Electric Field Integral Equation (EFIE) MoM

E8-202 Class 7

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Module 2: Method of Moments

- 2D vs 2.5D vs. 3D Formulations
- Electrostatic Formulation: Capacitance matrix extraction
- Magnetostatic Formulation: Inductance matrix extraction
- Electric Field Integral Equation (EFIE): S-parameter extraction
- Partial Element Equivalent Circuit (PEEC) Method
- Magnetic Field Integral Equation (MFIE) and Combined Field Integral Equation (CFIE)
- PMCHWT Formulation: Dielectric modeling
- Parallelization techniques

References

- S. M. Rao, D. R. Wilton and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape”, *IEEE Trans. Antennas Propagation*, vol. AP-30, pp. 409-418, May 1982.
- Walton C. Gibson: The Method of Moments in Electromagnetics, 1st Ed., Chapman and Hall, Chapter 3
- Roger F. Harrington: Field Computation by Moment Methods, 1993, Wiley-IEEE Press, Chapter 7

Equation and Boundary Condition

- Boundary Condition

$$n \times E = 0 \qquad E_{total}\Big|_{tangential} = 0$$

- Equation

$$E_{total} = E_{scattered} + E_{inc}$$

$$\vec{E} = -j\omega\vec{A} - \nabla\phi \qquad \nabla_s \cdot \mathbf{J}(\mathbf{r}) + j\omega\rho(\mathbf{r}) = 0 \ , \forall \mathbf{r} \in S_{EM}$$

Which Maxwell's Equation does it come from?

Green's Function and Potentials

- Green's Function

$$g_{\phi}(r,r') = \frac{1}{4\pi\varepsilon}\frac{e^{-jk|r-r'|}}{|r-r'|} \qquad g_{A}(r,r') = \frac{\mu}{4\pi}\frac{e^{-jk|r-r'|}}{|r-r'|}$$

x/y/z direction Cartesian Coordinate

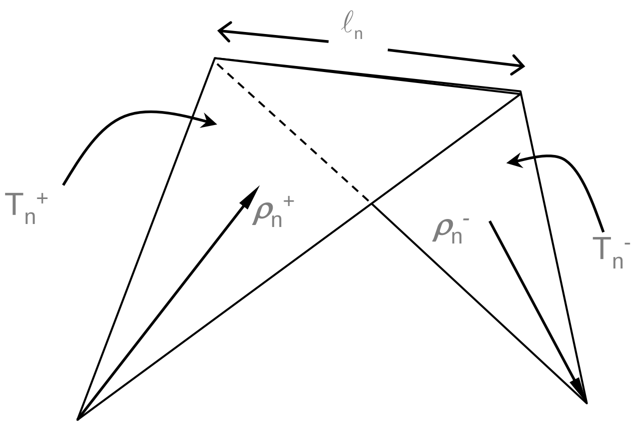
- Potentials

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\int_S \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}ds'$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int_S \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}ds'$$

Rao-Wilton-Glisson Basis Function

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \dfrac{l_n}{2A_{n+}}\boldsymbol{\rho}_{n+} & \mathbf{r} \in T_{n+} \\ \dfrac{l_n}{2A_{n-}}\boldsymbol{\rho}_{n-} & \mathbf{r} \in T_{n-} \end{cases}$$



S. M. Rao, D. R. Wilton and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape", *IEEE Trans. Antennas Propagation*, vol. AP-30, pp. 409-418, May 1982.

Properties of RWG Basis

- Normal component to the boundary = 0
- Normal component to the edge = 1
- Divergence is given by: $$\nabla \cdot \mathbf{f}_n(\mathbf{r}) = \begin{cases} \dfrac{l_n}{A_{n+}} & \mathbf{r} \in T_{n+} \\ -\dfrac{l_n}{A_{n-}} & \mathbf{r} \in T_{n-} \end{cases}$$

MoM

$$\bar{\mathbf{Z}}_{N_e \times N_e} \mathbf{x}_{N_e \times 1} = \mathbf{b}_{N_e \times 1}$$

$$\bar{\mathbf{Z}}_{ij} = \frac{j\omega\mu}{4\pi} \int_{T_i^{+,-}} \int_{T_j^{+,-}} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|} \mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \cdot \mathbf{f}_i(\mathbf{r})\, ds$$

$$+ \frac{1}{4j\omega\pi\varepsilon} \int_{T_i^{+,-}} \int_{T_j^{+,-}} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|} \nabla' \cdot \mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \, \nabla \cdot \mathbf{f}_i(r)\, ds$$

$$\mathbf{b}_i = \int_{T_i^{+,-}} f_i \cdot \boldsymbol{E}^i ds$$

MoM: Scalar and Vector Parts

$$\overline{\mathbf{Z}}^{sub}_{N_e^o \times N_e^s} = \overline{\mathbf{L}}^{++}_{N_e^o \times N_e^s} + \overline{\mathbf{L}}^{--}_{N_e^o \times N_e^s} + \overline{\mathbf{L}}^{+-}_{N_e^o \times N_e^s} + \overline{\mathbf{L}}^{-+}_{N_e^o \times N_e^s}$$
$$+ \overline{\mathbf{P}}^{++}_{N_e^o \times N_e^s} + \overline{\mathbf{P}}^{--}_{N_e^o \times N_e^s} + \overline{\mathbf{P}}^{+-}_{N_e^o \times N_e^s} + \overline{\mathbf{P}}^{-+}_{N_e^o \times N_e^s}$$

$$\overline{\mathbf{L}}^{+-}_{ij} = \frac{j\omega\mu}{4\pi} \int_{T_i^+} \int_{T_j^-} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\mathbf{f}_j^{T_j^-}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \cdot \mathbf{f}_i^{T_i^+}(\mathbf{r})\, ds$$

$$\overline{\mathbf{P}}^{+-}_{ij} = \frac{1}{4j\omega\pi\varepsilon} \int_{T_i^+} \int_{T_j^-} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\nabla' \cdot \mathbf{f}_j^{T_j^-}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, ds'\ \nabla \cdot \mathbf{f}_i^{T_i^+}(r)\, ds$$

$$\overline{\mathbf{Z}}^{sub}_{N_e^o \times N_e^s} = \overline{\mathbf{L}}_{N_e^o \times N_e^s} + \overline{\mathbf{B}}_{O_{N_e^o \times N_p^o}} \overline{\mathbf{P}}_{N_p^o \times N_p^s} \overline{\mathbf{B}}^T_{S_{N_p^S \times N_e^S}}$$

Integrations: Singularity Extraction

$$\frac{e^{-jkr}}{r} = \frac{e^{-jkr} - 1}{r} + \frac{1}{r}$$

Singular Non-Singular Singular

$$\frac{\rho e^{-jkr}}{r} = \frac{\rho\left(e^{-jkr} - 1\right)}{r} + \frac{\rho}{r}$$

Singular Non-Singular Singular

Data Structures

- N2XYZ
- P2N
- E2P
- E2N
- P2E