E2-202 Class 10 Dipanjan Gope

Dielectric formulation : Approx qs formulation.

$\nabla \cdot J = -j\omega\rho$ $\qquad$ $\nabla \cdot D = \rho$

free charge

$\phi = \int G \rho \, ds$

total charge

$$\text{free charge} = \frac{\epsilon(r)}{\epsilon_0} \text{ total charge}$$

eq^n 1 : $\langle j\omega A, f_t \rangle + \langle \nabla\phi, f_t \rangle = \langle E_i, f_t \rangle$

eq^n 2 : $\dfrac{(\epsilon^+ + \epsilon^-)}{2\epsilon_0(\epsilon^+ - \epsilon^-)} \sigma_T(r) + \dfrac{1}{4\pi\epsilon_0} \int \sigma_T(r') \left[\dfrac{(r - r')}{|r - r'|^3} \right] \cdot \hat{n} \, ds = 0$

How does Z_{cc} change?

Volumetric formulation :-

$$E^i = \frac{J}{j\omega(\epsilon_r - \epsilon_0)} + j\omega A + \nabla\phi$$

Term 1 :-

$$\left\langle \frac{J}{j\omega(\epsilon_r - \epsilon_0)}, f_t \right\rangle$$

$$= \left\langle \frac{f_s}{j\omega \epsilon_r \epsilon_0}, f_t \right\rangle$$

$$= \frac{1}{j\omega \epsilon_r \epsilon_0} \int_S f_s \cdot f_t \, ds$$

Term 2 :-

$$\langle j\omega A, f_t \rangle$$

$$= j\omega \int_T f_t \cdot \int_S G J \, ds \, dt$$

$$= j\omega \int f_t \cdot \int G \, j\omega \gamma D \, dt$$

$$= j\omega \int_T f_t \cdot \int_S G \, j\omega \gamma D \, ds \, dt$$

$$= j\omega\gamma \int_T f_t \cdot \int_S G f_s \, ds \, dt$$

$$= \frac{j\omega\gamma\mu}{4\pi} \int f_t \cdot \int \frac{e^{-jkr}}{r} f_s \, ds \, dt$$

$$= \frac{j\omega\gamma\mu A_t^+ A_s^+}{4\pi \, 3\times 3 V_t^+ V_s^+} \int_T f_t^+ \cdot \int_S \frac{e^{-jkr}}{r} f_s^+ \, ds \, dt$$

Term 3 :-

$$\langle \nabla\phi, f_t \rangle$$

$$\Rightarrow \int \phi f_m \cdot \hat{n} \, dt - \int \phi \, \nabla \cdot f_m \, dt = \int f \cdot \nabla\phi \, dt$$

Term 3a :-

$$-\int \phi \, \nabla \cdot f_m \, dt$$

$$T_{3a} = -\int \frac{A^+}{V^+} \int G \rho \, ds \, dt \, .$$

$$\rho = \frac{\nabla \cdot J}{-j\omega} = \frac{\nabla \cdot j\omega \gamma D}{-j\omega} = \frac{\nabla \cdot \gamma f_s}{-j\omega}$$

$$= \frac{\gamma \nabla \cdot f_s}{-j\omega} + \underbrace{\frac{f_s \cdot \nabla \gamma}{-j\omega}}_{\text{when is this } 0} \, .$$

~~Term 3a~~

$$T_{3a} = -\int \frac{A_t^+}{V_t^+} \int G \rho \, ds \, dt \, .$$

$$= + \frac{A_t^+ A_s^+}{V_t^+ V_s^+} \frac{\gamma^+}{j\omega 4\pi\epsilon_0} \int\int \frac{e^{-jkr}}{r} \, ds \, dt$$

$$+ \frac{A_t^+}{V_t^+} \frac{1}{j\omega 4\pi\epsilon_0} (\gamma^+ - \gamma^-) \int\int \frac{e^{-jkr}}{r} \, ds \, dt$$