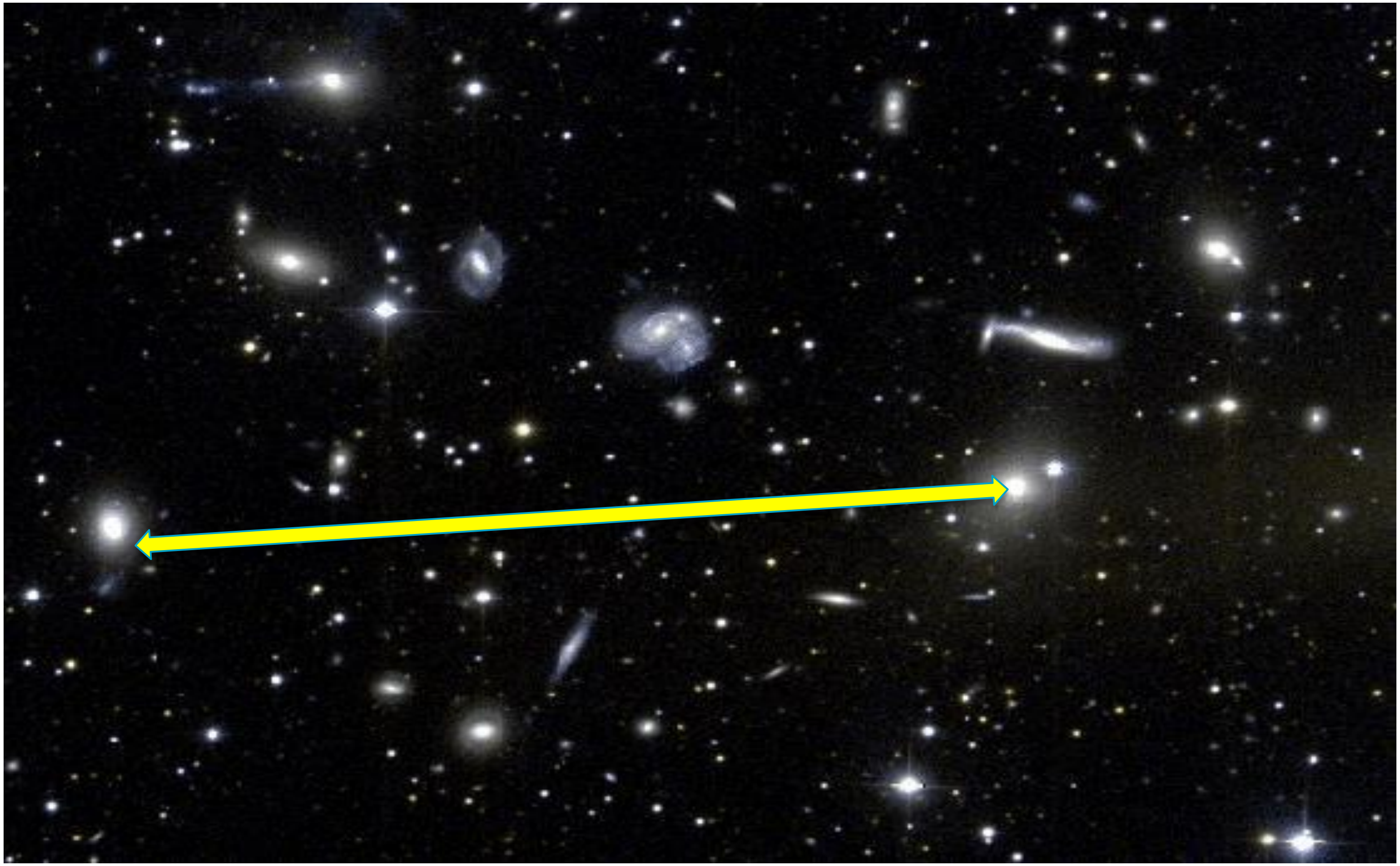
Scheme	Setup	Solve
Direct Solver		
Iterative Solver	?	
Fast Iterative Solver		

N= Matrix size = Number of basis = Number of triangles

r = Number of nets

p = Number of iterations for convergence

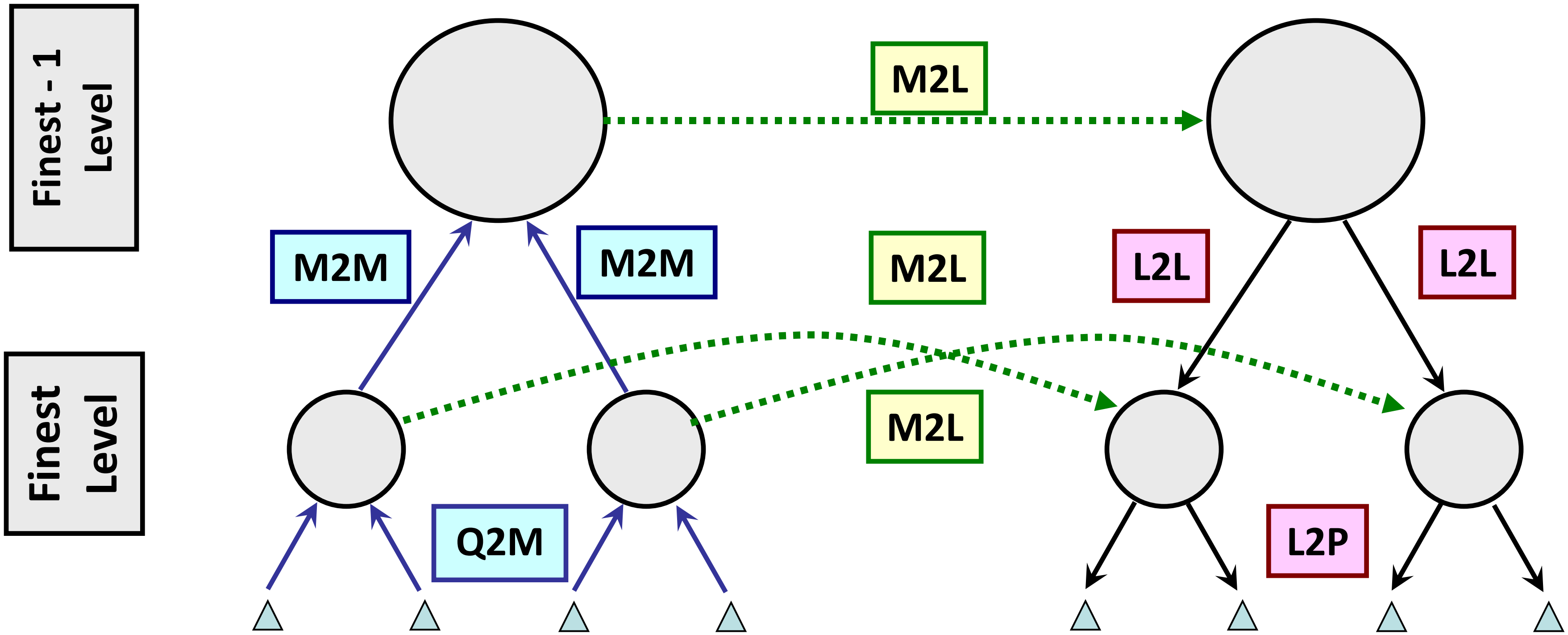
Preview of Fast Solvers



Astronomy Equivalent

FMM Fast Solver Algorithm

Multilevel Fast Multipole Algorithm

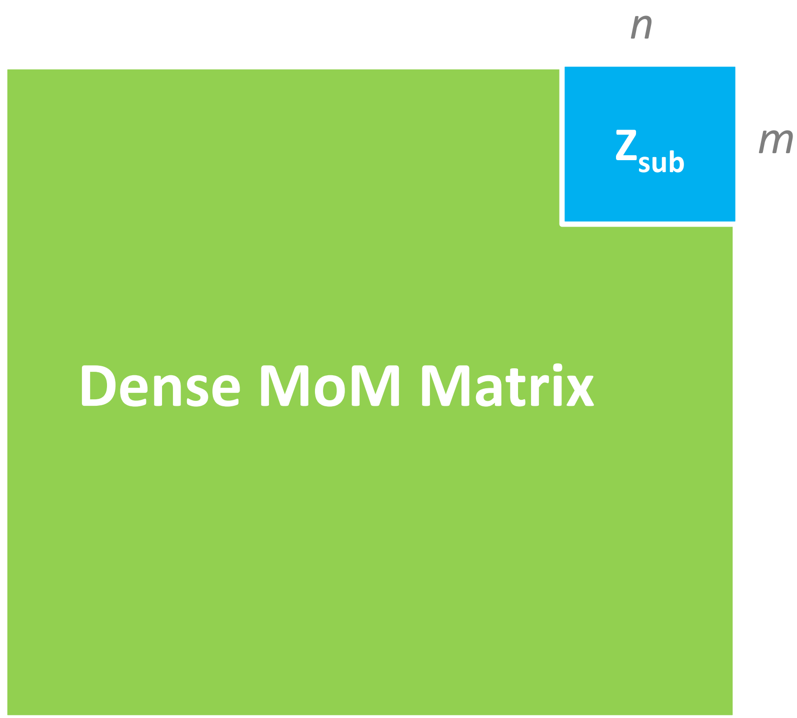


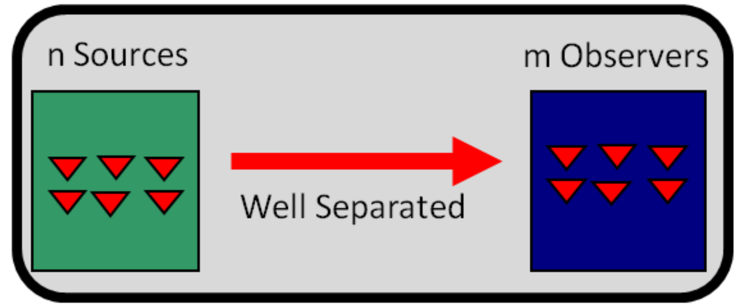
Q – charge
M – multipole
L - local expansion

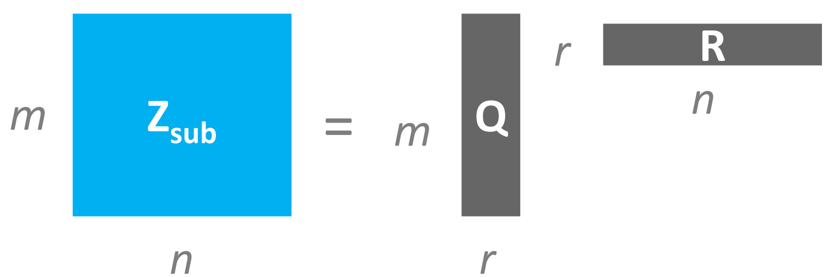
Up Tree Source | **Across Tree** | **Down Tree Observer**

SVD / QR Based Compression

S.Kapur, 1996







SVD/QR Decomposition Saving

$$\frac{m \times n}{(m+n) \times r}$$

Ortho-normalization

Electrostatic MoM: PEC + Dielectric

Boundary Condition

$$n \cdot D_2 - n \cdot D_1 = \rho_s = 0$$

Why?

Equation

$$E = -\nabla \phi$$

$$E^{+,-} = \frac{1}{4\pi\varepsilon_0} \int_{s'} \sigma_T(r') \frac{r - r'}{|r - r'|^3} ds' \pm n \frac{\sigma_T(r)}{2\varepsilon_0}$$

MoM Entries

$$\begin{pmatrix} \bar{\mathbf{Z}}_{CC} & \bar{\mathbf{Z}}_{CD} \\ \bar{\mathbf{Z}}'_{DC} & \bar{\mathbf{Z}}'_{DD} \end{pmatrix} \begin{pmatrix} \boldsymbol{\sigma}_C \\ \boldsymbol{\sigma}_D \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \end{pmatrix}$$

$$\bar{\mathbf{Z}}_{CC/CD}(j,i) = \left\{ \int_{t_j} ds\, h_j(\mathbf{r}) \int_{t_i} ds' g(\mathbf{r},\mathbf{r}') f_i(\mathbf{r}') \right\} \mid t_i \in S_{C/D}; t_j \in S_C$$

$$\bar{\mathbf{Z}}'_{CD/DD}(j,i) = \left\{ \begin{array}{l} \left[-\int_{t_j} ds\, h_j(\mathbf{r}) \int_{t_i} \hat{\mathbf{n}} \cdot ds' \nabla g(\mathbf{r},\mathbf{r}') f_i(\mathbf{r}') \right] \\ + \left[\dfrac{\varepsilon_r^+ + \varepsilon_r^-}{2\varepsilon_0 \left(\varepsilon_r^+ + \varepsilon_r^- \right)} \right] \delta_{ij} \end{array} \right\} \mid t_i \in S_{C/D}; t_j \in S_D$$

Capacitance Extraction

$$\bar{\mathbf{C}}_{ij} = \left\{ \int_{Y_i} \sigma_F(r')dr' \right\}$$

$$\sigma_F = \varepsilon_r \times \sigma_T$$

Electric Field Integral Equation

- Boundary Condition

$$n \times E = 0 \qquad E_{total}\Big|_{tangential} = 0$$

- Equation

$$E_{total} = E_{scattered} + E_{inc}$$

$$\vec{E} = -j\omega\vec{A} - \nabla\phi$$

Which Maxwell's Equation does it come from?

Electric Field Integral Equation

- Green's Function

$$g_\phi(r,r') = \frac{1}{4\pi\varepsilon}\frac{e^{-jk|r-r'|}}{|r-r'|} \qquad g_A(r,r') = \frac{\mu}{4\pi}\frac{e^{-jk|r-r'|}}{|r-r'|}$$

x/y/z direction Cartesian Coordinate

- Mesh