# E8-202 Lecture 15

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$$\nabla \times E = -j\omega B - M$$

$$\nabla \times H = j\omega D + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = \rho_m$$

$$E = -j\omega A - \nabla \phi - \frac{1}{\epsilon} \nabla \times F$$

$$H = -j\omega F - \nabla \phi_m + \frac{1}{\mu} \nabla \times A$$

MoM matrix entry:-

$$\cancel{\langle E, f_k \rangle} = \cancel{\langle -j\omega A, f_k \rangle} + \cancel{\langle -\nabla\phi, f_k \rangle}$$

$$n \times E^i = n \times \left(j\omega A_1 - j\omega A_2\right) + \hat{n} \times \left(\nabla \phi_1 - \nabla \phi_2\right) + \hat{n} \times \left[\frac{1}{\epsilon_1} \nabla \times F_1 - \frac{1}{\epsilon_2} \nabla \times F_2\right]$$

$$0 = n \times \left(j\omega F_1 - j\omega F_2\right) + n \times \left(\nabla \phi_1 - \nabla \phi_2\right) + n \times \left[\frac{1}{\mu_1} \nabla \times A_1 - \frac{1}{\mu_2} \nabla \times A_2\right]$$

$$E: \quad T_1 + T_2 + T_3$$

$$T_1 = \frac{j\omega\mu_1}{4\pi}\int f_t \cdot \int G_1 \overset{J}{f_s}\, ds\, dt - \frac{j\omega\mu_2}{4\pi}\int f_t \cdot \int G_2 \overset{J}{f_s}\, ds\, dt$$

$$T_2 = \frac{1}{j\omega 4\pi\epsilon_1}\int\int G_1 \overset{J}{ds}\, dt - \frac{1}{j\omega 4\pi\epsilon_2}\int\int G_2 \overset{J}{ds}\, dt$$

$$T_3 = \int f_t \cdot \int \nabla \times G_1 \overset{M}{f_s}\, ds\, dt - \int f_t \cdot \int \nabla \times G_2 \overset{M}{f_s}\, ds\, dt$$

$$H: \quad S_1 + S_2 + S_3$$

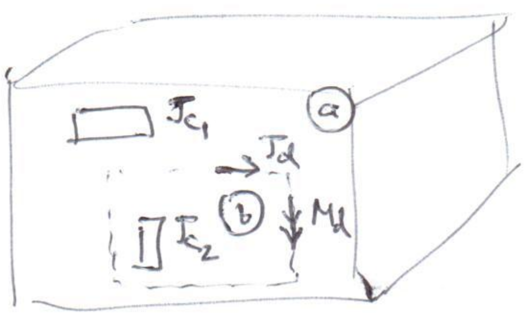
$$S_1 = \frac{j\omega\epsilon_1}{4\pi}\int f_t \cdot \int G_1 \overset{M}{f_s}\, ds\, dt - \frac{j\omega\epsilon_2}{4\pi}\int f_t \cdot \int G \overset{M}{f_s}\, ds\, dt$$

$$S_2 = \frac{1}{j\omega 4\pi\mu_1}\int\int G_1 \overset{M}{ds}\, dt - \frac{1}{j\omega 4\pi\mu_2}\int\int G_2 \overset{M}{ds}\, dt$$

$$S_3 = \int f_t \cdot \int \nabla \times G_1 \overset{J}{f_s}\, ds\, dt - \int f_t \cdot \int \nabla \times G_2 \overset{J}{f_s}\, ds\, dt$$

$$
\begin{bmatrix}
T_{1a}+T_{2a} & 0 & T_{1a}+T_{2a} & T_{3a} \\
0 & T_{1b}+T_{2b} & T_{1b}+T_{2b} & T_{3b} \\
T_{1a}+T_{2a} & T_{1b}+T_{2b} & T_{1a}+T_{2a}+T_{1b}+T_{2b} & T_{3a}+T_{3b} \\
S_{3a} & S_{3b} & S_{3a}+S_{3b} & S_{1a}+S_{2a}+S_{1b}+S_{2b}
\end{bmatrix}
\begin{bmatrix} J_{c_1} \\ J_{c_2} \\ J_d \\ M_d \end{bmatrix}
=
\begin{bmatrix} \langle E_i \cdot f_t \rangle \\ \langle E_i \cdot f_t \rangle \\ \langle E_i \cdot f_t \rangle \\ 0 \end{bmatrix}
$$

MoM matrix for :-




$$T_{3a} : \int f_t \cdot \int \nabla \times G \, f_s \, ds \, dt .$$

$$\boxed{\nabla \times sV = \nabla s \times V + s \nabla \times V}$$

$$\nabla \times GM = \underbrace{\nabla G \times M}_{-\nabla' G \times M \, = \, M \times \nabla' G} + \underbrace{G \nabla \times M}_{0}$$

$$\nabla' G = \left(jk + \frac{1}{R}\right)\frac{e^{-jkR}}{R}\hat{R}.$$

$$T_{3a} = \int f_t \cdot \int f_s \times \left(jk + \frac{1}{R}\right)\frac{e^{-jkR}}{R}\hat{R}\, ds\, dt$$

$$f_s \times \hat{R} = f_s \times \hat{R}_i \qquad \left[ f_s = \frac{l_s \rho}{2A} \right]$$

$$T_{3a} = \int f_t \cdot \vec{R}_i \times \int f_s \frac{(1 + jkR)}{R^3} e^{-jkR}\, ds\, dt.$$

$$I = \int f_s \frac{(1 + jkR)}{R^3} e^{-jkR}\, ds$$

$$= \int f_s \frac{\left[(1 + jkR)e^{-jkR}\right] - \left(1 + \frac{1}{2}k^2R^2\right)}{R^3}\, ds$$

$$+ \int \frac{f_s \left(1 + \frac{1}{2}k^2R^2\right)}{R^3}\, ds.$$