E8-202 Lecture 14

Dipayan Gope

$$\nabla \times E = -j\omega B$$

$$\nabla \times H = j\omega D + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$E = -j\omega A - \nabla\phi$$

+ Lorentz gauge

$$\nabla \cdot A = -j\omega\mu\epsilon$$

$$\text{①} \quad H = \frac{1}{\mu} \nabla \times A$$

$$\text{②} \quad (\nabla^2 + k^2) A = -\mu J$$

Boundary condn

$$n \times H = J_s$$

$$\Rightarrow n \times H^i + n \times H^s = J_s$$

$$\Rightarrow n \times H^i = J_s - \hat{n} \times H^s$$

$$= J_s - \hat{n} \times \frac{1}{\mu} \nabla \times A$$

$$= J_s - \hat{n} \times \dot{\nabla} \times (GJ$$

$$\boxed{\nabla \times sV = \nabla s \times V + s \nabla \times V}$$

$$\nabla \times GJ = \nabla G \times J + G \nabla \times J$$

$$\underbrace{\nabla}_{\text{obs coordinate}} \times J(\underbrace{r'}_{\text{src coordinate}}) = 0$$

$$\Rightarrow \nabla \times GJ = \nabla G \times J$$

$$= -J \times \nabla G$$

$$= -J \times (-\nabla' G) \qquad (\because \nabla G = -\nabla' G)$$

$$\Rightarrow \boxed{\frac{J_s}{2} = \hat{n} \times H_{inc} + \hat{n} \times \int J(r') \times \nabla' G(r, r')\, ds}$$

$$\nabla' G = \left(jk + \frac{1}{R}\right) \frac{e^{-jkR}}{R} \hat{R}$$

$$H_i = \frac{J}{2} - \int J \times \left(jk + \frac{1}{R}\right) e^{-jkR} \hat{R} \, ds$$

RHS $\int_T H_i \cdot f_t \, dt$

Term 1 $\int \frac{J}{2} \cdot f_t \, dt$

$\Rightarrow \int \frac{f_s}{2} \cdot f_t \, dt$

Term 2

$$\int \frac{f_t l}{2A} \cdot \int \frac{f_s l}{2A} \times \left(jk + \frac{1}{R}\right) \frac{e^{-jkR}}{R} \hat{R} \, ds$$

$$\int \left(jk + \frac{1}{R}\right) \frac{e^{-jkR}}{R} f_s \times \hat{R} \, ds$$

$$f_s \times \hat{R} = f_s \times \frac{\vec{R}_1}{R}$$

$$I = \int f_i \frac{(1 + jkR)}{R^3} e^{-jkR} \, ds$$

$$= \int f_i \frac{\left[(1 + jkR) e^{-jkR} - \left(1 + \frac{1}{2} k^2 R^2\right)\right]}{R^3} ds$$

$$+ \frac{f_i \left(1 + \frac{1}{2} k^2 R^2\right)}{R^3} ds$$

Combined Field

$$\alpha \, \mathrm{EFIE} + (1-\alpha) \frac{j}{k} \, \mathrm{MFIE}$$